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PROCEEDINGS

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Design of an Internal Combustion Engine for Shell Eco-marathon Competition Vehicle

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Abstract: The article describes the design of a custom internal combustion engine (ICE) for a Prototype-class vehicle participating in the Shell Eco-marathon. It is a follow-up to the article “Development of an Internal Combustion Engine for Shell Eco-marathon Competition Vehicle” published in 2025. The IC engine described in this article was designed using engine parameters determined through experimental testing and research based on various educational and technical sources. Critical engine components such as the crankshaft and connecting rod were validated through finite element method (FEM) strength analyses. The engine is somewhat based on the Honda GX35 that was previously used in the vehicle. The resulting engine is lighter, more compact, and better suited to the vehicle than the previous solution, providing a solid foundation for further development and testing.

Keywords: INTERNAL COMBUSTION ENGINE, ENGINE DESIGN, FUEL EFFICIENCY, SHELL ECO-MARATHON, HONDA GX35,

1. Introduction

The objective of the project is to develop a custom internal combustion engine for a vehicle competing in the Shell Eco-marathon competition series. The engine must comply with all applicable competition regulations while improving the overall energy efficiency of the vehicle. Building upon the Honda GX35 currently in use, the new engine is intended to provide a solution that is better suited to the vehicle's design objectives and operating requirements.

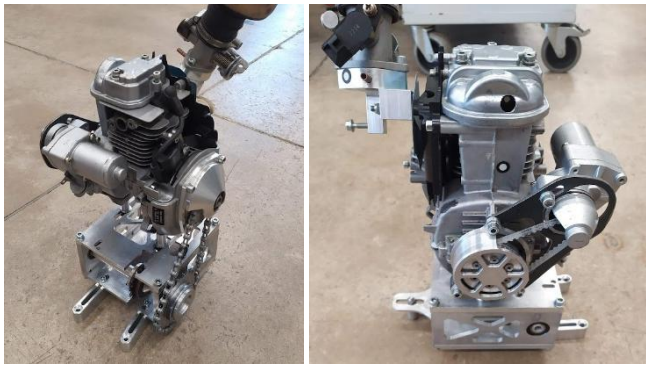


Fig.1 Honda GX35, the previous engine of the vehicle.

The previous stage of this project focused on the collection and analysis of the data required for the development of a custom internal combustion engine intended for use in a Shell Eco-marathon Prototype class vehicle. A dedicated test methodology was developed to evaluate the performance and fuel efficiency of small displacement internal combustion engines, and extensive testing was carried out on a Honda GX35 engine, which had previously been used to power the vehicle. Based on the results of these tests, together with information gathered from literature and reports published by competing teams, the principal design parameters – including cylinder dimensions, compression ratio and connecting rod geometry - of the new engine were determined.

The selected parameters indicated that a purpose-built engine could offer significant advantages over the modified GX35. While the previous engine provided a reliable and practical platform for vehicle development, its design and operating characteristics were originally intended for general-purpose industrial applications rather than ultra-high-efficiency vehicle propulsion. Consequently, several compromises had to be accepted in terms of compactness, transmission design, operating speed range, and overall fuel economy.

The final parameters selected for the new engine design are summarised in Table 1. The equivalent parameters of the Honda GX35 are shown alongside to highlight the differences between the two engines. The most notable differences are the connecting rod length (increased from 51 mm to 150 mm) and the compression ratio (increased from 8:1 to 12:1).

Table 1: Main parameters of the GX35 and the new engine. [1]

Parameter	Honda GX35	Custom Engine
Cylinder bore (mm)	39	39
Stroke (mm)	30	39
Displacement (cm ³)	35,84	46,59
Bore to stroke ratio	1,3	1
Compression ratio	8:1	12:1
Combustion chamber volume (cm ³)	5,12	4,24
Connecting rod length (mm)	51	150
Rod to stroke ratio	1,7	3,85

The objective of the present stage of the project is to transform the previously defined design parameters into a complete and manufacturable engine design. The new engine is intended to meet all Shell Eco-marathon technical regulations while achieving higher efficiency. Emphasis is placed on reducing mechanical losses and designing the engine around the specific operating conditions encountered during competition. In addition, one of the most important design requirements is the compactness of the engine, which would allow the vehicle to use a single-stage transmission.

This article describes the engineering design process of the new engine and the development of its main assemblies. The design methodology, component selection process, and manufacturing considerations are presented and discussed. Special attention is given to the crankshaft assembly, crankcase design, cylinder head configuration and the valvetrain system. The resulting design represents the next step toward the construction and experimental validation of a dedicated high-efficiency power unit for the TTK UAS Shell Eco-marathon vehicle.

2. Preparations for Design

The type of the newly designed engine remained the same as that of the Honda GX35. Consequently, the new engine is also a four-stroke single-cylinder air-cooled overhead-camshaft gasoline engine. Changing the engine type was not considered reasonable, as all previous development work had focused exclusively on this type of engine.

The design of the manufactured components was guided primarily by the principle that they should be as simple as possible to produce and be manufacturable almost entirely within the machining laboratory of TTK University of Applied Sciences. Several components used in the new engine were selected from the existing GX35 because manufacturing some of these parts with the available equipment would be unreasonably complex or time-consuming.

One of the primary reasons for designing a new engine was the unsuitable dimensions and overall shape of the GX35, which prevented it from fitting within the vehicle body. To ensure that the new engine would satisfy all design requirements, an accurate model of the vehicle's engine compartment had to be created before the design process could begin.

For this purpose, a 3D scanner was used to scan the rear end of the vehicle. Based on the resulting scan data, a precise CAD model of the engine bay was made.

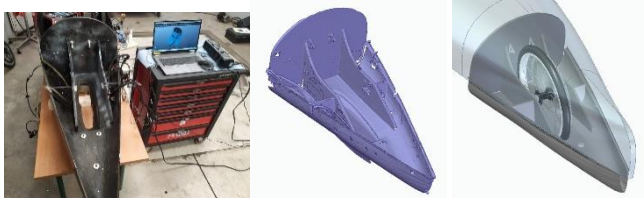


Fig. 2. Engine compartment, scanned surface, and completed CAD model.

The most important dimensions obtained during the scanning process were the position and geometry of the rear wheel mounting points, the locations of the three engine mounting holes, and the shape of the engine compartment walls. The rear wheel mounting geometry was used to determine the location and principal dimensions of the transmission system being designed simultaneously with the engine. This, in turn, fixed the engine position and the direction of rotation of the output shaft. The walls of the engine compartment, the upper body cover, and the engine mounting points defined the maximum allowable engine dimensions and strongly influenced both the engine layout and the placement of its components.

3. Crankshaft Assembly

The crankshaft was the first assembly designed for the new engine. The primary design objective was minimizing its axial length to enable the use of a single-stage chain transmission. Furthermore, the crankshaft largely determined the placement of the remaining engine components and the overall shape of the engine.

The first step was defining the crankshaft dimensions. To determine the maximum allowable crankshaft length, the positions of the new transmission components were fixed within the engine compartment, and the sprocket mounted on the engine output shaft was aligned with the sprocket attached to the vehicle's rear wheel. It was concluded that a single-stage chain transmission was feasible but required a significantly shorter crankshaft than that of the GX35.

The designed crankshaft consists of two identical crank webs connected by a crank pin. Using identical crankshaft halves considerably simplifies the design, balancing procedure, and manufacturing process. To reduce complexity and development time, the crank pin from the GX35 was reused. Its dimensions consequently determined the spacing between the crankshaft halves and the thickness of the counterweights.

The distance between the crank pin and the main journal, i.e. the crank radius, was determined by the previously selected piston stroke. Since the piston stroke equals twice the crank radius, and the selected stroke length was 39 mm, the crank radius is 19.5 mm.

The dimensions of the main journals were determined by the selected bearings and the external components mounted on the shaft. As with the GX35, 6202-RSH deep-groove ball bearings were selected. [2]. To make the design more compact and reduce frictional losses, no additional radial shaft seals are used. Crankcase sealing is therefore provided solely by the integrated seals of the bearings.

The general geometry of the crankshaft counterweights was based on the GX35 crank webs. Their final dimensions were determined during the balancing process. A solid-web crankshaft design was also considered; however, because the engine uses splash lubrication, a cut-out counterweight design was ultimately selected.

The piston, piston rings, and piston pin are reused from the GX35, resulting in a piston diameter of 39 mm. A centrifugal clutch

adapter flange is mounted on the output side of the crankshaft, while a flywheel is attached to the opposite side.

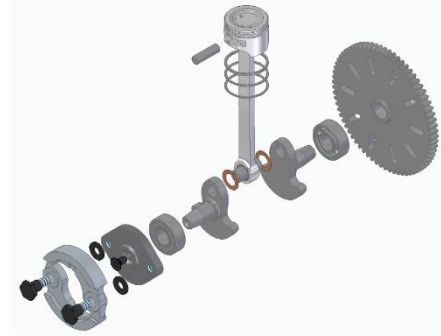


Fig. 3. Crankshaft assembly.

To reduce engine vibration and increase component service life, proper crankshaft balancing is essential. Due to the inherently asymmetric configuration of single-cylinder engines, perfect balance cannot be achieved. While rotating masses can be fully balanced, reciprocating masses can only be compensated partially. Nevertheless, vibration levels can be reduced to an acceptable level. The crankshaft balancing procedure was performed according to following Equation 1 [3]:

$$m_{rot}\omega^2 R = m_b\omega^2 R_b, \quad (1)$$

where m_{rot} – rotating mass, consisting of the crank pin mass and two-thirds of the connecting rod mass;
 ω – crankshaft angular velocity;
 R – crankshaft radius;
 m_b – counterweight mass;
 R_b – distance from the crankshaft centerline to the center of gravity of the counterweight.

According to the equation, the objective was to position the combined center of gravity of the crankshaft and rotating masses directly on the crankshaft axis. This was most conveniently achieved in CAD software, where the center of gravity of a component or assembly can be displayed at any time. A crank pin representing the equivalent rotating mass was added to the crankshaft assembly model. The shape of the counterweights was then gradually modified until the center of gravity coincided with the crankshaft centerline.

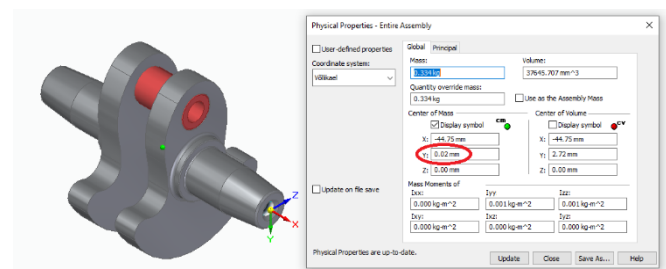


Fig. 4. Crankshaft balancing in CAD software.

Final balancing will be performed during engine assembly once the actual masses of all manufactured components are known.

4. Cylinder Head and Valvetrain Assembly

In the Honda GX35, the engine housing consists of two main components: an upper and a lower half. Although this design significantly simplifies both the construction and mass production of the engine, it completely prevents modifications and improvements to the combustion chamber and intake/exhaust ports. For this reason, the upper portion of the combustion chamber in the new engine was designed as a separate cylinder head rather than being integrated into the engine housing.

The dimensions of the cylinder head were primarily determined by the components mounted to it, as well as the size and orientation of the intake and exhaust ports. The angle between the ports and the valves was selected to be as small as possible in order to minimize air flow resistance. In the GX35, the intake and exhaust ports are arranged approximately perpendicular to the combustion chamber (Fig. 5), forcing the airflow to undergo a 90-degree change in direction as it enters and exits the cylinder. From a fluid-flow perspective, this arrangement is unfavorable. The redesigned cylinder head allowed the port angle to be reduced to approximately 50 degrees, which is expected to improve airflow characteristics and increase volumetric efficiency.

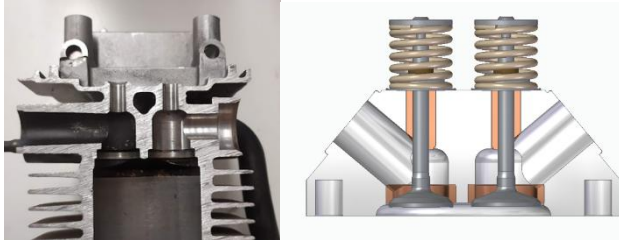


Fig. 5. Cross-section of the new cylinder head (right) compared to the Honda GX35 combustion chamber (left).

The intake manifold of the new engine consists of two aluminum tubes joined at a common flange. One tube serves as the air passage while the other accommodates the fuel injector. Unlike conventional internal combustion engines, the designed engine does not incorporate a driver-operated throttle valve. This is a characteristic feature of many Shell Eco-marathon powertrains and is intended to minimize the influence of the driver on fuel consumption.

During competition, the driver's task is limited to starting the engine and steering the vehicle. The engine is only permitted to start when the vehicle is stationary or when its speed falls below a predetermined threshold. Once the target speed has been reached, the engine control unit shuts the engine off and allows the vehicle to coast until the speed decreases sufficiently for another start cycle. As a result, the engine operates in short, cyclic intervals and therefore does not require conventional throttle control.

When designing the combustion chamber, the primary objective was achieving the previously specified compression ratio of 12:1, corresponding to a combustion chamber volume of 4.24 cm³. The chamber volume was verified within the CAD environment by creating a solid body representing the combustion chamber and progressively modifying its geometry until the target volume was obtained. Particular attention was paid to avoiding sharp edges and abrupt transitions within the chamber. Smooth surfaces and gradual radii were employed wherever possible to promote favorable combustion characteristics and reduce the likelihood of localized hot spots. The spark plug is installed at an angle of 30 degrees relative to the cylinder axis.

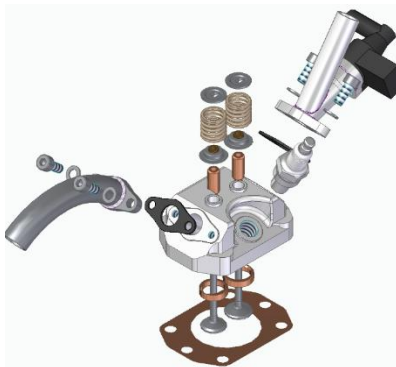


Fig. 6. Cylinder head assembly.

The cylinder head gasket is manufactured from 0.25 mm thick CuZn37 brass sheet. Material selection was primarily based on

machinability and availability. The final gasket geometry, including all openings and mounting holes, is produced by laser cutting.

Since the intake and exhaust valves are reused from the GX35, the dimensions of the valve guides and valve seats were designed to match the original components. The valve guides are also initially taken from a GX35, while the valve seats are manufactured specifically for the new cylinder head. Aluminum bronze CuAl10Fe5Ni5-C (JM7) was selected as the valve seat material due to its favorable wear resistance and thermal properties. [4]

Due to project time constraints, the initial version of the engine employs a valve train system largely based on the GX35 design. However, the valve train mounting arrangement was designed in such a way that a custom solution can be implemented in future development stages without requiring major modifications to the engine structure.

One of the most significant differences compared to the GX35 is that the entire valvetrain assembly is located outside the engine housing. Consequently, these components are not actively lubricated by engine oil and must instead be periodically lubricated manually using grease. This arrangement significantly reduces the dimensions and mass of the engine housing. It also provides substantially easier access to the valvetrain and simplifies valve timing adjustments during testing and development.

The camshaft mounting was also redesigned to improve compactness. In the GX35, the camshaft rotates on an axle supported at both ends. In the new design, the camshaft axle is essentially a cantilever beam. The camshaft pulley is manufactured from a commercially available standard timing pulley. Elongated mounting slots are incorporated into the pulley, allowing it to be rotated relative to the camshaft and thereby enabling adjustment of valve timing. To provide engine speed information to the electronic control unit, a magnet is press-fitted into a radial hole in the pulley. During operation, the magnet passes a Hall-effect sensor mounted adjacent to the pulley, generating the signal required for engine speed measurement and ignition control.



Fig. 7. Valvetrain assembly.

5. Engine Housing Assembly

The housing of the newly designed engine differs significantly from that of the Honda GX35 previously used in the vehicle. As previously mentioned, the GX35 consists primarily of two major cast components, an upper and a lower housing half, which are split horizontally along the crankshaft centerline. In contrast, the new engine consists of four major structural components, with the crankcase itself being divided into two vertically split halves. Although this solution increases the total number of components, it is considerably easier to manufacture using the available equipment. Furthermore, this design simplifies future modifications and maintenance procedures. For example, access to the combustion chamber and cylinder head can now be achieved without completely disassembling the engine.

The crankcase assembly consists of a crankcase body and a crankcase cover. Both components share essentially identical internal and external profiles but differ in depth. The geometry of the crankcase was largely determined by the crankshaft assembly

and the components mounted to it. When defining the internal shape of the housing, several factors had to be considered, including the rotational envelope of the crankshaft, the location of the crankcase ventilation system, and the mounting requirements for various external components. Components attached directly to the crankcase include the electric starter motor, clutch housing, crankcase breather tube, engine mounting brackets, and the Hall-effect sensor used for engine speed measurement. Sealing between the crankcase halves is achieved using a liquid gasket compound.

An important design decision concerned the crankshaft bearing arrangement. Several solutions were evaluated before selecting the simplest option from an assembly perspective. In the final design, the bearings are first press-fitted onto the crankshaft and subsequently inserted into their respective bearing seats from the inside of the crankcase. The bearing seat machined into the crankcase cover is 1 mm deeper than the corresponding seat in the crankcase body. This arrangement provides the crankshaft with the required axial clearance. Final axial play is adjusted using DIN 988 shim washers. The number and thickness of the shims are determined during assembly and compensate for manufacturing tolerances while also accounting for thermal expansion of the crankshaft during engine operation.

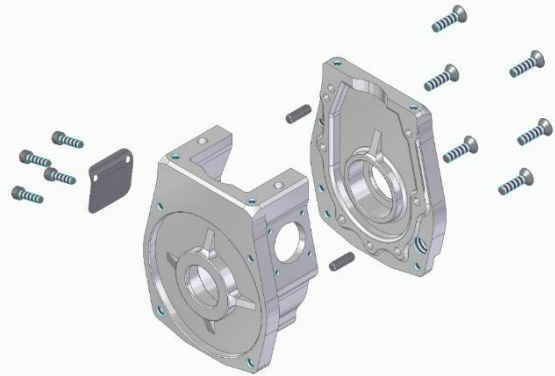


Fig. 8. Engine housing assembly.

As with the Honda GX35, lubrication of the engine is based on the relatively simplistic splash lubrication principle. In a splash-lubricated engine, the rapidly rotating crankshaft passes through the oil reservoir located at the bottom of the crankcase and disperses lubricant throughout the engine. In addition to the direct splashing effect, the reciprocating motion of the piston and the rotation of the crankshaft generate considerable turbulence within the crankcase. This turbulence creates an oil mist that further assists in distributing lubricant to critical engine components. [5] Despite its simplicity, it is well suited to the intermittent engine operating conditions encountered in the Shell Eco-marathon.

6. Strength Analysis of Critical Components

Finite element method (FEM) analyses were carried out on several critical engine components to verify their structural integrity under the loads expected during engine operation.

The first component to be analysed was the crankshaft. With the diameter of the main journals remaining unchanged compared to the GX35, the increased displacement and expected performance of the new engine warranted verification of the proposed design.

The loading conditions were determined using the previously developed kinematic and dynamic model of the crank mechanism. The highest load acting on the crankshaft occurs at top dead center when the engine operates at its maximum permitted speed of 10,000 rpm. Under these conditions, the maximum force transmitted through the connecting rod was calculated to be 2370 N. The FEM model was constrained at the bearing locations, while the calculated connecting rod force was applied to the crank pin.

The analysis showed a maximum crankshaft deflection of 0.0029 mm and a maximum equivalent stress of 77.76 MPa

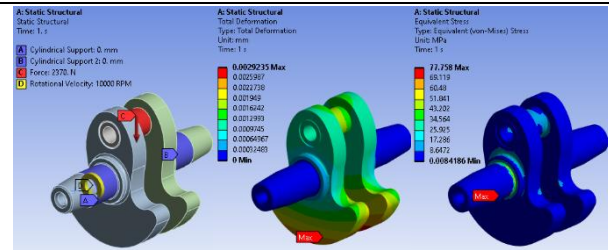


Fig. 9. Crankshaft FEM model and analysis results.

Alloy steel 42CrMo4 was selected for the crankshaft material. The material has a yield strength of approximately 650–750 MPa, an ultimate tensile strength of 900–1200 MPa, and a hardness of approximately 241 HB. [6] The selected material provides suitable wear resistance and a substantial safety factor, ensuring long-term durability of the assembly.

A strength analysis was also performed on the connecting rod. The connecting rod of the new engine is approximately three times longer than that of the GX35 and is also subjected to significant forces during operation. The analysis focused on determining the stresses and deformations generated by the maximum calculated connecting rod load. In addition, buckling resistance was evaluated because the connecting rod is relatively slender compared to its length.

The FEM results indicated a maximum equivalent stress of 56.36 MPa and a maximum deformation of 0.0577 mm. Buckling analysis showed that instability would occur only if the applied load increased by approximately a factor of 4.7.

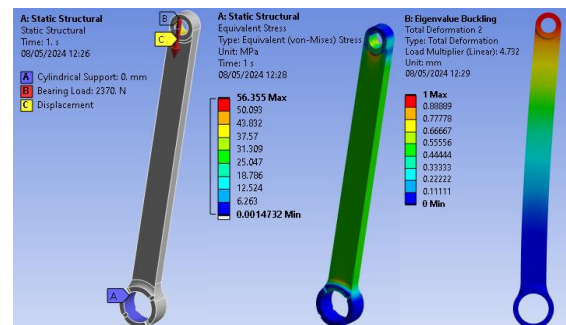


Fig. 10. Connecting rod FEM model and analysis results.

The connecting rod was designed from EN AW-7075-T6 aluminum alloy, which has a yield strength of approximately 550 MPa. [7] Consequently, the calculated stresses remained well below the material limits and no further design modifications were required.

A further FEM analysis was conducted on the axle supporting the camshaft assembly. Due to the compact dimensions of the camshaft, the largest suitable ball bearing had dimensions of $16 \times 8 \times 5$ mm, resulting in a bearing journal diameter of 8 mm. Since the axle is a cantilever beam and its unsupported length exceeds twice its diameter, the assembly was considered a potential structural concern and was therefore selected for detailed analysis.

The torque acting on the upper timing pulley was derived from the estimated engine output torque. A design torque of 3 Nm was assumed for the crankshaft. Since the camshaft rotates at half crankshaft speed, the corresponding torque acting on the camshaft pulley was calculated as 6 Nm.

The timing belt tension was determined from the belt profile, belt width, center distance, pulley tooth count, and operating speed of the driving pulley. The resulting maximum force acting on the axle was calculated to be 72.67 N.

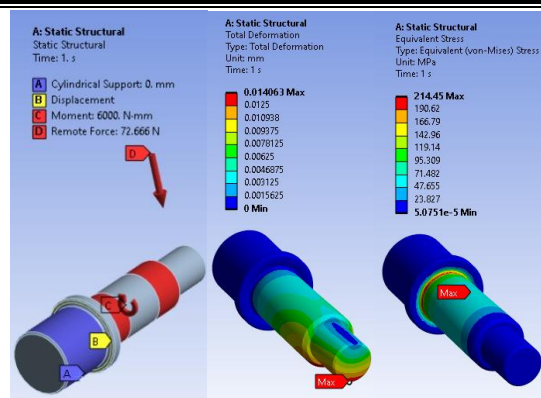


Fig. 11. Camshaft support axle FEM model and analysis results.

Initial simulations indicated stress concentrations approaching 300 MPa at the transition between the bearing journal and the axle shoulder. To reduce these stresses, the sharp corner was replaced with a fillet and a relief groove was added.

Following these modifications, the maximum axle deflection was reduced to 0.014 mm, while the maximum equivalent stress decreased to 214.45 MPa. These values were considered acceptable for the intended application.

Based on the calculated loads and stresses, alloy steel 42CrMo4 was selected as the axle material. Its high yield strength provides an adequate safety margin and ensures reliable operation of the valve train assembly under all anticipated operating conditions.

7. Engine Assembly

The complete engine assembly consists of 97 unique components, of which 27 are custom-manufactured parts, 22 are reused from the Honda GX35, 18 are standard commercially available components, and 30 are various fastening elements. The total number of individual parts within the assembly is 180.

The final dimensions of the engine are $158 \times 220 \times 326$ mm. The total mass of the assembled engine, excluding oil, is 4.55 kg. A significant proportion of this mass is attributable to the electric starter motor and the steel flywheel. As this engine represents the team's first fully custom prototype power unit, the primary objective during development was to ensure functionality, reliability, and ease of manufacturing rather than achieving the lowest possible mass. Despite this approach, the completed engine assembly is substantially lighter than the previous powertrain which had a mass of 5.7 kg. Compared to the previous assembly, access to individual components is significantly improved, and installation of the engine into the vehicle is considerably more convenient.



Fig. 12. Complete engine assembly in CAD (left) and after manufacturing (right).

8. Results and Discussion

The objective of this stage of the project was to convert the previously selected engine parameters into a complete and manufacturable internal combustion engine optimized for Shell Eco-marathon competition use. The resulting design differs significantly from the Honda GX35 previously used in the vehicle and addresses several limitations identified during earlier development.

All design goals were achieved, as a complete and functional 3D model of new engine was created. The new engine is lighter, more compact, and better suited to the competition's operating conditions. The compact crankshaft arrangement and revised housing design make it possible to fit the engine inside the vehicle and implement a single-stage chain transmission.

The engine is made up of custom parts with certain components being reused from the GX35. During the design process, strength analyses were carried out on the most critical parts and subassemblies. Based on the FEM analyses, suitable materials were selected and, if necessary, changes were made to the design.

Some engine subassemblies, such as the intake and exhaust system, along with the valvetrain, require further in-depth analysis which was not possible due to limited time. This will be part of future development and will take place once the engine is manufactured and tested. The design of the engine supports this as several design features support future modifications. Components such as the cylinder head, valve train assembly, and crankcase can be modified or replaced without requiring extensive redesign of the remaining engine structure. This modularity enables rapid implementation and evaluation of future improvements, including alternative combustion chamber geometries, revised valve timing strategies, and further weight optimization measures.

To conclude, the goal of the project can be considered fulfilled. The designed engine is the result of a year and a half long research and development done by the authors and other project members. The next phase of the project will focus on manufacturing and assembling the prototype engine and experimentally validating its performance.

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Thermal-EHL Tribodynamic analysis of a spur-gear pair, using correction factors

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Abstract. A thermal-tribodynamic model was developed to study the produced vibrations in a pair of spur gears, considering rough anisotropic tooth surfaces, the use of Newtonian and non-Newtonian lubricants, and friction induced temperature effects. It consists of two sub-models, the contact model and the dynamic model. Outputs of the contact model for each pair of the meshing gears' teeth are their respective contact stiffnesses and produced friction forces. The load-sharing method of Gelinck and DJ Schipper was used to avoid solving the complex and computationally expensive Reynolds equation. Correction factors were applied to the calculated lubricant thickness to account for the aforementioned phenomena. The dynamic response is determined by solving the equations of motion represented in state-space using the fourth-order Runge-Kutta numerical method. The total gear meshing stiffness and elasticities of bearings and shafts are modelled as forces from equivalent compression and torsional springs. For the total gear meshing stiffness, in addition to the commonly used terms and the contact stiffness, the structural coupling root-fillet stiffnesses of neighboring meshing teeth of each gear are also considered. These terms account for the deformation/movement of one tooth when force is exerted on an adjacent tooth. The effect of rotational speed, load, gear ratio, surface roughness, different lubricants, surface anisotropy coefficients, and oil bath temperature was also evaluated.

1 Introduction

Unwanted vibrations compromise system performance and reduce the service life of mechanical components. Particularly on geartrains, these accelerate wear by overloading the contacting surfaces. The opposite phenomenon has also been observed. As wear degenerates the gears' shape, operation becomes more unstable. Under certain conditions, cyclic bending of the tooth can form fatigue cracks in its root or break it off completely [1]. Excessive noise generated by industrial gearboxes also creates health and safety concerns for operators [2]. For these issues and several others, gearboxes seldom work without lubrication and surface properties are carefully selected. Hence, a dynamic model of a geartrain is incomplete if the tribological properties of the mating surfaces are not considered.

The simpler dynamic consider only movement in the direction of the line of action and torsion to a shaft whose mass is neglected. [3] made such a model with the gear mesh stiffness being developed in a Fourier series. Backlash was also included. [4] expanded to a two-stage geartrain considering the shaft's inertia and bearing flexibility [5] decomposed the response and meshing stiffness to static and alternating components to reveal how vibrations in gear-pairs are excited. [6] incorporated gravity, eccentricity, and bearing clearance into their model [7] and [8] tackled this problem using FEA.

[9] made a significant contribution in gear meshing stiffness modeling. He created stiffness analytical equations for all loading modes on a gear's tooth (axial, bending and shear), including also the stiffness of each gear's body and Hertzian contact stiffness. However, these equations could be applied only when addendum and base radiuses are equal. [10] lifted this restriction. [11] introduced additional stiffness equations to include structural coupling of tooth deformations. He followed the same approach as [12] using polynomials. He validated his findings with FEA simulations.

All the above models employ the Hertzian stiffness equations to model the contact and ignore friction. As mentioned earlier, this is insufficient for actual geartrains. With an appropriate thermal-elastohydrodynamic contact model the effect of lubricant and rough surfaces can be instilled to the contact stiffness, and the end friction force can be determined more accurately.

These two factors are primarily governed by the lubricant film thickness (h) and the RMS surface roughness (σ). There are three are the main methods for calculating lubricant thickness in elastohydrodynamic lubrication. All three rely on [13] rough contact models but they differ significantly.

One method involves solving the Reynolds flow equation of the lubricant along with the supplementary equations needed for describing elastohydrodynamic lubrication [14-16]. Surface roughness is incorporated in the geometric equation of surface.

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The second approach relies on experimentally derived lubricant thickness equations for the center of the contact and near the outlet region, where the film is [17-19] coupled with appropriate correction factors to accommodate for non-Newtonian lubricant, generation of heat, anisotropy and distribution the contact load between asperities and lubricant. [20] provides an outline of correction factors. This approach can be further divided depending on the required level of detail. One can either solve for the whole length of the contact treating the surface profile similarly to the first method (Reynolds approach) or assume contact pressure and lubricant thickness constant along the length of the contact and equal to the central values [21]. The latter assumption is valid when mating surfaces are in relative motion such as in gears [18]. Also, it is significantly faster thus more appropriate for practical use [22]. [23] pioneered the load sharing concept and modeled a full EHL contact by using Greenwood's model [13] and Dowson's lubricant thickness equation [17]. Greenwood's model is not the only rough contact model adopted in EHL models. A short overview of rough EHL-contact models using different contact and roughness equations and methods is provided by [24].

Utilizing curve-fit expressions from conducting numerous numerical experiments is the third and newest method. These expressions are derived mainly by following the Reynolds approach [25]. Lubricant thickness and portion of the load carried by the asperities can be directly determined from the geometry, surface and lubricant properties.

Heat is generated inside the contact from asperity scraping, lubricant shearing and compression. There is a rise in lubricant temperature which affects its properties and reduces its thickness [26]. This reduction was first studied by [15]. Based on that work [27] derived a correction factor which compares film thicknesses at bath and inlet temperatures. [28] using that coefficient and one for non-Newtonian lubricants produced an algorithm which calculates film thickness, central contact temperature and friction coefficient using previously derived analytical equations.

EHL contact models for Spur-Gears is already a well-studied topic. Gears are modeled as cylinders of radius equal to the radius of the curvature varying along the line of action. Notable is the work of [29-31]. [32] added a tangential component for the stiffness of a non-Newtonian lubricant.

The first two procedures for determining lubricant thickness have been applied in EHL contact models used in meshing stiffness models. The Reynolds approach was employed by [33]. Additionally, they have accommodated for alterations of contact point positions due to gear deformations, especially at the start and the end of the meshing period. A more accurate calculation of the distribution of load between the pairs of meshing teeth was made possible. [34] applied the load-sharing concept of [21] using Moes's, and Greenwood's equations.

In a tribodynamic model the EHL the contact model is mated with the dynamic model through the Meshing stiffness.

Notable works on Tribodynamic models include the earliest work of [35] who included, gear-pairs with damping and backlash, the contact model with the Reynolds approach and the effect of shaft and bearing elasticity but focused primarily on the dynamic response. [36] applied the Reynolds approach using, however, the dynamic meshing force. They extracted a similar lubricant curve with [30,31], but with an additional vibrating component. [37] and [38] conducted a full-on FEA on a pair of rotors with spur gears. The second team of researchers added the effects of a broken tooth, at different stages of damage.

What is missing from current works is a proper middle ground between overly simplified models and accurate full-detail models which are computationally expensive. During the literature review a universal analysis of all the effects in operation from different operational parameters, lubricants and surfaces was not found.

This analysis aims to do so by first developing a computationally inexpensive but sufficiently accurate tribodynamic model. Starting with the contact model and moving to the gear mesh model before finally applying it to the dynamic model.

For the contact model, the simple tried and tested second method will be used with correction factors. The actual temperature at the center of the contact will be found to determine more accurately required lubricant properties used to calculate the friction force. The application of the Structural coupling in a gear mesh of a dynamic model will be also a first. Finally, the dynamic model will incorporate, bearings, lateral and torsional movement as well as shafts.

The concept of the model is to start with a simplified model and add innovations of previous researchers, such as the correction factors, to be closer in accuracy to the more complicated models without affecting computational speed. Especially, the contact model will be called many times, since the dynamic's model's time step must be small enough for its numerical methods to be stable. Also, this approach will allow for further research into other aspects of contact related phenomena like wear evolution of the contacting surfaces and optimization problems. These problems require efficient algorithms. to produce meaningful results in a reasonable amount of time. This study is limited to ideal and healthy gear pairs operating in steady state conditions, meaning faults as backlash and eccentricity and other such faults and transient phenomena are not included.

2 Gear Properties

2.1 Gear Meshing Mechanism

The angle covered by each gear's tooth during meshing is called the meshing angle and is equal to:

$$\phi_{\text{meshing},i} = \frac{\phi_i}{\phi_{\text{gearmesh}}} = \frac{2^*}{z_i}, \quad i=1,2 \quad (1)$$

Two neighboring teeth are separated by one meshing angle when compared at the same point of the involute curve. Properties used in the next sections are measured relative to the start of a meshing period of the input gear. The observed pair of meshing teeth will be called from now on “current”. The one leading by $(2\pi/z_1)$ will be called “leading” and the one lagging by the same amount, “following”. When the “current” gear pair enters the “leading” pair has already covered some part of the gear mesh as shown in Fig. 1. The remaining angle of the “leading” gear pair to exit is also the first period of double gear pair contact. It is also known as the lowest point of single tooth contact *LPSTC* because after that angle single pair tooth contact begins. That angle is equal to (2):

The angle when single point contact ends is when the “current” gear pair reaches where the “leading” gear pair was when meshing started. This point is also called the highest point of single pair contact *HPSTC*. It is the remaining angle between double tooth state and the whole gear mesh angle.

$$\square_{HPSTC} = \square_{gearmesh} - \square_{LPSTC} \quad (3)$$

2.2 Gear Geometry

For studying gear teeth contact, these can be considered as cylinders with a time-variant radius equal to the current curvature radius of the involute curve. Those are provided by [39]:

$$R_1(t) = r_{b1} * \tan(\square_0) + X \quad (4)$$

$$R_2(t) = r_{b2} * \tan(\square_0) - X \quad (5)$$

Where X equals to:

$$X = \frac{L_{oc}}{T_{gearmesh}} * t - L_{ap}, \quad t \in [0, T_{gearmesh}] \quad (6)$$

Where $\square_0$ is the pressure angle and r_b is the base radius of each gear. Length of contact (L_{oc}) is the part of the line of action, in which the teeth are in contact. Length of approach (L_{ap}) is the part of the L_{oc} from the start of the contact up to the contact point coinciding with the pitch diameter. The contact between two cylinders can be simplified by assuming contact of a cylinder of an equivalent radius to a flat plane. The equivalent radius equal to:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad (7)$$

$$\square_{LPSTC} = 0.5 * \tan \left[\arccos \left(\frac{z_1 * \cos(a_0)}{\sqrt{(z_2+2)^2 + (z_1+z_2)^2 - 2 * (z_2+2) * (z_1+z_2) * \cos \left(\arccos \left(\frac{z_2 * \cos(a_0)}{z_2+2} \right) - a_0 \right)}} \right) \right] \quad (2)$$

Angular and linear variables used in this model are based on the work of [9,39]. To save space the reader is directed to these sources for further reading.

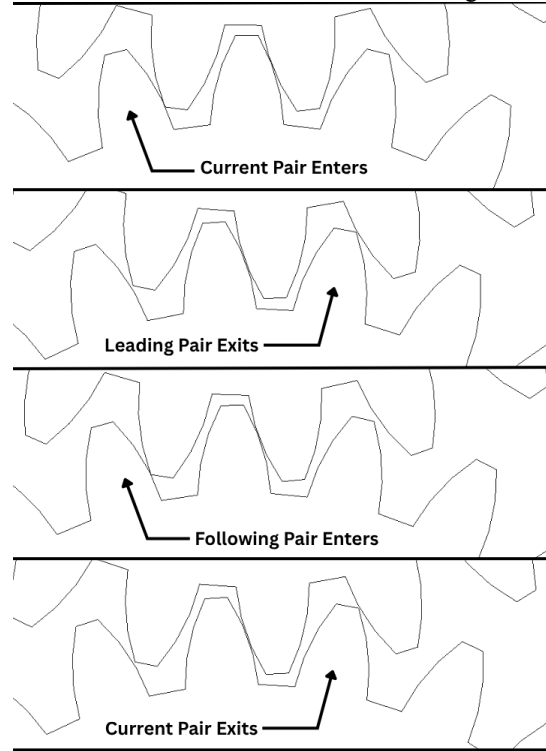


Fig. 1. Meshing Procedure for clockwise rotation of the bottom gear

2.3 Gear Kinematics

The tangent speed of the contact point relative to the gear's surface is:

$$u_i = V * \left(X * \frac{\cos(a_0)}{r_{b1}} + \sin(a_0) \right), \quad i=1,2 \quad (8)$$

While V is the tangential speed of each gear's pitch circle

$$V = \square_1 * r_{p1} = \square_2 * r_{p2} \quad (9)$$

Depending on the contact geometry a different “type” of speed is used in lubricant thickness calculations. Gear teeth contact is non-conformal thus the average, or entrainment speed u_r is appropriate.

2.4 Load Sharing Factor

Gears transmit a certain amount of torque for constant operating conditions, distributed between the meshing pairs of teeth. Depending on the number of pairs and the stage of meshing each pair carries a different amount of the total load. The AGMA's coefficient of load sharing (LSF) determines the contact force of the "current" tooth pair. The load sharing factor (LSF) is equal to:

$$LSF = \frac{F_{\text{contact},1}}{F_{\text{meshing}}} \quad (10)$$

For spur gears the equation of the LSF factor is the following (12) [40] and the curve produced is displayed in Fig. 2.:

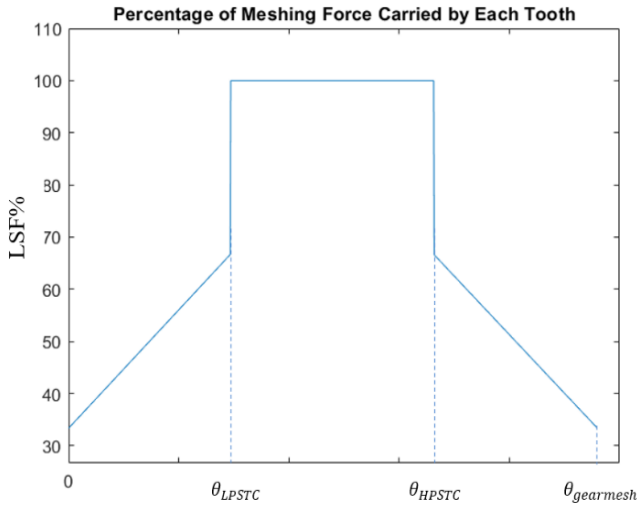


Fig. 2. LSF during Meshing

3 Surface Properties

3.1 Main Surface Parameters

The focus will be on the ones used in Greenwood's models. Most of those are measured from a median level of reference. For a single surface, the asperity density n_s is the number of asperity peaks per sampled area. The peaks are assumed to be spherical with a radius $\square$ and same for all asperities as per [13,23,41].

As with the equivalent radius of curvature, the approach to analyzing contact between two rough surfaces, is to assume an equivalent pair of surfaces one rough and one smooth. For the new "equivalent" rough surface, its RMS roughness is equal to [21]:

$$LSF(\square) = \begin{cases} \frac{1}{3} + \frac{\square}{3 * \square_{LPSTC}}, & 0 \leq \square < \square_{LPSTC} \\ 1, & \square_{LPSTC} \leq \square \leq \square_{HPSTC} \\ \frac{1}{3} + \frac{\square_{gearmesh} - \square}{3 * (\square_{gearmesh} - \square_{HPSTC})}, & \square_{HPSTC} < \square \leq \square_{gearmesh} \end{cases} \quad (11)$$

$$\square_{eq} = \sqrt{\square_1 + \square_2} \quad (12)$$

The equivalent asperity radius ($\square_{eq}$) is half the asperity radius of the original surfaces [41]. As for the equivalent asperity density, the three parameters n_s , $\square_s$, and $\square$ are related by the equation [23].

$$n_s * \square * \square = 0.05 \quad (13)$$

For calculating some contact parameters, a second plane of reference is utilized [23], the mean level of peaks. It has a standard deviation of [23].

$$\square_s = 0.7 * \square \quad (14)$$

The distance between the mean and peak planes (d_{deq}) is constant and equal to [21,23,42].

$$d_{\text{deq}} = 0.82 * \square \quad (15)$$

3.2 Surface Anisotropy

Surface topology from grinding, a very common method of finishing gear teeth is strongly dependent on machining direction [43]. To assess a surface's directionality the surface anisotropy coefficient (or surface pattern parameter in other sources) ($\square$) is used.

The Roughness Mean Spacing of profile irregularities (RSM_i) is the number of points coinciding with the mean plane of roughness measurements relative to a given sampling length.

The surface pattern parameter ($\square$) is the ratio of roughness mean spacing of profile irregularities, perpendicular to the machining direction (RSM_1) to that of the parallel direction (RSM_2) [44].

$$\square = \frac{RSM_1}{RSM_2} \quad (16)$$

Similarly, in EHL lubrication, a surface is deemed anisotropic when its asperities appear to have a specific orientation relative to lubricant flow [52,53]. When $\square > 1$ the asperities orientation is more in line with lubricant flow, and is considered longitudinal. On the opposite, asperities that have a transverse orientation when $\square < 1$. When $\square$ is equal to unity the surface is deemed isotropic. In extreme cases of $\square$ being infinite or $\square$ becoming zero surface asperities are parallel and perpendicular to the lubricant flow respectively. The surface pattern parameter is tied to the operating conditions of the surface. The surface's anisotropy can be transverse or longitudinal if lubricant flow changes direction.

4 Lubricant Properties

Contact parameters exhibit great sensitivity to changes in lubricant characteristics [45]. When two gears mesh the trapped lubricant is not only sheared but also compressed and heated. Despite the high pressures subjected, it is assumed that the lubricant always remains liquid [45].

The two parameters of interest are the lubricant viscosity and the Bulk modulus. The former affects mainly its thickness, friction produced while the later affects the stiffness of the contact.

4.1 Lubricant Viscosity

4.1.1 Roelands lubricant model for Newtonian Lubricants

Lubricant viscosity behavior is controlled mostly by the contact temperature and the pressure. The Roelands viscosity equation (17) has seen widespread use in EHL contact analyses [31,45].

Where the subscript 0 indicates conditions at ambient pressure (atmospheric) and temperature (25°C). T is the temperature at K, P_H is the manometric Lubricant Pressure at Pa, Z is the viscosity-pressure index and S_0 is the rate of lubricant Viscosity Index change with Temperature for Atmospheric Pressure [33,46].

4.1.2 Carreau power-law lubricant model for Non-Newtonian Lubricants

The Carreau power-law model is chosen for modeling non-Newtonian lubricants because it predicts accurately the pseudoplastic behavior of various common lubricants linking viscosity to operating conditions with shearing [28]. Also, this model is more suitable for EHL contacts employing shear thinning lubricants than the Eyring Shear thinning model [47]. The viscosity calculated by the Roelands' model is corrected with the following formula.

$$\eta = \eta_{\text{newt}} * \left[1 + \left(\frac{\eta_s * u_s}{G * h_c} \right)^2 \right]^{\frac{n-1}{2}} \quad (18)$$

G corresponds to the lubricant's shear modulus. n is the power-law exponent of the lubricant. For Newtonian lubricants G is infinite and n equals to unity.

4.2 Lubricant Bulk Modulus

The Bulk Modulus of a fluid represents its resistance to compression. The rate of meshing does not allow for a considerable amount of heat to be transferred between the

lubricant-meshing gears system and the outside environment. Thus, the compression of the lubricant is thought to be an adiabatic process, and the adiabatic Bulk modulus will be used [48-51].

$$B_0 = B_{00} * \exp(-\alpha_k * \Delta T) \quad (20)$$

The Tait-Doolittle model, used extensively in literature is chosen for calculating the Isentropic Bulk modulus [49].

Where B'_0 is the pressure rate of change of the Bulk modulus at ambient pressure. B_{00} is the bulk modulus of the lubricant at ambient pressure and temperature. α_k is the Tait-Doolittle temperature coefficient.

5 Thermal Contact Model

The main outputs of the Thermal Contact Model are the contact stiffness and produced friction, for use in the dynamics model. The idea of load distribution between asperities and lubricants was introduced by [23]. Following the same principle the contact stiffness and friction are a sum of an asperity part and a lubricant part.

[23] proved that the gap between the two rough surfaces is almost completely flooded with lubricant. The distance between the two rough surfaces is considered equal to the lubricant thickness. This assumption is valid for contacts under high loads, such as gears [21]. What controls both outputs are the lubricant thickness and the surface properties. For this reason, proper modeling of lubricant behavior, increase in contact temperature and surface topology were accounted for using correction factors.

5.1 Greenwood - Williamson Asperity Deformation Model

For modeling asperity behavior, the Greenwood family of rough contact models is the benchmark. Continuing from the work of previous researchers, Greenwood approached the asperities of a surface as a statistical population with their properties following a probability distribution [13].

Both mating tooth surfaces are considered rough with each surface having its own properties. To simplify the problem, it is safe to approach this as contact of an "equivalent" rough surface with a smooth flat plane [41]. However, with this approach however, for the same asperity pressure the separation between the rough surfaces is overestimated. This stems from treating the rough surfaces as contact between the peaks of asperities. In other words, asperity misalignment is not accounted for.

$$\eta_{\text{newt}}(T, P_H) = \eta_0 * \exp \left\{ \ln(\eta_0 + 9.67) * \left[-1 + (1 + 5.1 * 10^{-9} * P_H)^{\eta_0} * \left(\frac{T-138}{T_0-138} \right)^{-S_0} \right] \right\} \quad (17)$$

$$\eta = \left\{ 1 - \frac{1}{1+B'_0} * \ln \left[1 + \frac{P_H}{B_0} * (1+B'_0) \right] \right\} * [B_0 + P_H * (1+B'_0)] \quad (19)$$

Since the separation between the surfaces depends primarily on lubricant properties. For the same separation actual asperity pressure is less compared to the “equivalent” model due to misalignment. In the work of [41] it is shown that

$$P_{c-2rough}(h_c) = P_{c-eq}(h_c + 0.5 * \square_s) \quad (21)$$

When there is misalignment part of the load carried by the asperities, passes through the lubricant. Hence in the “two rough” surfaces model the portion of load carried by the asperities will be less than in the “equivalent” surface model

5.2 Moes Lubricant Equations

Following Gelnick's [21] methodology Moes's experimental equations for lubricant thickness are used. [19] his type of equations, assume Newtonian lubricant and constant temperature inside the contact. Lubricant properties are taken for ambient temperature and pressure. Lubricant thickness from Moes's equations will use the symbol $h_{c-isothermal}$ while the corrected value will use the term h_c .

$$\frac{h_{c-isothermal}}{R_{eq}} = H_c * U^{\frac{1}{\square}} \quad (22)$$

$$H_c = \left[\left(H_{RI}^{\frac{7}{3}} + H_{EI}^{\frac{7}{3}} \right)^{\frac{3}{7s}} + \left(H_{RP}^{\frac{7}{2}} + H_{EP}^{\frac{7}{2}} \right)^{\frac{2}{7s}} \right]^{s-1} \quad (23)$$

H_c is the dimensionless central lubricant thickness. $H_{RI}, H_{EI}, H_{RP}, H_{EP}$ are Koet's asymptotes used in Moes's diagram for different Lubricant behaviors. The subscripts RI, RP, EI, EP stand for rigid/isoviscous, rigid/piezoviscous, elastic/isoviscous, and elastic/piezoviscous areas respectively.

$$H_{RI} = 3 * M^1 \quad (24)$$

$$H_{EI} = 2.621 * M^{\frac{1}{5}} \quad (25)$$

$$H_{RP} = 1.287 * L^{\frac{2}{3}} \quad (26)$$

$$H_{EP} = 1.311 * M^{\frac{1}{8}} * L^{\frac{3}{4}} \quad (27)$$

M is a dimensionless load parameter and L is a dimensionless material parameter. Here, G , W , and U are dimensionless material elasticity, load, and velocity respectively. a is the pressure viscosity coefficient and s is a supporting variable.

$$M = W * U^{\frac{1}{2}} \quad (28)$$

$$L = G * U^{\frac{1}{4}} \quad (29)$$

$$U = \frac{\square(T_{bath}, 0) * u_r}{E_{eq} * R_{eq}} \quad (30)$$

$$G = a * E_{eq} \quad (31)$$

$$s = \frac{1}{5} * \left(7 + 8 * \exp \left(-2 * \frac{H_{EI}}{H_{RI}} \right) \right) \quad (32)$$

5.3 Lubricant thickness correction factors

5.3.1 Correction coefficients for surface anisotropy

[52] simulated over 2000 EHL models with different surface pattern coefficients ranging from 1/9 to 9, then curve-fitted the solutions to create the following correction coefficients. It is expected for surfaces with longitudinal asperities to carry less lubricant, since asperities more in line with the direction of the flow to provide less resistance. On the contrary, asperities with a transverse orientation restrict lubricant flow, thus lubricant thickness will be higher. More information on this model on [25,45, 52-54]. The researchers extracted three correction coefficients. One for the minimum lubricant thickness (h_m), which is not used in this model. The second is for the central lubricant thickness and the third one is for the portion of the load supported by asperities $1/\square_2$.

$$\square_{h_c} = \begin{cases} 1 + 0.354 * \square^{-1.351} * (1 - \square)^{2.261}, & \square \leq 1 \\ 1 - 0.135 * \square^{-1.430} * (\square - 1)^{0.329}, & \square > 1 \end{cases} \quad (33)$$

$$\square_{1/\square_2} = \begin{cases} 1 - 0.459 * \square^{-0.196} * (1 - \square)^{3.028}, & \square \leq 1 \\ 1 + 0.354 * \square^{-0.026} * (\square - 1)^{0.34}, & \square > 1 \end{cases} \quad (34)$$

These come with some limitations. Acceptable surface pattern values must be between 1/9 to 9. Outside of these values, the model might not be accurate. There is also a valid range for the $\square$. Even then, the results should be used cautiously if the portion of the load supported by asperities is greater than 70%.

Table 1 Validity range of different $\square$ values [52].

$\square$	Valid range
1/9	$\square > 1$
1/6	$\square > 1$
1/3	$\square > 0.75$
1	$\square > 0.5$
3	$\square > 0.5$
6	$\square > 0.5$
9	$\square > 0.5$

5.3.2 Correction Coefficients for Heat Generation and Contact Temperature increase

Heat is generated inside the contact from asperity scraping, shearing and compressing the lubricant which increases its temperature [26]. It is transferred from the lubricant to the mating surfaces through conduction despite the lubricant being fluid, because the thickness of the film is very small and the pressure inside the contact very high [26,47].

Lubricant thickness depends on its properties at inlet region only [28]. The validity of assuming constant lubricant thickness despite the temperature changes inside the contact still stands. Lubricant pressure in the inlet region is assumed equal to ambient. Inlet temperature is

higher though. It is unknown while bath temperature can easily be measured. The isothermal lubricant thickness is determined from properties at bath temperature thus coming with a small error. Thickness is overestimated because the actual viscosity is smaller than the one used in calculations. [27] suggested a correction coefficient, which compares thicknesses at bath and inlet temperatures. [28] applied that coefficient in their study and validating their findings with ball on disc and triple contact between steel discs experiments.

$$K_T = \frac{h_{c-\text{isothermal@inlet-temp}}}{h_{c-\text{isothermal@bath-temp}}} \quad (35)$$

$$K_T = \frac{1 - 13.2 * \left(\frac{P_{\text{Hertzian}}}{E_{\text{eq}}} \right) * L_T^{0.42}}{1 + 0.213 * (1 + 2.23 * SRR^{0.83}) * L_T^{0.64}} \quad (36)$$

SRR is the slide to roll ratio and L_T is the thermal loading factor. The latter links the lubricant's ability to expel heat to the mating surfaces (η_{lube} is its thermal conductivity) with the lubricant's viscosity and one of the sources of heat generation, entrainment speed.

$$SRR = \frac{u_s}{u_r} \quad (37)$$

$$L_T = \frac{\eta_{\text{lube}} * (\eta_{\text{bath},0}) * u_r^2}{\eta_{\text{lube}}} \quad (38)$$

η_{lube} is the opposite of the rate of change of lubricant viscosity with temperature at ambient environment conditions [27,28].

The actual inlet temperature can be found from lubricant viscosity at that temperature. Taking as equals the bath thickness with its correction coefficient with that of the inlet region, and inserting the equation for lubricant thickness on either side will yield the following relationship [28]. Solving for η_{inlet} we get inlet temperature at K.

5.3.3 Correction coefficients for Non-Newtonian Lubricant

Limiting the analysis to only Newtonian lubricants would rule out many commercial synthetic lubricants. Relying on the work of [55] who conducted a regression analysis on shear-thinning lubricants, the following coefficient K_{ST} was extracted [28], equation (40). This factor is based on the Carreau-power-law lubricant model and uses lubricant thickness at inlet temperature.

$$\eta_{\text{inlet}} = 138 + (T_o - 138) * \left[\frac{\ln(K_T)}{0.692 * \ln(\eta_o + 9.67)} + \left(\frac{T_{\text{bath}} - 138}{T_o - 138} \right)^{-S_o} \right]^{\frac{1}{S_o}} \quad (39)$$

$$K_{ST} = \left[1 + 0.79 * \left((1 + SRR) * \frac{u_r * \eta_{\text{inlet}}}{h_{c@inlet-temp} * G_{\text{lube}}} \right)^{\frac{1}{1 + 0.2 * SRR}} \right]^{-3.6 * (1 - n)^{1.7}} \quad (40)$$

5.3.4 Final Corrected Lubricant Thickness

After introducing correction coefficients for surface anisotropy, heat generation and shear thinning behavior, the end lubricant thickness is equal to:

$$h_c = h_{c-\text{iso}} * K_h * K_T * K_{ST} \quad (41)$$

5.4 Gelinck & DJ Schipper Load Sharing Model

Gelinck and D.J. Shipper's load-sharing model predicts the friction coefficient under EHL lubrication without solving the Reynolds equation. Instead, they rely on analytical equations of Greenwood and Moes using a clever trick. They came up with an experimentally derived asperity pressure equation $P_{c\text{exp}}$, equation (43) linking the Hertzian contact pressure with surface properties. When in EHL conditions the asperities take a portion of the load (F_T/η_2), and n_g equals to:

$$n_g = n_{s-\text{eq}} * R_{\text{eq}} * \sqrt{R_{\text{eq}} * \eta_{\text{eq}}} \quad (42)$$

$P_{c\text{exp}}$ must be equal to the asperity pressure calculated by Greenwood and Williamson equation for separation equal to the corrected lubricant thickness $P_{c\text{GWM}}(h_c)$. Lubricant thickness depends on the carried load F_T/η_1 , with the η_1 term inserted in the Moes's equation. All in all, there are three equations with three unknowns η_1, η_2, h_c . All equations used can be found on their works [21].

The complexity of this non-linear system of equations makes an analytical solution impractical. An iterative process is the approach taken by the writer to solve these equations shown as flowchart in Fig 3. Starting with η_1 equal to unity (i.e., the lubricant carries all the load), $h_{c-\text{iso}}$ and η_2 are then calculated. The term " $h_c + 0.5 * \eta_s$ " is added onto $h_{c-\text{iso}}$ to account for asperity misalignment. This step pushes the algorithm to converge to a smaller η_1 which in essence means a higher portion of load carried by the lubricant. The end result is a smaller final corrected h_c by that margin and at the same time a smaller P_c . The next step is the application of the correction factors to the lubricant thickness to get h_c .

Afterwards, from h_c , $P_{c\text{GWM}}$ is determined, using the Simpson numerical Scheme and $P_{c\text{exp}}$ from η_2 . The values of $P_{c\text{GWM}}$, $P_{c\text{exp}}$ are compared. If the difference is greater than a certain limit, η_1 is increased by a $\Delta\eta_1$ and the process is repeated.

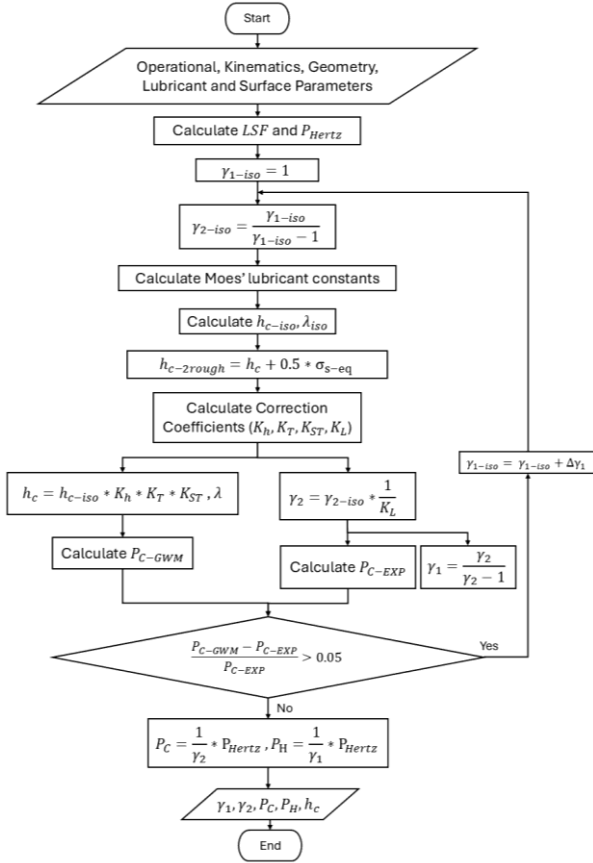


Fig 3. Load Sharing Model Solution Algorithm.

5.5 Contact Temperature and Friction

5.5.1 Contact Temperature and Friction Equations

Total friction is the sum of friction between asperities and lubricants. [21], [30]. The friction coefficient of asperity contact is almost constant and depends on the contacting pair of materials. Total asperity friction is the product of the said coefficient and the time variant contact force [21]. Lubricant friction changes also with time because gear kinematics, contact pressure, lubricant thickness and viscosity are time-variant. Frictional force from the lubricant is generated by its shearing at the hertzian contact area.

$$F_{fr} = F_{fr_c} + F_{fr_h} \quad (44)$$

$$f_{fr} = \frac{F_{fr}}{F_T} \quad (45)$$

$$P_{c_{exp}} = \frac{1}{\zeta_2} * \left[1 + \left(A_1 * n_g^{A_2} * \zeta_{eqn}^{A_3} * W^{A_2-A_3} * \zeta_2^{A_2} \right)^{A_4} \right]^{\frac{1}{A_4}} * P_{Hertzian} \quad (43)$$

$$\zeta_1 = \frac{0.53 * f_{fr} * \left(\frac{F_T}{W_{contact}} \right) * u_s}{\sqrt{\zeta_1 * (0.9 * \zeta_1 + b_h * u_1 * \zeta_1 * cp_1)} + \sqrt{\zeta_2 * (0.9 * \zeta_2 + b_h * u_2 * \zeta_2 * cp_2)}} \quad (49)$$

$$F_{fr_c} = f_c * F_c \quad (46)$$

$$F_{fr_h} = \frac{2 * w_g * b_{hertz} * \zeta(T_{central}, P_H) * u_s}{h_{c-corrected}} \quad (47)$$

On EHL analyses, lubricant thickness is calculated with properties at inlet area while friction generated by the lubricant requires properties from inside the Hertzian region. In this model knowledge of the temperature at the center of the contact is needed.

Inside the contact, temperature increases from two phenomena, heat from asperity friction and heat from lubricant shearing and compression. From the inlet temperature the contact temperature can be calculated from [28]:

$$T_c = T_{inlet} + \zeta_1 + \zeta_2 \quad (48)$$

$$\zeta_{lube-mech} = \frac{f_{fr} * \left(\frac{F_T}{W_{contact}} \right) * h_{c-corrected} * u_s}{16 * b_h * \zeta_{lube}} \quad (50)$$

The second term refers to the average flash temperature. It is the increase in temperature from heat generated by “rubbing” of the contacting asperities. Those points of contact act as heat sources. [56]. The third term is the increase in temperature lubricant from shearing and compressing. For line contacts there are analytical equations for each of the two terms found in [28] skipping setting up and solving complex energy balance equations each time. ζ_1, ζ_2, cp_1 are the thermal conductivity, density and specific heat of each gear.

5.5.2 Contact Temperature and Friction Calculation Algorithm

For calculating temperature at the center of the contact the methodology of [28] is followed shown in Fig 4. The algorithm starts with an assumed contact temperature equal to the inlet temperature which is the lowest possible value. Then, the lubricant viscosity based on the assumed temperature is determined using Roelands' and Carreau's equations. The third step is to find the friction coefficient using the equations mentioned in this section. The third step is to find the final contact temperature. Using a step ζ_1 the contact temperature was increased until the central contact temperature calculated by the friction coefficient matched the assumed one.

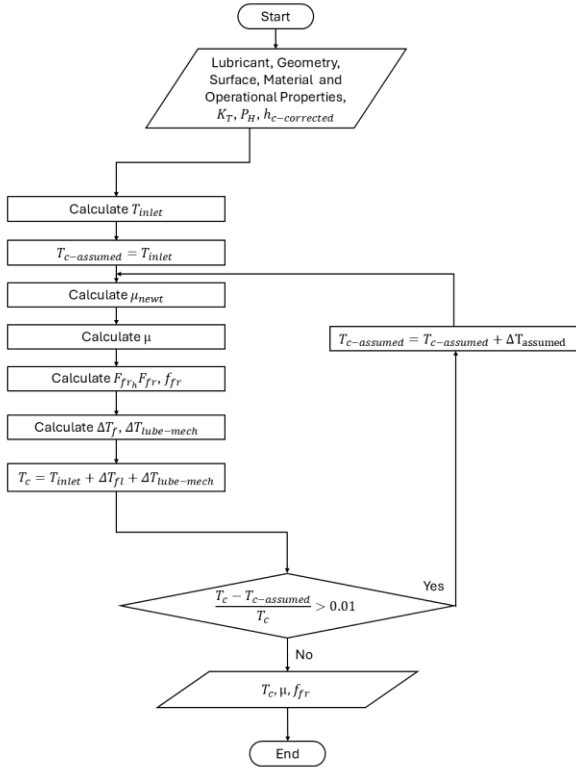


Fig. 4. Contact Temperature and Friction Calculation Algorithm.

5.6 Contact Stiffness

Contact stiffness is the sum of the asperity and lubricant stiffnesses as two springs connected in parallel. The term i refers to the i th pair of meshing teeth.

$$K_{contact,i} = K_{H,i} + K_{asp,i} \quad (51)$$

5.6.1 Lubricant Stiffness

The equation for the stiffness of the lubricant film is found in [34,49]:

$$K_H = \frac{B}{h_c} * A_{Hertzian} \quad (52)$$

5.6.2 Asperity Stiffness

Using Greenwood's model for calculating asperity load and deformation [34] extracted the equation (53) for asperity stiffness. They used also a correction coefficient $\square_c$ for curved surfaces because the number of asperities in contact between curved surfaces is less than in flat ones and the way force is exerted from one asperity to the other is out of line with the previously mentioned assumptions [13]. In the same manner as in Gelinck and DJ Schipper the separation of the surfaces is taken equal to the corrected lubricant thickness.

$$K_{asp} = 2 * n_{seq} * A_{Hertzian} * \square_c * E_{eq} * (\square_{eq} * \square_{eq})^{\frac{1}{2}} * \int_{h_n - d_{deqn}}^T [z_n - (h_n - d_{deqn})]^{\frac{1}{2}} * \left(\frac{\square_{eq}}{\square_{seq}}\right) * \exp\left(-\frac{1}{2} * \left(\frac{\square_{eq}}{\square_{seq}}\right)^2 * z^2\right) * dz \quad (53)$$

In the stiffness equation E_{eq} is the equivalent Young Modulus. The subscript $\square_{eq}$, indicates that a variable has been normalized to the equivalent radius of curvature. $A_{nominal}$ is the cumulative surface area for the two gear teeth. $b_{Hertzian}$, $A_{Hertzian}$ are the contact half-width and area, for line-contact. w is the width of the surface, in our case the width of the gears. " $h_n - d_{deqn}$ " shows that the plane of reference is the plane of the asperity peaks.

$$A_{nominal} = 2 * \sqrt{(R_{in} + R_{out}) * w_{contact}} \quad (54)$$

$$b_{Hertzian} = \sqrt{\frac{4 * |F_T| * R_{eq}}{w_{contact} * E_{eq}}} \quad (55)$$

$$A_{Hertzian} = 2 * b_{Hertzian} * w_{contact} \quad (56)$$

$$\square_c = \left(\frac{A_{Hertzian}}{A_{nominal}}\right)^{R_{eq}} \quad (57)$$

6 Gear Meshing Stiffness and Damping

The Gear meshing stiffness combines contact elasticity with tooth elasticity to create a spring connection between the two gears. Experiments have also shown energy losses through meshing in addition to frictional losses. Those losses were discovered to be related with the meshing stiffness [9]. Both meshing and damping forces are exerted in the direction of line of contact.

6.1 Gear Meshing Stiffness

The two parts of the meshing stiffness are modeled as springs connected in a series. To determine deflection under load each gear tooth is considered as a non-prismatic cantilever beam. When meshing occurs, a gear's tooth is compressed, bent, and sheared at the same time. Each mode is modeled as an individual spring connected in series with the others. There is also a fourth stiffness called the "Root Fillet Radius" stiffness. It accounts for the elasticity of the gear wheel. All terms change with the position of the point of contact. Also, the total gear mesh stiffness is affected by the entering and exiting of other pairs of gear teeth. This procedure was described in detail in Section 2

6.1.1 Gear Meshing Stiffness Formulas of a single tooth

Teeth stiffness equations differ when root and base diameters are equal or one larger than the other [10]. In this model both are considered equal. K_a , K_b , K_s are provided by [9].

$$\frac{1}{K_g} = \frac{1}{K_a} + \frac{1}{K_b} + \frac{1}{K_s} + \frac{1}{K_f} \quad (58)$$

6.1.2 Root Fillet Stiffness of a single tooth

For a full analysis of the meshing stiffness of the gear-pair the stiffness of the body of the gears should not be neglected. Even more so when modeling gears of large diameter. The theory of Muskhelishvili for calculating strains on circular rings [57] was used by Sainsot [12] to determine static deflections of gears bodies and extract equations of stiffness in polynomial form. The Root Fillet stiffness of a gear's tooth at each moment can be found in the following works [9,12,58].

6.1.3 Gear Teeth Structural Coupling Root fillet Stiffness

When two pairs of teeth mesh the deformation of one tooth affects the deformation of the other [11]. Using only the equations of the previous section is sufficient only in single tooth pair period. Using the same theory of elastic ring strain [57], and formulation as in [12], [11] extracted similar analytical equations for predicting the rigid movement of one tooth when force is exerted on another.

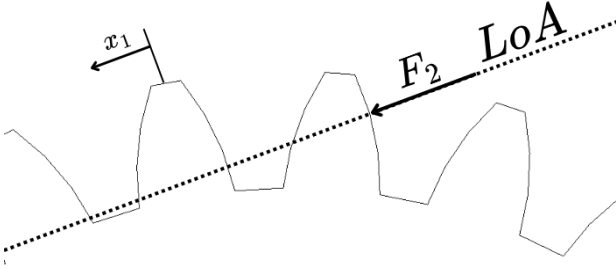


Fig. 5. Cross Coupling Deformation of Gear Teeth.

Since having two double pair meshing periods $[0, \square_{LPSTC}]$ and $[\square_{HPSTC}, \square_{gearmesh}]$ one must account for two sets of equations for each period. $K_{f,1,2}$ is refers to the stiffness of the tooth pair 1 (i.e. “current”) when force is exerted on tooth pair number two (i.e. “leading”). The same principle is applied to the rest of the equations.

6.1.4 Total Gear Meshing Stiffness of a tooth pair

The gear stiffness of a gear tooth i^{th} pair is the cumulative stiffness of the driving, driven gears and their contact. Each pair has its own contact stiffness.

$$K_{gearmesh}(\square) = \begin{cases} \frac{K_{g,1} + K_{g,2} - K_{g,1} * K_{g,2} * \left(\frac{1}{K_{f12}} + \frac{1}{K_{f21}} \right)}{1 - \frac{K_{g,1} * K_{g,2}}{K_{f21} * K_{f12}}} & [0, \square_{LPSTC}] \\ K_{g,1} & [\square_{LPSTC}, \square_{HPSTC}] \\ \frac{K_{g,1} + K_{g,3} - K_{g,1} * K_{g,3} * \left(\frac{1}{K_{f13}} + \frac{1}{K_{f31}} \right)}{1 - \frac{K_{g,1} * K_{g,3}}{K_{f31} * K_{f13}}} & [\square_{HPSTC}, \square_{gearmesh}] \end{cases} \quad (61)$$

$$\frac{1}{K_{g,i}} = \frac{1}{\frac{1}{K_{contact,i}} + \frac{1}{K_{g1,i}} + \frac{1}{K_{g2,i}}}, \quad i=1,2,3 \quad (59)$$

$$K_{contact,i} = K_{H,i} + K_{asp,i} \quad (60)$$

When meshing of multiple pairs of teeth and structural coupling is factored in the total gear mesh of a gear-pair becomes [58] (61).

To avoid calculating the gear meshing stiffness for each pair. First, the stiffness of the “current” pair can be calculated and then make use of the angular relationship between teeth ($2\pi/z_1$). The last step is to superimpose the three stiffnesses appropriately for each period. For the “leading” and “following” pair these apply only during $[0, \square_{LPSTC}]$ and $[\square_{LPSTC}, \square_{gearmesh}]$ respectively. Otherwise $K_{g,2}$ and $K_{g,3}$ are equal to zero.

$$K_{g,2}(\square_1) = K_{g,1} \left(\square_1 + 2\pi \frac{1}{z_1} \right) \quad (62)$$

and

$$K_{g,3}(\square_1) = K_{g,1} \left(\square_1 - 2\pi \frac{1}{z_1} \right) \quad (63)$$

6.2 Meshing Damping

A Rayleigh damper, independent of the gears' masses ($\square \neq 0$), is used to model losses in the direction of the line of action. $\square_k$ is so far unknown. m_{eq} is the equivalent mass of the gear pair [9]. However, $\square_k$ must be equal to $c_{g,avg}/K_{g,avg}$. The second ratio can be easily determined from damping ratio. Experiments on gear pairs have shown that $\square$ is equal to 0.07 [9,59].

$$c_{g,avg} = 2 * \square * \sqrt{K_{g,avg} * \frac{m_{r1} * m_{r2}}{m_{r1} + m_{r2}}} \quad (64)$$

$$c_g(t) = \frac{c_{g,avg}}{K_{g,avg}} * K_g \quad (65)$$

7 Dynamic Model

Problems of geartrain dynamics used for studying the meshing stiffness are solved with force equations and a numerical scheme for predicting the response. [5] provided the framework for setting up the equations for the geartrain's vibrating stages, which is experimentally validated. In their approach, the meshing stiffness and response are split into static and alternating parts.

7.1 Vibration Mechanism on geartrains

When plotting the Fast Fourier Transform (FFT) of the time response in a healthy pair of gears the main contributing amplitudes to the response are an integer of the Gear Meshing Frequency (*GMF*) [5]. These are known as the higher harmonics of the response. The gears' meshing frequency is the impact rate of their teeth, when it rotates at a certain speed.

$$GMF = z_1 * f_{rot1} = \frac{z_1 * \omega_1}{2\pi} \quad (66)$$

Contrary to intuition, those impacts alone do not excite the system when the system is in a steady state. When meshing starts, the speed difference between the driving and the driven gear is small. Hence, contact happens in a "gentle", quasi-static manner.

Also, the meshing stiffness equations do not contain impact terms, only geometry and material variables. Its value, however, significantly varies when changing between single and double meshing periods. This happens once every meshing period, with the frequency of this phenomenon being equal to the *GMF*. What changes with different gear geometries is the portion of the meshing time done in single-pair and double-pair teeth states. Thus, the actor of vibrations is actually the change in the number of the meshing pairs of teeth.

The meshing stiffness consists of a static part $K_{g_{avg}}$ and a vibrating part $K_{g_{var}}(t)$ [5]. Each part contributes to its respective response. Having determined the $K_g(t)$ in the previous chapter, $K_{g_{avg}}$ is its mean value and $K_{g_{var}}(t)$ comes from:

$$K_{g_{var}}(t) = K_g(t) - K_{g_{avg}} \quad (67)$$

7.2 Dynamic Model Assumptions

Some simplifying assumptions were made to the dynamic model. The system operates in steady state conditions. The rotational speed of the input shaft and load exerted on the output shaft are constant. The gears are allowed to move in the direction of the meshing force and to twist relative to their respective shafts. There is no backlash between the meshing teeth. The mass of the shafts is considered. Gears are wedged into their shafts, with their connection is deemed rigid. In other words, gears and shafts move together as a single entity but there is relative torsion. The gears are considered as rigid cylinders in the dynamic model with

their elasticity included in the meshing stiffness and tooth geometry omitted. Shafts and bearings are considered elastic elements.

7.3 Machine Element Behavior

Shafts act as torsional springs and bearings act as compression springs deforming in the direction of the line of action. The equation of shaft stiffness is provided by [60].

$$K_s = \frac{G_{sh} * I_{p_{sh}}}{L_{sh}} \quad (68)$$

Where G_{sh} is the shear modulus of the shaft's material $I_{p_{sh}}$ is the polar moment of the shaft's cross-section, and L_{sh} is the distance of the mounted gear to the free edge. Based on [61] the bearing stiffness formula is:

$$K_{b_{ver}} = \kappa_b * N_{bl}^{\frac{2}{3}} * d_{bl}^{\frac{1}{3}} * F_b^{\frac{1}{3}} * \cos(\alpha_{ber}) \quad (69)$$

From the above equation, bearings act as non-linear springs with time-variant stiffness. κ_b is a constant equal to $13 * 10^6$ with units $N^{\frac{2}{3}} * m^{\frac{4}{3}}$. N_{bl} is the number of balls inside the bearing cage, and d_{bl} is the diameter of each ball. F_b is the force exerted on each bearing at each moment. For a two-bearing set-up per shaft, placed on equal distances from the respective gear, each bearing carries half of the load. α_{ber} is the contact angle of the bearing. For deep grooved ball bearings that angle is null.

Bearings display different stiffnesses in the vertical (direction of gravity) and horizontal directions which depends on the number of rolling elements. [61] provides such ratios $(K_{b_{hor}})/(K_{b_{ver}})$ for ball bearings. In this analysis the total bearing's stiffness is considered the average of the two directions.

Table 2. Stiffness Ratios for different ball numbers

Number of rolling elements	8	12	16
$(K_{b_{hor}})/(K_{b_{ver}})$	0.46	0.64	0.73

7.4 Meshing force and Meshing Stiffness from a dynamics perspective

Motion is transmitted from one gear to another through the meshing force. It is the total force distributed to all contacting teeth at a time as the sum of all the contact forces. The gear connection is viewed macroscopically as a spring connecting two cylinders with its direction coinciding with the line of action and tangent to the gear cylinders. The meshing force is the product of the meshing stiffness times the transmission error (ϵ). The transmission error is the relative position of its

connected points and is not related to any specific gear tooth.

$$F_{\text{mesh}} = K_g * \square + c_g * \dot{\square} \quad (70)$$

$$\square = (u_1 + d_{p,1} * \square_2) - (u_2 - d_{p,2} * \square_3) \quad (71)$$

To further simplify the algorithm the static meshing force will be used in the contact model, which is equal to $T_1/d_{\text{pitch},1}$. This allows the decoupling of the dynamic and contact models. If not then, both models would have to be solved simultaneously at every time step, because both the contact stiffness and dynamic response are unknown. When using the dynamic terms, the meshing force at every moment is not equal to $T_1/d_{\text{pitch},1}$, only its average for the whole meshing period.

$$F_{\text{contact}} = T_1/d_{\text{pitch},1} * LSF \quad (72)$$

7.5 Torque Generated by friction

The frictional force between the meshing pair of teeth moves along the line of action. Therefore, an opposing torque is also generated. The frictional force is always perpendicular to the line of action and the produced torque is the product of that force times the covered distance along the line of action relative to the center of the gear tooth [62,63]. Its thickness must be accounted, because the line of action is measured from the first contact point not the center of the tooth. The frictional torque of the “current” pair of teeth for the input and output gears is.

$$T_{\text{fr},1}(t) = F_{\text{fr}} * \left(X(t) + L_{\text{ap}} + \frac{L_{\text{arc},1}}{2} \right) \quad (73)$$

$$T_{\text{fr},2}(t) = F_{\text{fr}} * \left[L_{\text{oc}} - \left(X(t) + L_{\text{ap}} + \frac{L_{\text{arc},2}}{2} \right) \right] \quad (74)$$

The friction of the other two pairs of teeth also creates torque ($T_{\text{fr},2}(\square), T_{\text{fr},3}(\square)$). One easy way to determine their torque is to make use of the angle between meshing pairs of teeth ($2 * |z_1|$), in the same manner as in the gear meshing stiffness equations. The same equations can be applied for the equations for output gear.

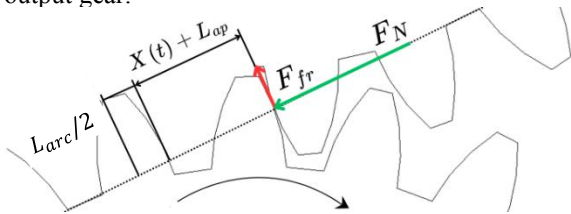


Fig. 6. Torque produced by Friction

The total opposing torque for the input gear by at every period of the meshing duration is:

$$T_{\text{fr},1}(t) = \begin{cases} T_{\text{fr},1} + T_{\text{fr},2} & [0, t_{\text{LPSTC}}] \\ T_{\text{fr},1} & [t_{\text{LPSTC}}, t_{\text{HPSTC}}] \\ T_{\text{fr},1} + T_{\text{fr},3} & [t_{\text{HPSTC}}, t_{\text{gearmesh}}] \end{cases} \quad (75)$$

7.6 Static and Dynamic Response

Following the methodology [5] the response is broken to static and dynamic parts. The static part is solved first by solving the linear system equations of the static response in the physical space. Then, in each motion equation the static and dynamic parts are separated on each side of the equation. The whole dynamic system is converted into state space form for easier numerical solution and compatibility with other MATLAB functions. For a single stage geartrain, there are in total six degrees of freedom. For each rotor, there is one for lateral movement (u_1, u_2), one for torsion of the shaft ($\square_1, \square_4$), and one for the torsion of its gear ($\square_2, \square_3$). The total response is calculated by summing the dynamic and static parts.

7.7 Numerical Scheme

Problems such as this one are modeled by a system of stiff differential equations. The end solutions of stiff differential equations are functions of variables which change at very different rates. To acquire an accurate solution the time-step must accommodate the fastest changing independent variable [64]. A stiff problem requires a “large” simulation time to catch the “slower” components but a very “small” time-step to catch the “faster” ones.

The behavior of the meshing stiffness excitation makes this problem stiff. As said above, the meshing stiffness has terms changing with the rate of the meshing frequency and its higher harmonics. Despite only the first few being significant in magnitude, higher harmonics of smaller amplitude contribute to the overall solution and cannot be neglected. Also, the contact model requires an even smaller time-step to converge.

Many numerical methods have been evaluated. The end choice was between the Runge-Kutta and Gear numerical methods. Both have similar accuracy [65]. For both schemes the state space formulation can be applied directly. The Runge-Kutta method was chosen, even though the second being considered more appropriate for such problems. The main advantage of the Runge-Kutta method is that it requires only one point to start. On the other hand, the Gear method needs six. When only the initial conditions are known, the Runge-Kutta algorithm must be utilized first to first get those six points. This mixture of numerical methods was tested but no additional speed or accuracy were obtained. Thus, this idea was abandoned.

8 Tribodynamic Model Solution Algorithm

The total solution algorithm is produced from combining the equations and methods mentioned in the previous chapters displayed in Fig. 7. Each time step starts with calculating K_b, K_a, K_s, K_f and the structural coupling fillet stiffness using the equations in chapter 4. Then contact stiffness and friction produced are determined using the equations and methodology mentioned in chapters 2 and 3. The total meshing

stiffness and damping are calculated with the equations. stated in chapter 6. Finally, the state equations are solved. and the algorithm proceeds to the next timepoint. This process is repeated until reaching the end of the simulation time.

For the numerical method to converge, a thousand time-points per period were chosen. The time step is of the 10^{-5} ~ 10^{-7} s order for the studied rotational speeds. The periodicity of meshing and stable conditions of the environment and gear-pair are utilized to skip solving the same equations multiple times. The contact variables, frictional forces and Meshing stiffness are calculated only during the first meshing period. Afterwards, these are superimposed for the next meshing periods until simulation time ends. The end result is a solution time of only few minutes, making a parametric analysis feasible on a home computer.

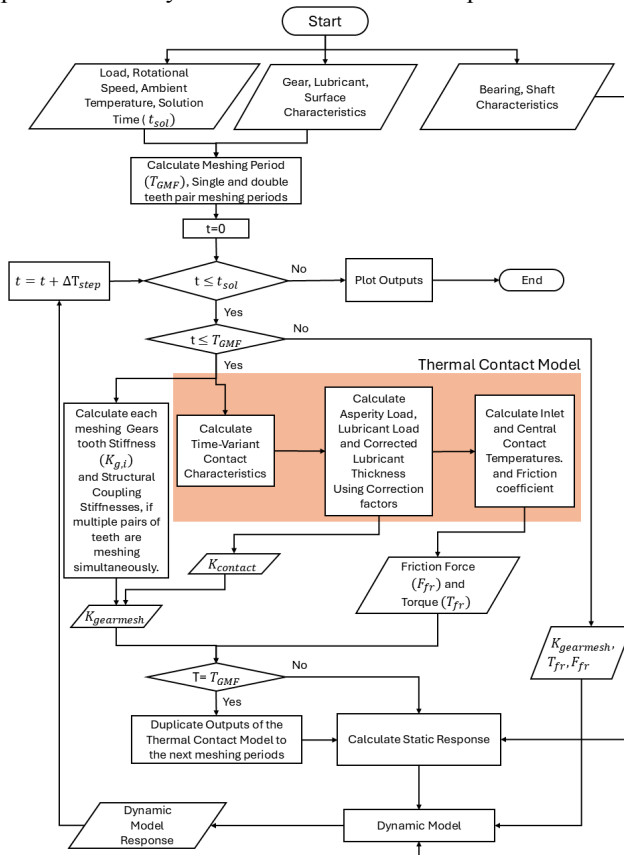


Fig. 7. Tribodynamic Model Solution Algorithm

9 Data used for the model

The geometry of the gear pair relies on the following primary parameters. All-others are calculated from the equations shown in chapters 2 and 3. For testing different gear ratios the load was altered accordingly to produce the same contact force.

Table 3. Geometrical Properties of the input gear

module (m)	0.004		
α (deg)	20		
z_1	25		
w_{gear} (m)	0.05		
Gear Ratio	1:3	1	3:1

	(Reference case)	
--	------------------	--

The chosen material is AISI-4340 Nitrided Steel mainly for its high contact strength. Useful mechanical and thermal properties were extracted from [66-68].

Table 4. Gear Material Properties

E (Pa)	$200 \cdot 10^9$
ν	0.27
ρ (kg/m ³)	7800
cp (J/ kg*°C)	475
α (W/m*K)	44.5

The mounting shafts have a diameter of 0.04m with a sufficient length of 0.106m to fit each gear and bearings. Shaft diameter remained unchanged when testing different gear ratios. Deep groove bearings were chosen due to the absence of axial loads. Thermal effects on bearing performance were neglected.

Table 5. Bearing Properties

d_{bore} (m)	0.04
d_{out} (m)	0.09
w_b (m)	0.023
N_b	8
d_{ball} (m)	$7.541 \cdot 10^{-3}$
α_{contact} (deg)	0
ω_{max} (RPM)	11000

Three surfaces were used for testing contact and dynamic [69].

Table 6. Properties of tested surfaces

Property	Surface #1 (Reference Case)	Surface#2	Surface #3
R_a (μm)	0.258	0.036	0.450
R_q (μm)	0.321	0.046	0.579
R_z (μm)	1.643	0.275	3.4

And three surface anisotropy coefficients, $\beta=1/5$ correlates to a grinded surface [70].

Table 7. Tested anisotropy coefficients

	Pattern #1 (Reference Case)	Pattern #2	Pattern #3	Pattern #4	Pattern #5
β	1	1/9	1/5	5	9

Lubricant properties for the reference case and the test cases were taken from [28,45,54,71-75]. The SAE-80 GL4 is a mineral lubricant with Newtonian behavior. VG-68 has the same behavior, differing only in viscosity. PAO-100 is a synthetic lubricant with viscosity close to SAE-80 GL4, but it is non-Newtonian.

Table 8. Properties of tested lubricants

Lubricant	SAE-80 GL4 (Reference Case)	VG-68	PAO-100
$\eta @ 30^\circ\text{C}$ (Pa*s)	0.153	0.095	0.1803
$a(1/\text{Pa})$ @30°C	$24 * 10^{-9}$	$25.1 * 10^{-9}$	$17.5 * 10^{-9}$
η_0 (GPa)	1.675	9	11.804
$\eta_0 \dot{U}$	10	11	11.74
η_1 (1/k)	$6.181 * 10^{-3}$	$6.5 * 10^{-3}$	$8.655 * 10^{-3}$
S_0	1	1	1.1
G (MPa)	1	1	1.5
N	1	1	0.625
α (W/m*K)	0.12	0.12	0.15

10 Parametric Analysis

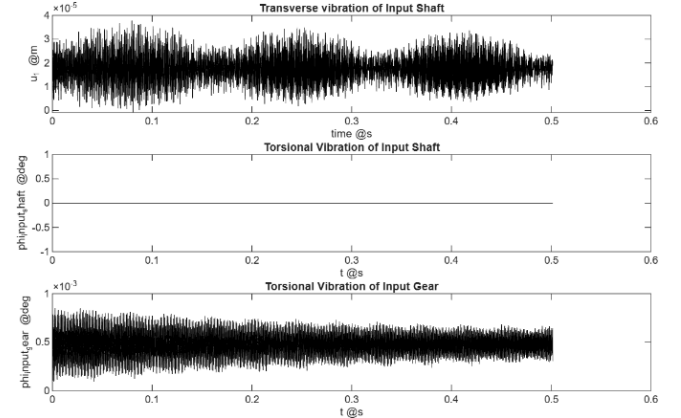
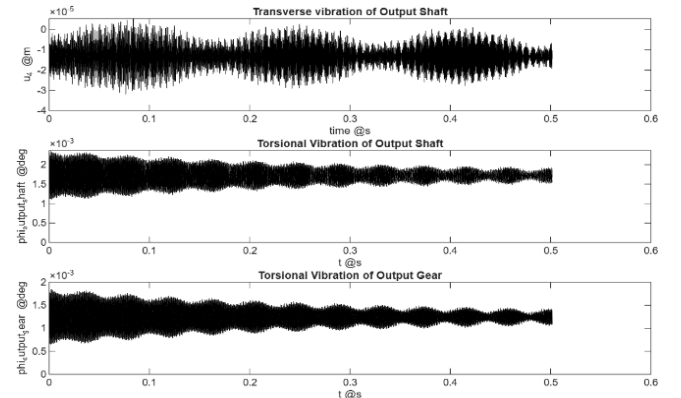
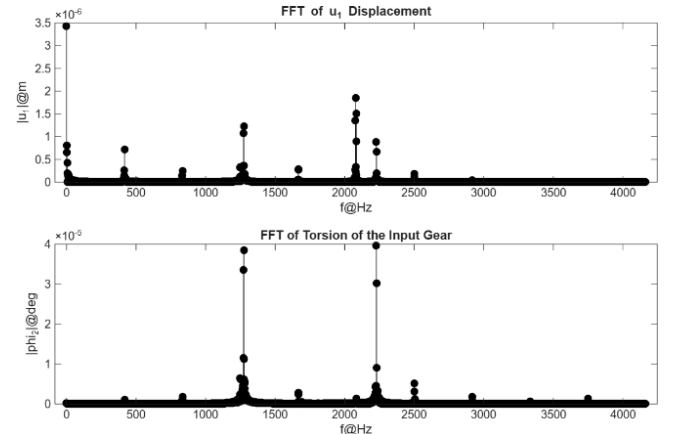
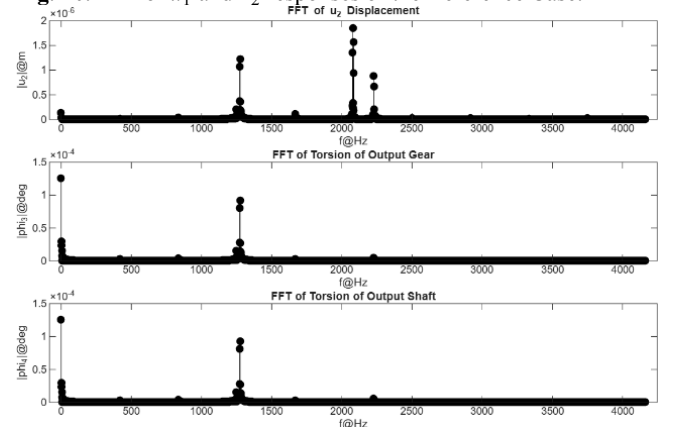
The computational speed of the model allows for testing of multiple cases. The effect of different operational conditions and system parameters, presented in the previous section were assessed relative to a reference case. In all test cases, both dynamic and tribological behavior were studied for a deeper evaluation of the tribodynamic model.

Table 6. Tested Operating Conditions

Speed (RPM)	Torque (Nm)	Bath Temperature (°C)
100	10	30 (Reference case)
500	50	50
1000 (Reference case)	100 (Reference case)	70
2000	200	90
5000	500	120

10.1 Reference Case

In the reference case the operational conditions of the geartrain are 1000RPM, 100Nm for an ambient temperature of 30°C. Isotropic tooth surfaces of $\sigma=0.321\mu\text{m}$ and SAE-80 GL-4 lubricant are used.

**Fig. 8.** Time Domain Response of u_1 , ϕ_1 and ϕ_2 of the Reference Case.**Fig. 9.** Time Domain Response of u_2 , ϕ_3 and ϕ_4 of the Reference Case.**Fig. 10.** FFT of u_1 and ϕ_2 responses of the Reference Case.**Fig. 11.** FFT of u_2 , ϕ_3 , and ϕ_4 responses of the Reference Case

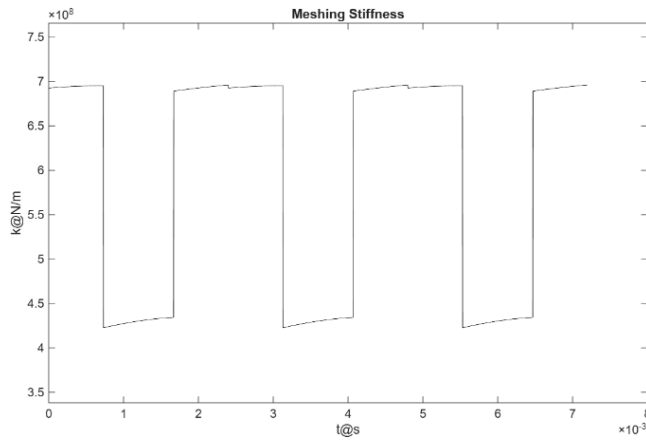


Fig. 12. Meshing Stiffness of the Reference Case.

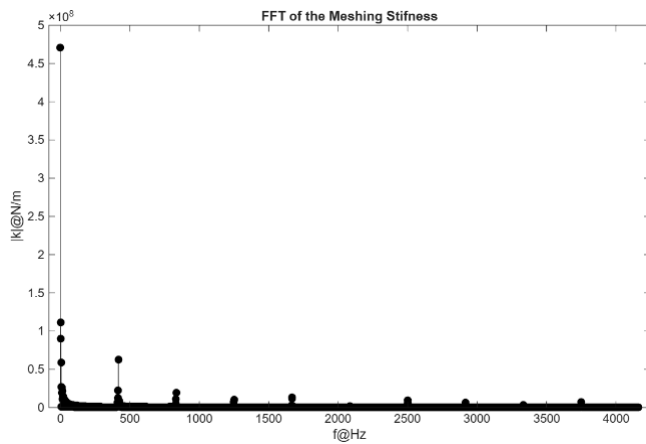


Fig. 13. FFT of the Meshing Stiffness of the Reference Case.

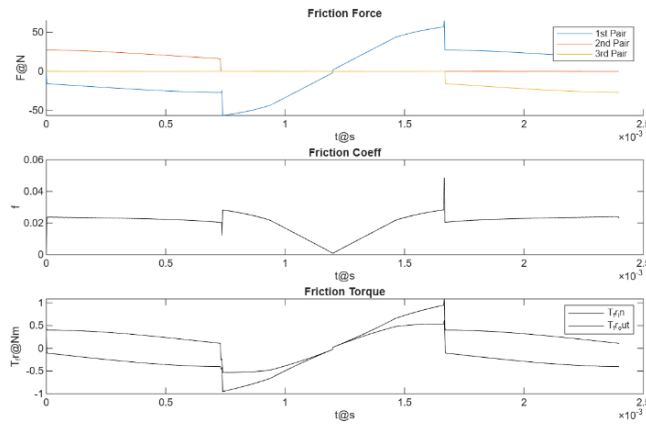


Fig. 14. Friction Force, Friction Coefficient, and Friction Torque of the Reference Case.

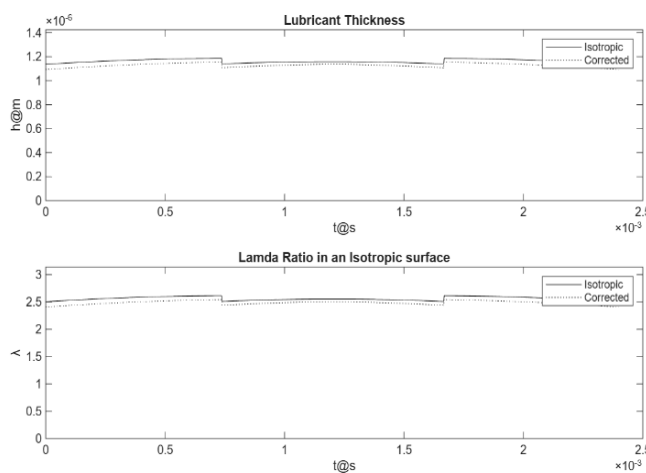


Fig. 15. Lubricant Thickness and Lambda Ratio of the Reference Case.

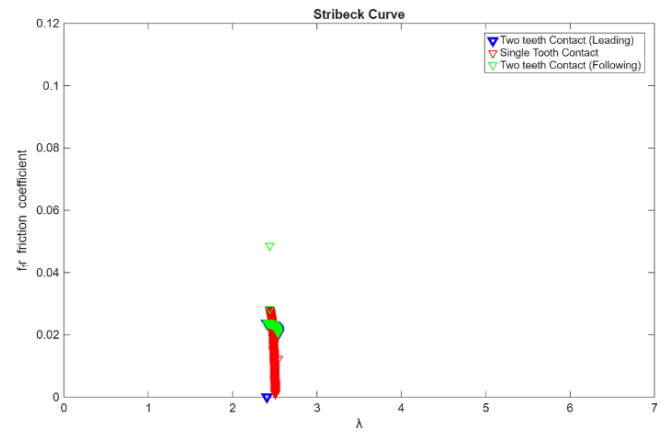


Fig. 16. Stribeck Curve of the Reference Case.

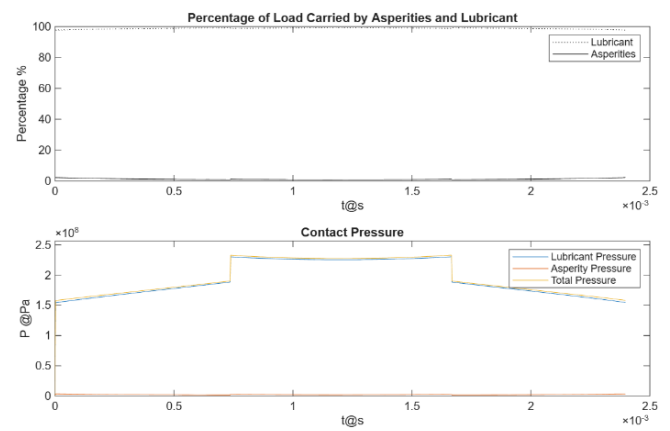


Fig. 17. Load and Pressure distribution of the Reference Case.

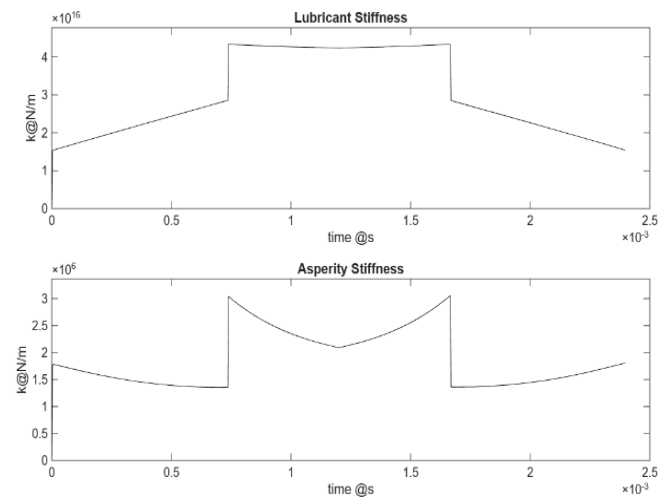


Fig. 18. Stiffness Comparison Between Lubricant and Asperities of the Reference Case.

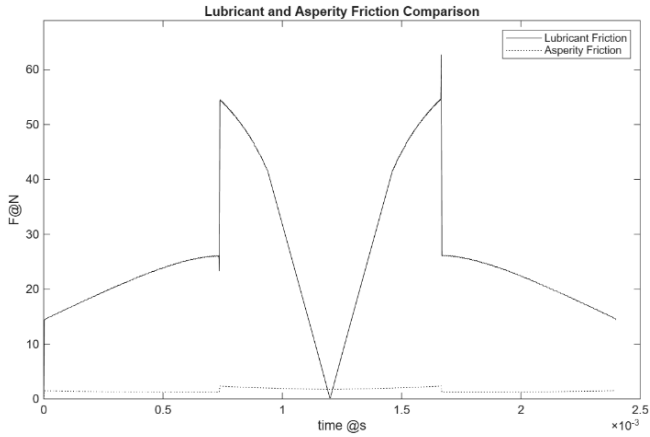


Fig. 19. Friction Comparison Between Lubricant and Asperities of the Reference Case.

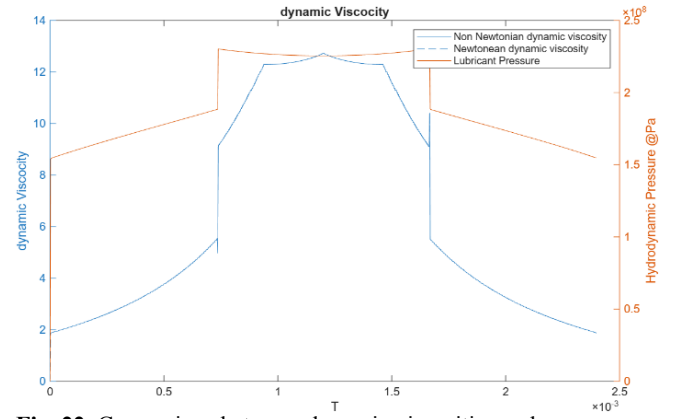


Fig. 22. Comparison between dynamic viscosities and Lubricant pressure of the Reference Case.

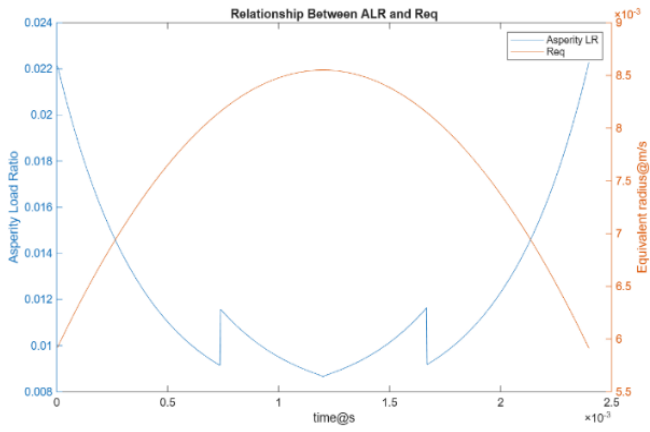


Fig. 20. Comparison Between Asperity Load Ratio and Radius of Curvature of the Reference Case.

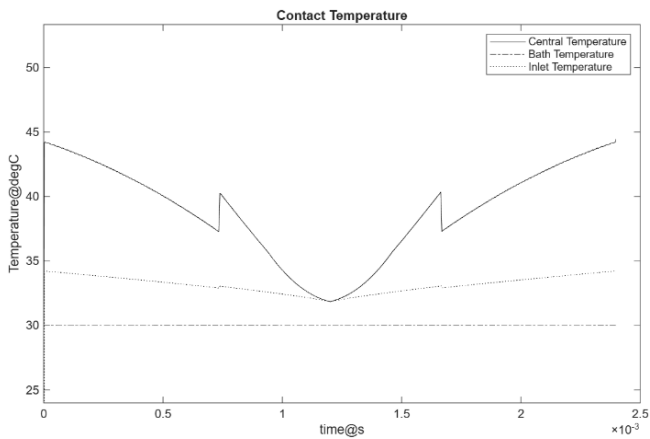


Fig. 21. Comparison Between Bath, Inlet and Contact Temperatures of the Reference Case.

10.2 Rotational Speed

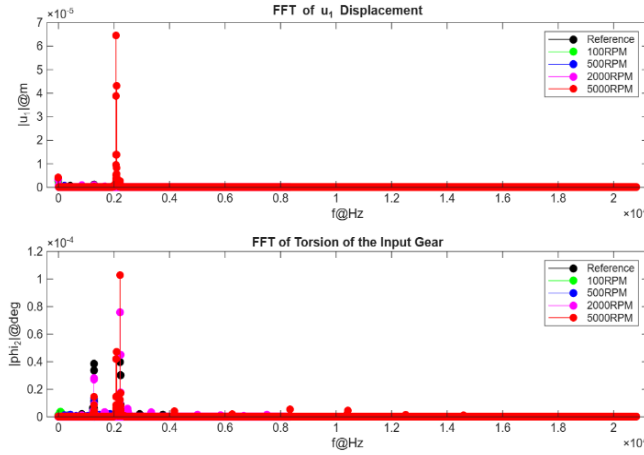


Fig. 23. FFT of u_1 and u_2 responses for different rotational speeds.

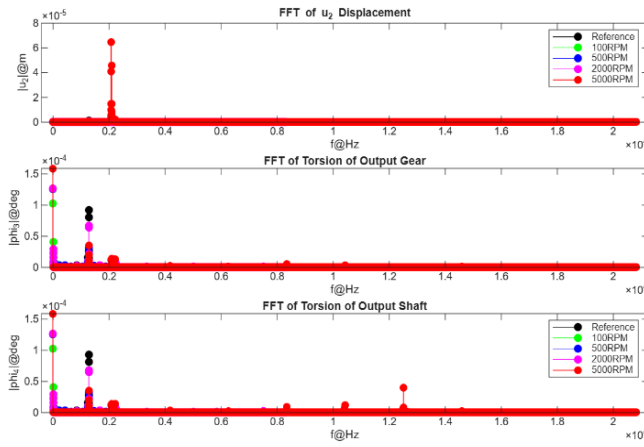


Fig. 24. FFT of u_2 , ϕ_3 , and ϕ_4 responses for different rotational speeds.

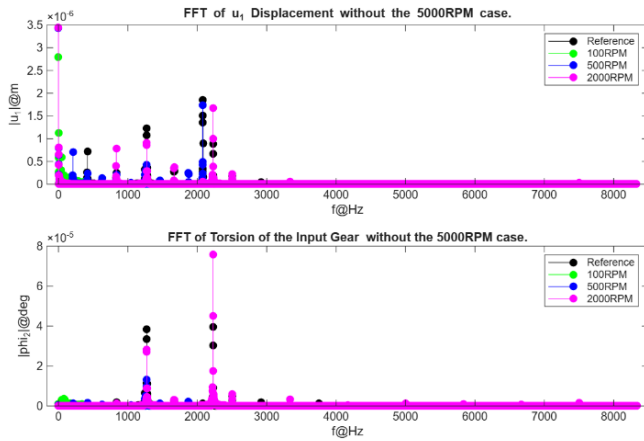


Fig. 25. FFT of u_1 and ϕ_2 responses excluding the 5000 RPM case.

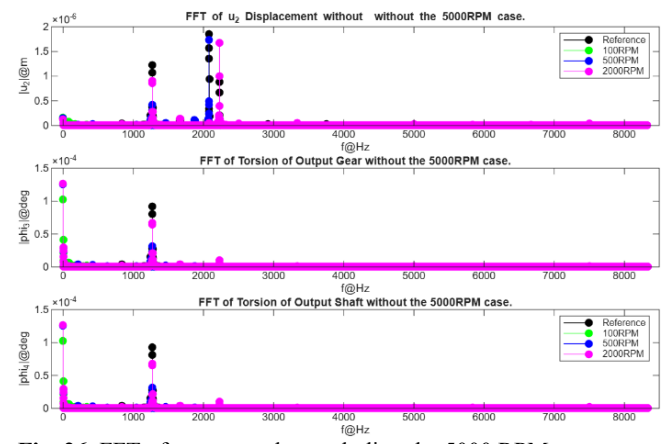


Fig. 26. FFT of u_2 , ϕ_3 , and ϕ_4 excluding the 5000 RPM case.

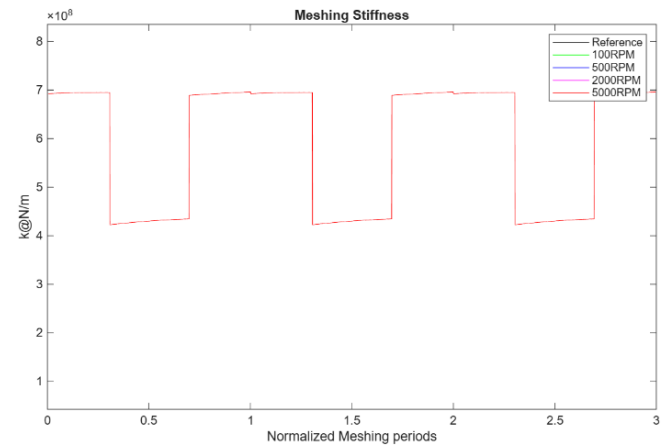


Fig. 27. Meshing Stiffness for different rotational speeds.

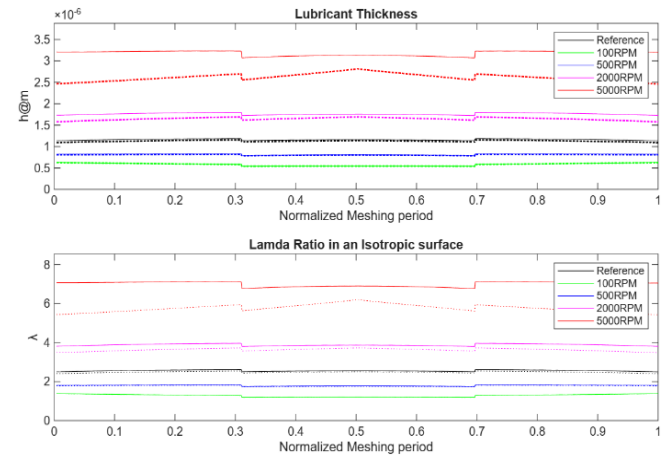


Fig. 28. Lubricant Thickness and Lambda Ratio for different rotational speeds.

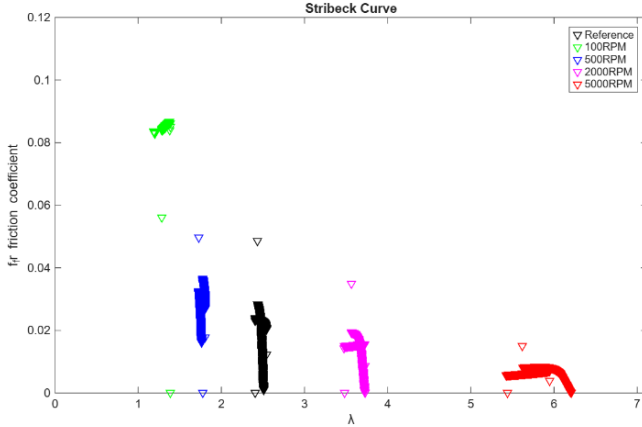


Fig. 29. Stribeck Curve for different rotational speeds.

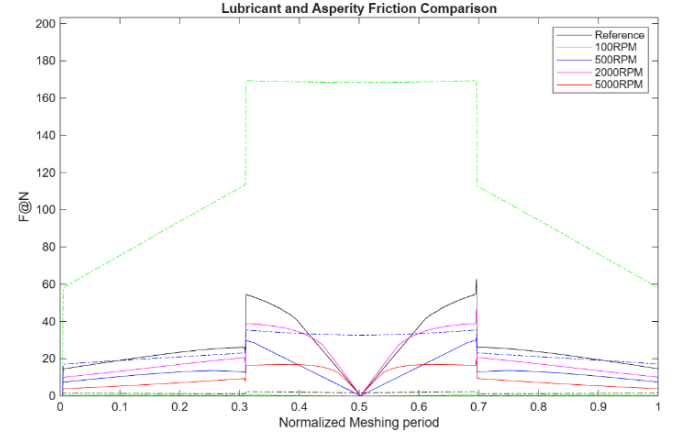


Fig. 32. Friction Comparison Between Lubricant and Asperities for different rotational speeds.

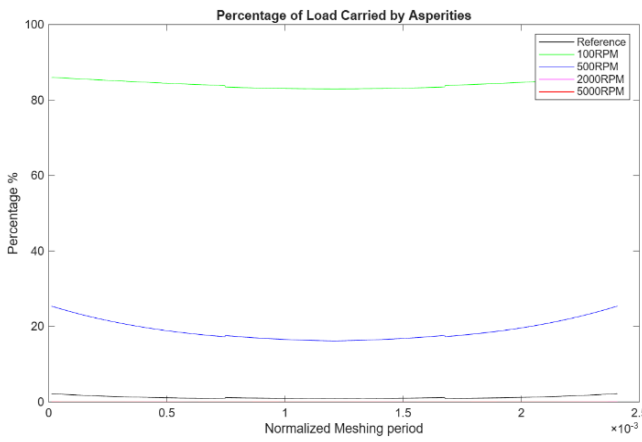


Fig. 30. Load and Pressure distribution for different rotational speeds.

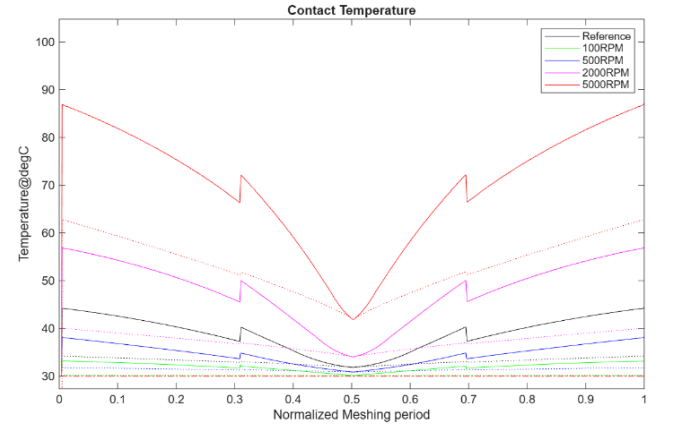


Fig. 33. Comparison Between Bath, Inlet and Contact Temperatures for different rotational speeds.

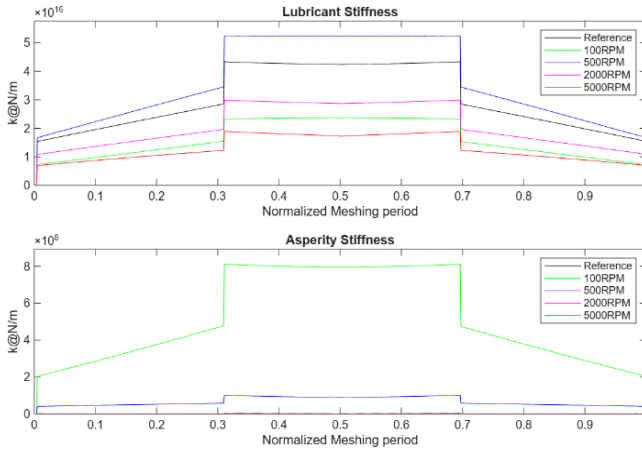


Fig. 31. Stiffness Comparison Between Lubricant and Asperities for different rotational speeds.

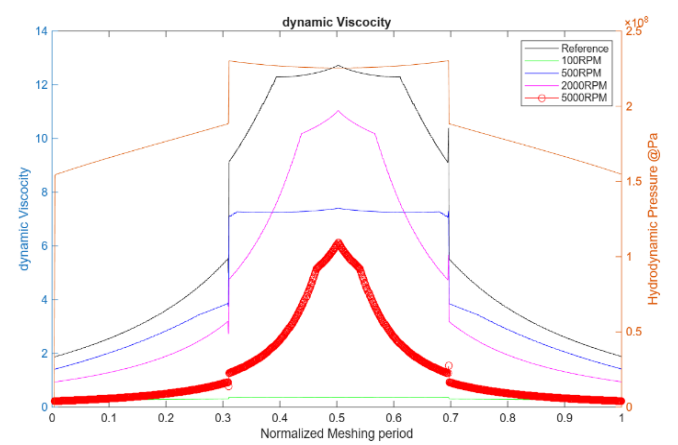


Fig. 34. Comparison between dynamic viscosities and Lubricant pressure for different rotational speeds.

When the excitation is far from the natural frequencies of the system all DoF behave the same. The exception being the 5000RPM where the magnitudes of u_1 and u_2 DoFs are significantly larger than other cases, indicating that resonance has occurred. It also shows that at around 2000Hz, the GMF of 5000RPM there is the natural frequency of the two DoFs. Further proof is that the 2000Hz signal in the FFT dominates the FFT response. When increasing the rotational speed, two opposite phenomena occur in lubricant viscosity. Lubricant pressure (P_H) increases, increasing viscosity. On the other hand, lubricant temperature increases from shearing, reducing the dynamic viscosity. These

opposing forces can be observed in the lubricant thickness diagram

Overall thickness increases but when factoring the K_T , that increase lessens with the isotropic and the corrected curves deviating more in higher rotational speeds. In over 4000RPM there is a reduction of 20% in lubricant thickness. Thus, when dealing with such speeds using the K_T correction factor is necessary. With the increase in temperature the lubricant becomes softer. Lastly, for the range of speeds chosen, all lubrication schemes can be observed.

10.3 Torque

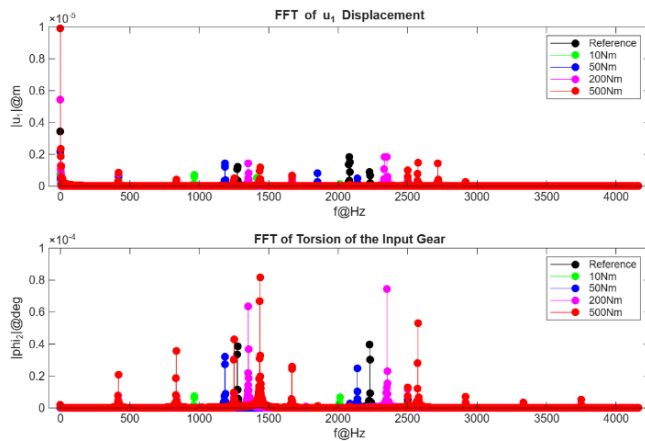


Fig. 35. FFT of u_1 and u_2 responses for different loads.

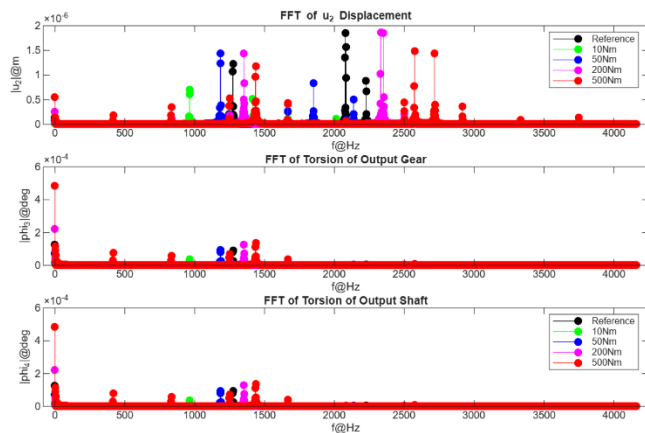


Fig. 36. FFT of u_2 , u_3 , and u_4 responses for different loads.

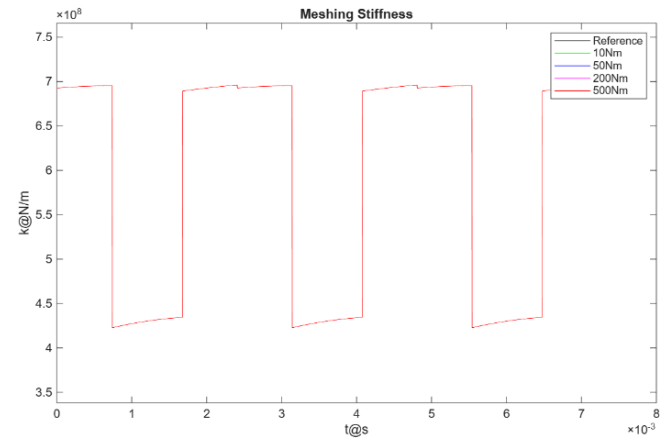


Fig. 37. Meshing Stiffness for different loads.

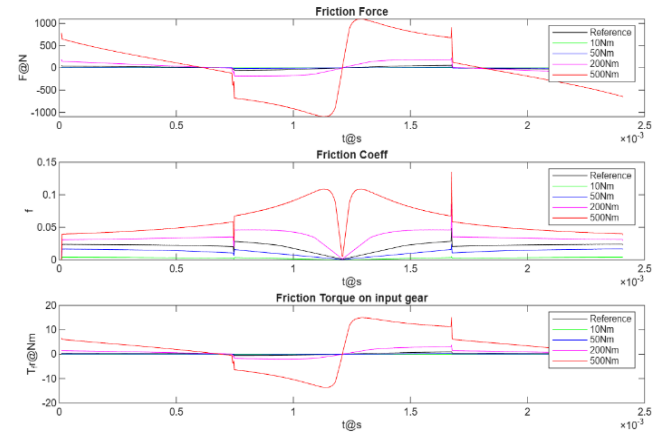


Fig. 38. Friction Force, Friction coefficient, and Friction Torque for different loads.

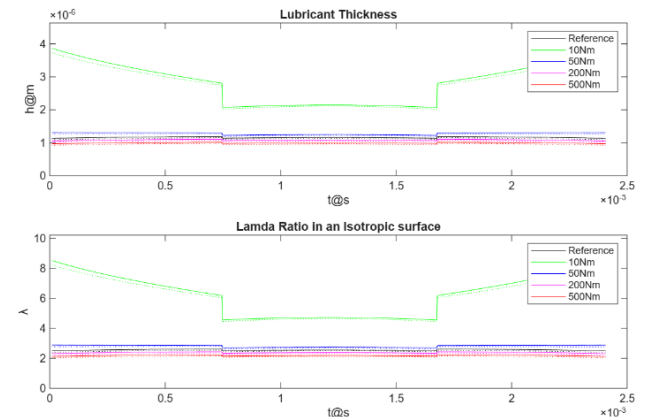


Fig. 39. Lubricant Thickness and Lamda Ratio for different loads.

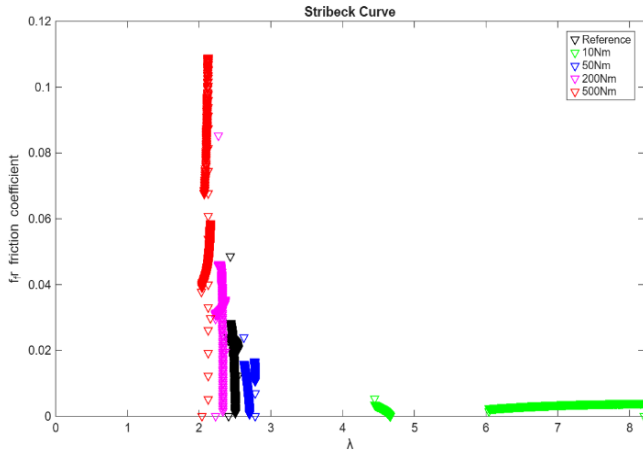


Fig. 40. Stribeck Curve for different loads.

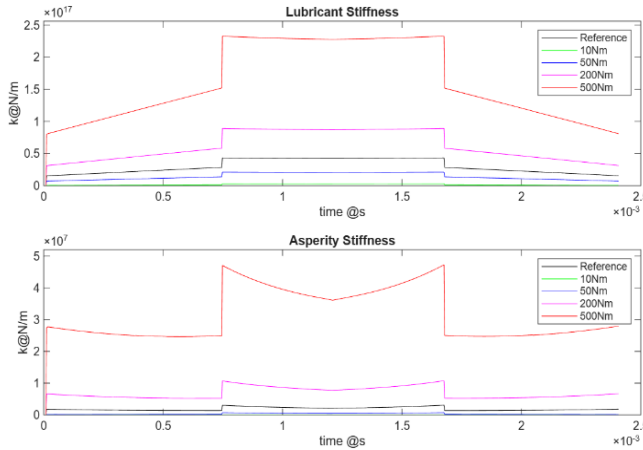


Fig. 41. Stiffness Comparison Between Lubricant and Asperities for different loads.

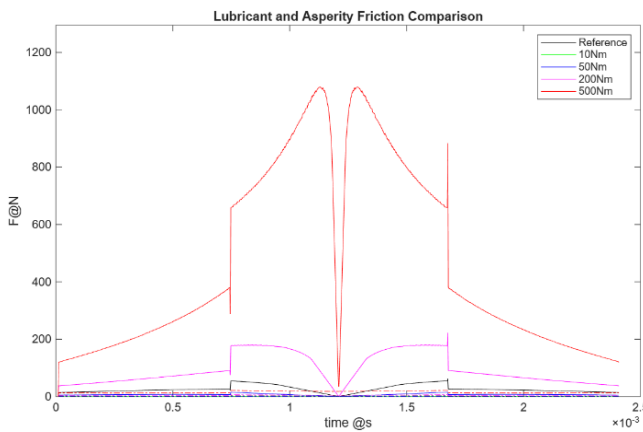


Fig. 42. Friction Comparison Between Lubricant and Asperities for different loads.

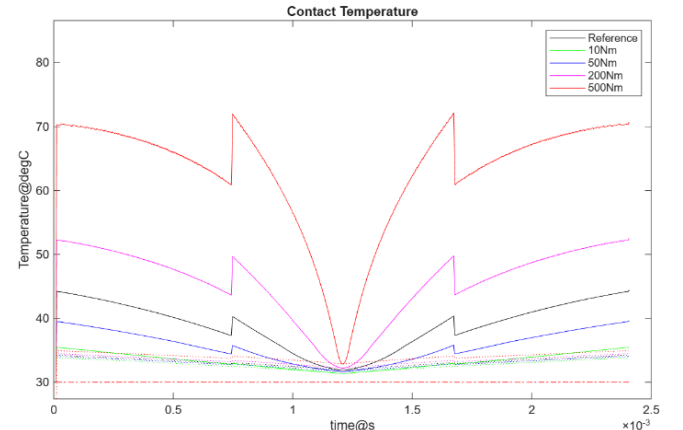


Fig. 43. Comparison Between Bath, Inlet and Contact Temperatures for different loads.

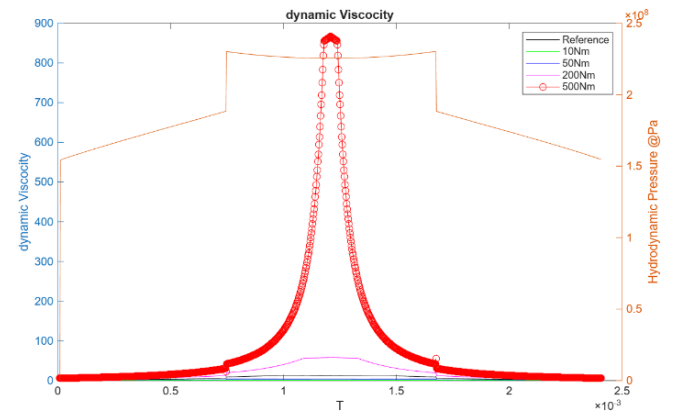


Fig. 44. Comparison between dynamic viscosities and Lubricant pressure for different loads.

Increasing the load builds higher hydrodynamic which affects mostly the lubricant's viscosity and load bearing capabilities. In very high contact forces viscosity can increase so much that the generated lubricant friction can be higher than in boundary friction. Thus, the validity of Roelands' viscosity equation must be investigated for very high hydrodynamic pressures. Asperity load, despite increasing the load, is not sufficient to alter the lubrication scheme in this case, partly due to this effect being counteracted by the lubricant's viscosity. Increasing the load reduces sensitivity to the sliding speed because the duration of the dip in lubricant friction lasts less. Before and after that dip this curve follows the trend of the contact force. When the load is very low, such as the case of 10Nm, in the lubricant thickness curve, the effect of tooth curvature can be observed. Contact temperature increases from the larger lubricant frictional forces.

10.4 Gear ratio

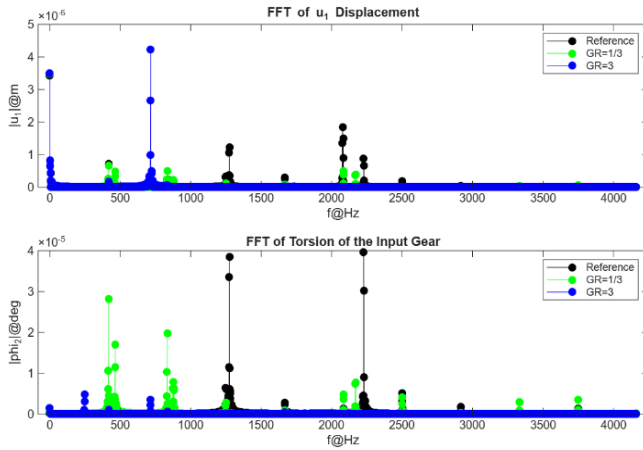


Fig. 45. FFT of u_1 and ϕ_1 responses for different gear ratios.

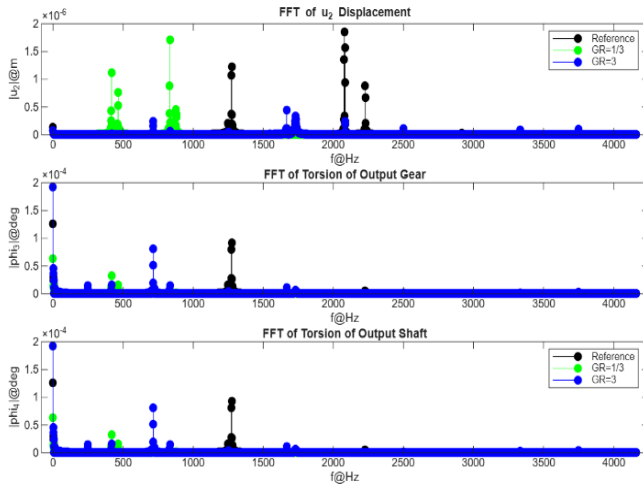


Fig. 46. FFT of u_2 , ϕ_3 , and ϕ_4 responses for different gear ratios.

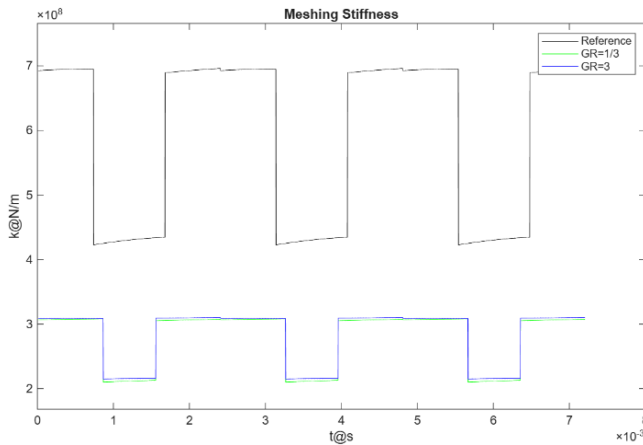


Fig. 47. Meshing Stiffness for different gear ratios.

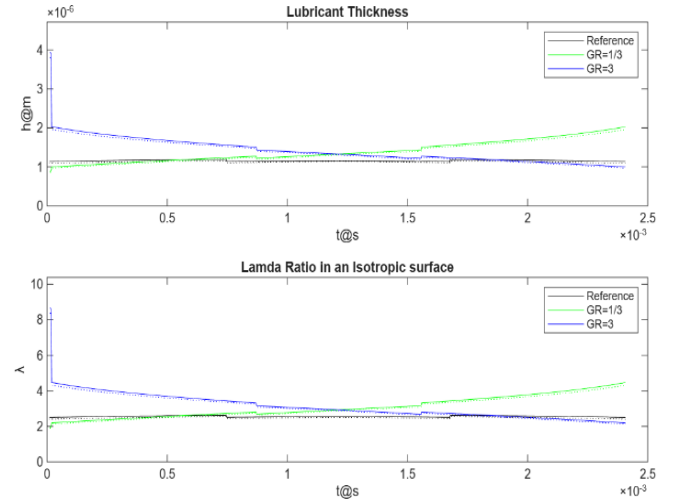


Fig. 48. Lubricant Thickness and Lamda Ratio for different gear ratios.

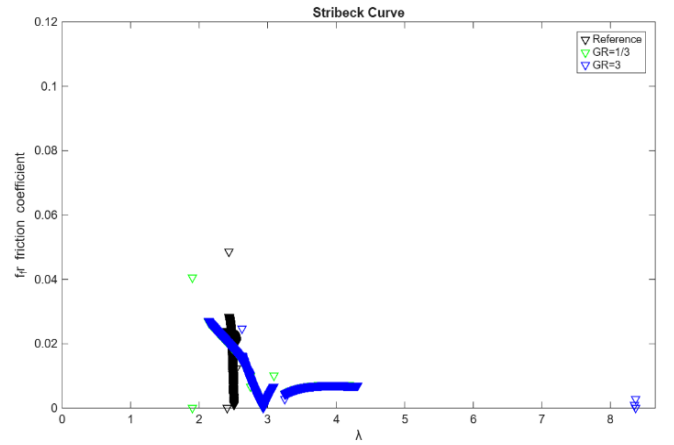


Fig. 49. Stribeck Curve for different gear ratios.

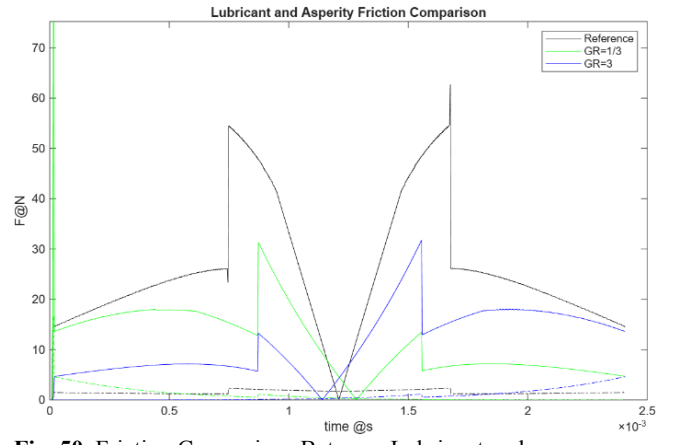


Fig. 50. Friction Comparison Between Lubricant and Asperities for different gear ratios.

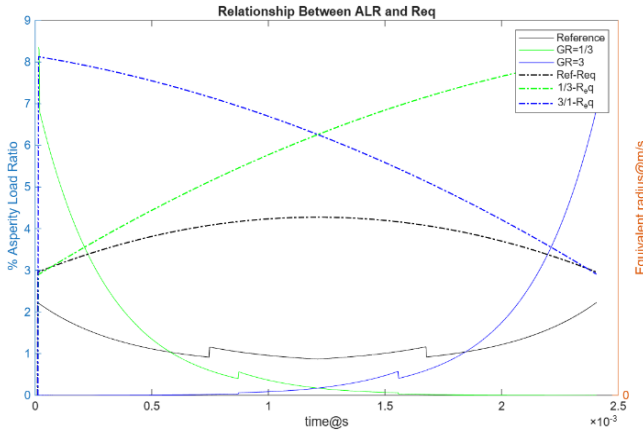


Fig. 51. Comparison Between Asperity Load Ratio and Radius of Curvature for different gear ratios.

Different gear ratios display the effect of the gears' teeth radius of curvature on the outputs of the contact model. When the ratio is not 1:1 symmetry of the radius of curvature in relation to the pitch point stops existing. On ratios larger than one R_{eq} reduces over time. Consequently, adding to all other effects on the tribological parameters, an upwards trend is observed. The opposite behavior is observed in ratios less than one. This behavior of the tribological system in inverse gear ratios can be used in validating the kinematic and dynamic equations. Observing the curves in 3:1 and 1:3 ratios they are exactly the same but with opposite starting points. In other words, they act as they have opposite ways of rotation which they have.

What is different however is the vibrational response. Firstly, the total meshing stiffness is significantly smaller, not because of the contact parameters but because the gear pair is different. The responses of the input-output gears are not reversed like in the contact model, not even opposite in sign, because of the boundary conditions. It was assumed at the start of chapter 5 that the input shaft does not vibrate. If in that system the opposite gear ratio is archived by reversing the direction of motion transmission, the input shaft turns into the output and vice versa. What is considered fixed does not change and now the “new” system has its output shaft fixed. In essence, these kinematic constraints do not allow, a 3:1 pair with opposite rotational speed to behave the same way as an “original” 1:3 pair.

10.5 Roughness

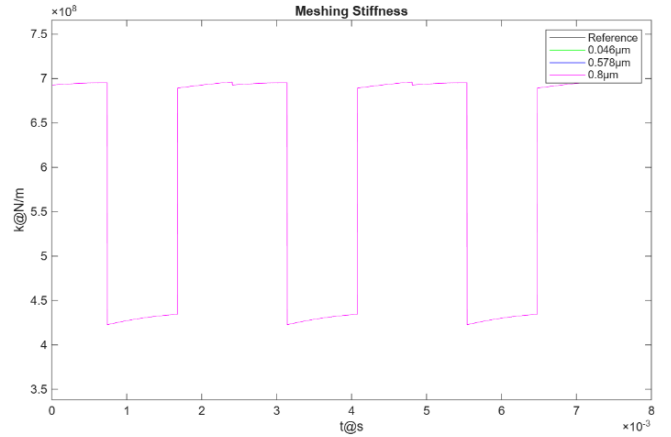


Fig. 52. Meshing Stiffness for different surface roughnesses.

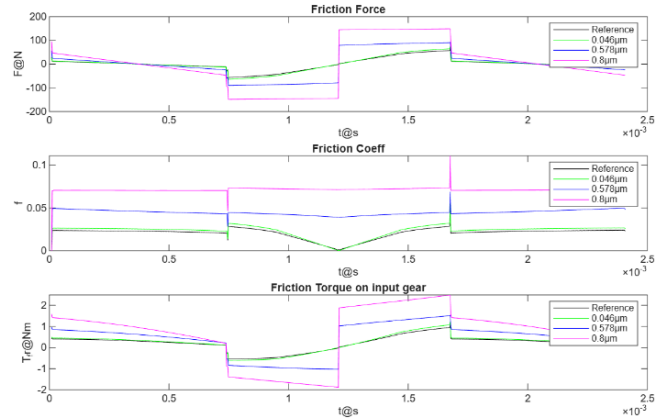


Fig. 53. Friction Force, Friction coefficient, and Friction Torque for different surface roughnesses.

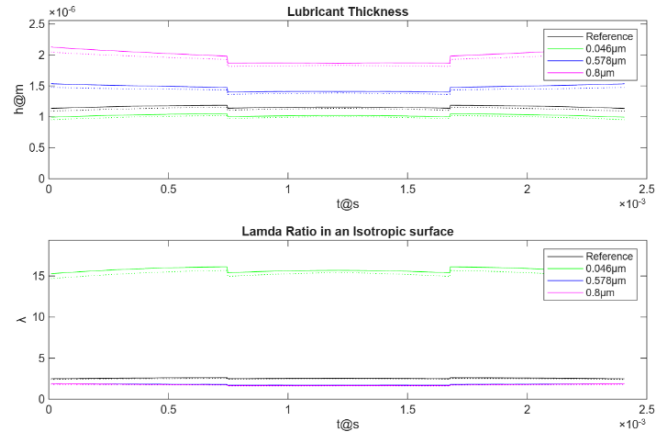


Fig. 54. Lubricant Thickness and Lamda Ratio for different surface roughnesses.

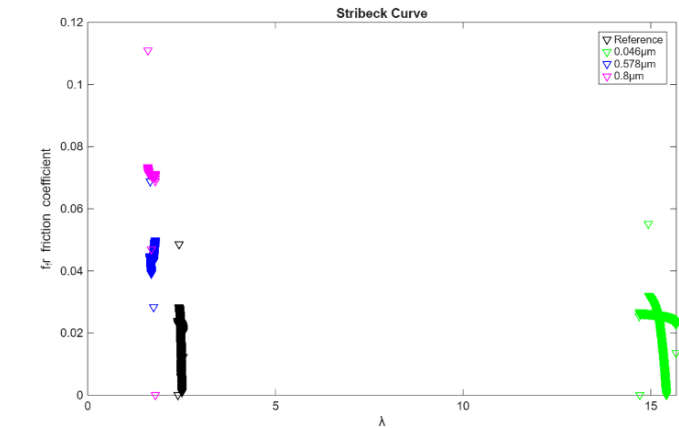


Fig. 55. Stribeck Curve for different surface roughnesses.

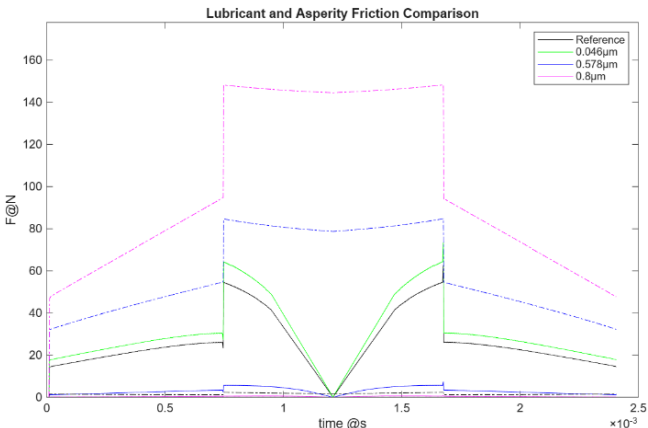


Fig. 58. Friction Comparison Between Lubricant and Asperities for different surface roughnesses.

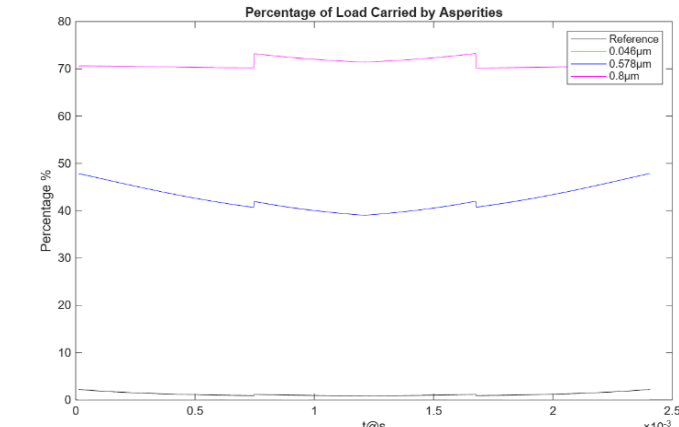


Fig. 56. Load and Pressure distribution for different surface roughnesses.

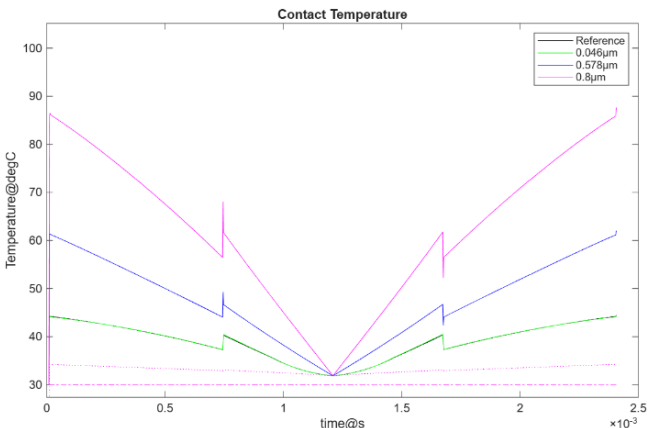


Fig. 59. Comparison Between Bath, Inlet and Contact Temperatures for different surface roughnesses.

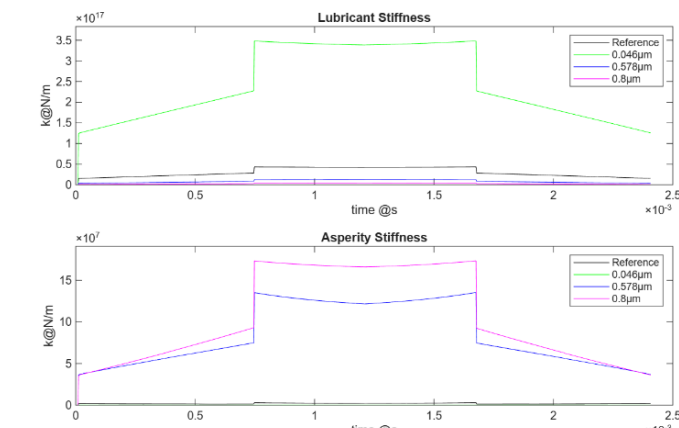


Fig. 57. Stiffness Comparison Between Lubricant and Asperities for different surface roughnesses.

Rougher surfaces lead to higher friction values and significant heat generation from flash (□□). What was not expected is that lubricant thickness was highest in the roughest surfaces, with □ and □ having values appropriate for boundary friction. This can be attributed to rougher surfaces having higher capacity for trapping/storing lubricant. Also, this shows that lubricant thickness is not by itself what leads to boundary friction, but the □ ratio and the portion of the load carried by the asperities. Contrary to intuition this does not need to be close to 100% of the contact load, a ratio of asperity to lubricant load of 60:40 was enough for the friction to be entirely generated by the asperities.

10.6 Lubricant

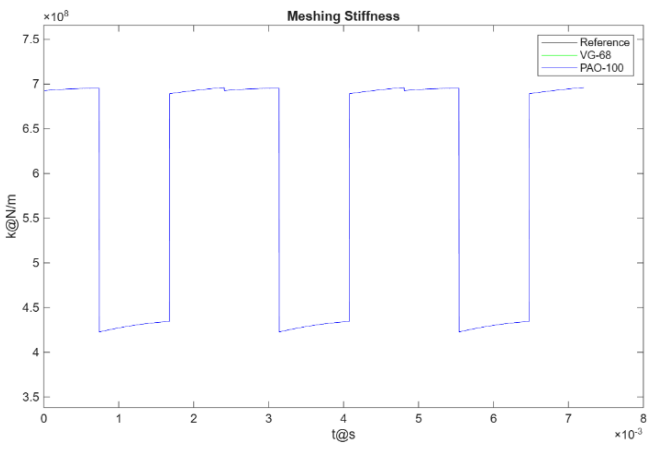
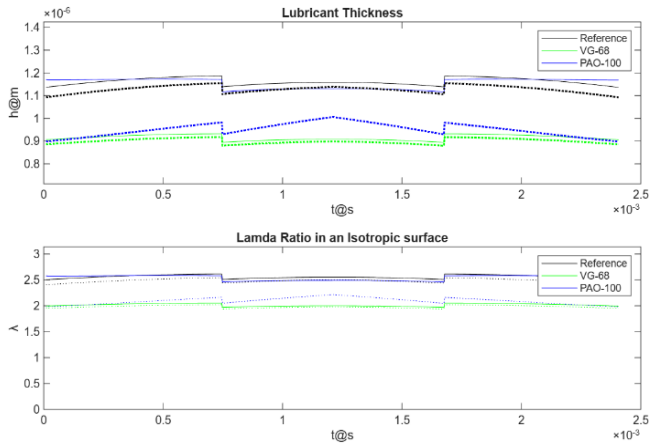
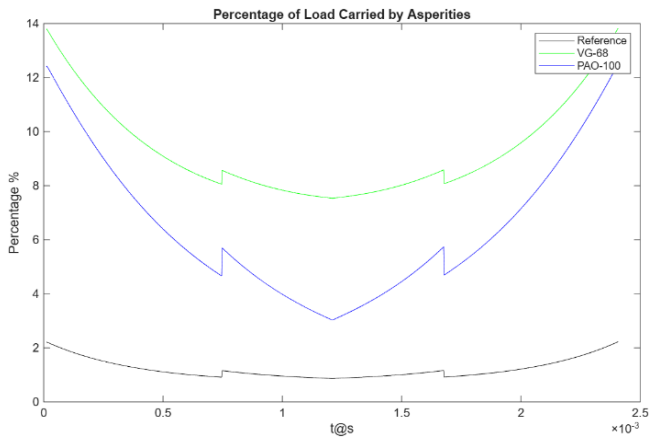
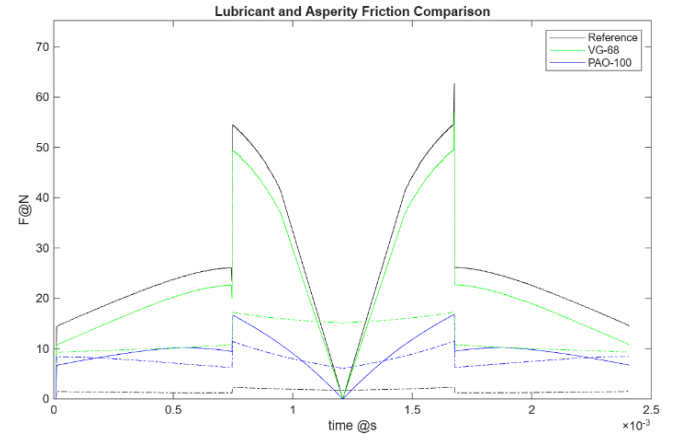
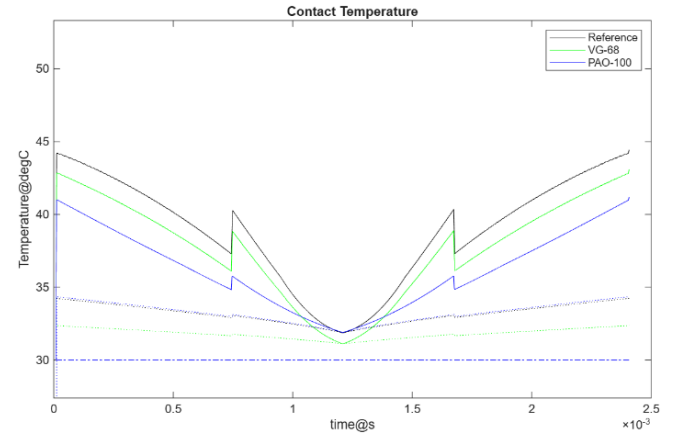
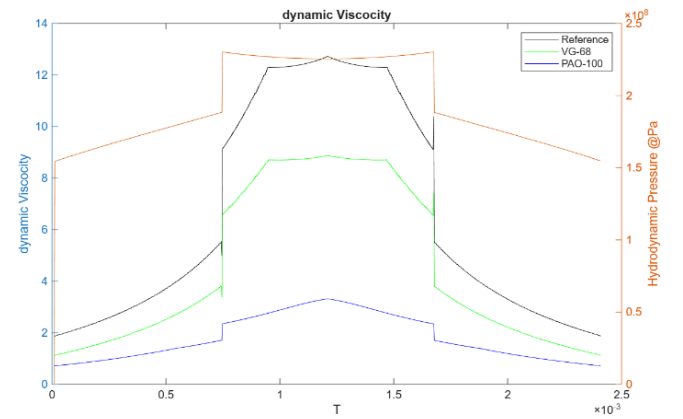


Fig. 60. Meshing Stiffness for different lubricants.**Fig. 61.** Lubricant Thickness and Lamda Ratio for different lubricants.**Fig. 62.** Load and Pressure distribution for different lubricants.**Fig. 63.** Friction Comparison Between Lubricant and Asperities for different lubricants.**Fig. 64.** Comparison Between Bath, Inlet and Contact Temperatures for different lubricants.**Fig. 65.** Comparison between dynamic viscosities and Lubricant pressure for different lubricants.

Lubricant compressibility does not affect the Meshing stiffness, since in all three cases lubricant stiffness is significantly higher than the gears' stiffness.

Both lower viscosity and shear thinning behavior lead to asperities carrying a higher portion of the load. Despite that, there is a smaller increase in contact temperature compared to the reference case. Lower viscosity creates a smaller lubricant frictional force and in turn a smaller increase in $\square_{lube-mech}$. The increase in flash temperature is not enough to counteract this phenomenon. When dealing with lubricants of the same Newtonian viscosity primarily controls friction from controlling the load capacity of the lubricant. Total

friction of the second lubricant is larger than in the reference case. In the first case, almost all the load is carried by the lubricant and friction of asperities is negligible. In the case of the VG-68 lubricant, due to smaller viscosity the lubricant cannot support the load as well as the SAE – 80 and even if viscosity is less, thickness is just as reduced and lubricant friction is almost the same. Additionally, since asperities now carry more load, for the same lubricant friction there is extra friction from that part.

For the case of the Non-Newtonian Lubricant, even at relatively low rotational speeds the shear thinning lubricant had 20% lower thickness compared to the reference case, even if its initial viscosity was slightly higher ($0.18 \text{ Pa}\cdot\text{s}$ to $0.15 \text{ Pa}\cdot\text{s}$). The gap is expected to increase with higher rotational speeds given the increased shear rate. K_{ST} cannot be neglected. Despite the lower thickness (but larger than the VG-68), this time the reduction in viscosity is more dramatic, nearly a third of the Newtonian, leading to a reduction of equal measure in lubricant friction. Even though, just as in the second case, asperities produce some friction, total friction is overall reduced.

In conclusion, the main advantage observed in Synthetic (Non-Newtonian) lubricants is the decoupling of lubricant thickness to viscosity. This type of lubricant maintains sufficient lubricant film for a fraction of the viscosity having the best of both worlds.

10.7 Treatment

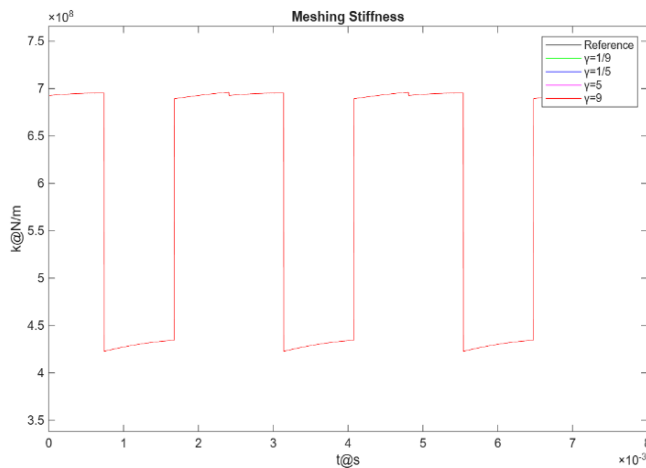


Fig. 66. Meshing Stiffness for different surface treatments.

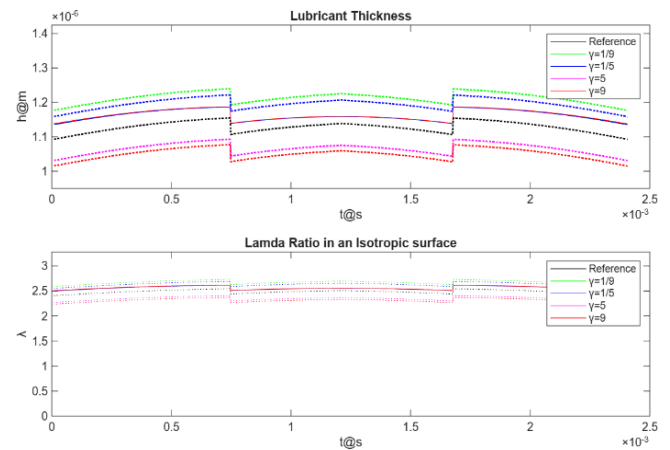


Fig. 67. Lubricant Thickness and Lamda Ratio for different surface treatments.

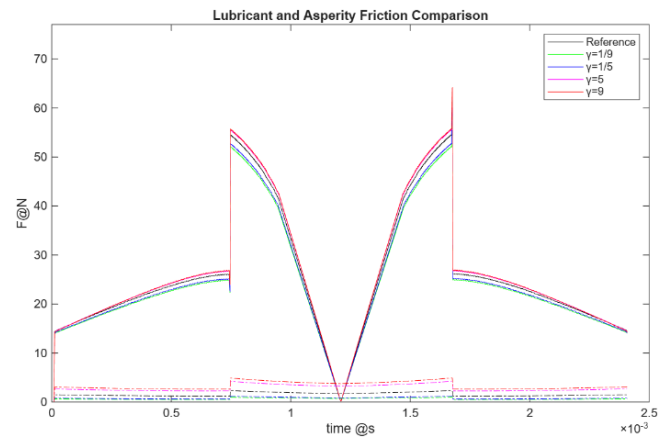


Fig. 68. Friction Comparison Between Lubricant and Asperities for different surface treatments.

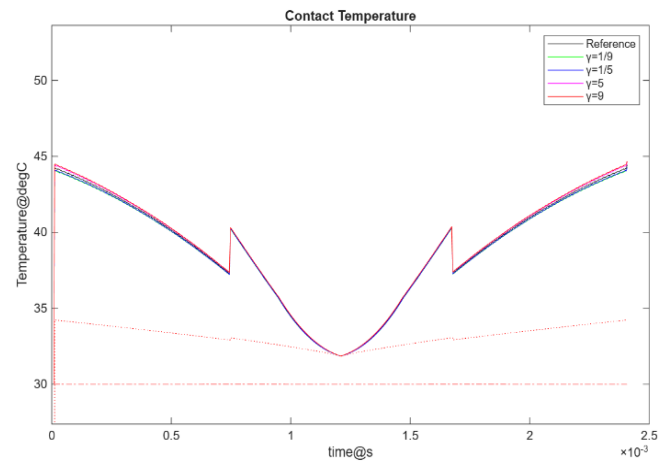


Fig. 69. Comparison Between Bath, Inlet and Contact Temperatures for different surface treatments

There is 10% difference in lubricant thickness between the reference case and each extreme case. The effect on other parameters seems insignificant for the operational conditions used in the reference case. However, if conditions push the system closer to the Boundary lubrication the effect of the surface directionality will be more pronounced. These results also demonstrate the importance of the parameters analyzed so far. The surface pattern parameters affect primarily lubricant thickness, the overall tribo-system converges to similar states for varying lubricant thicknesses.

10.8 Ambient Temperature

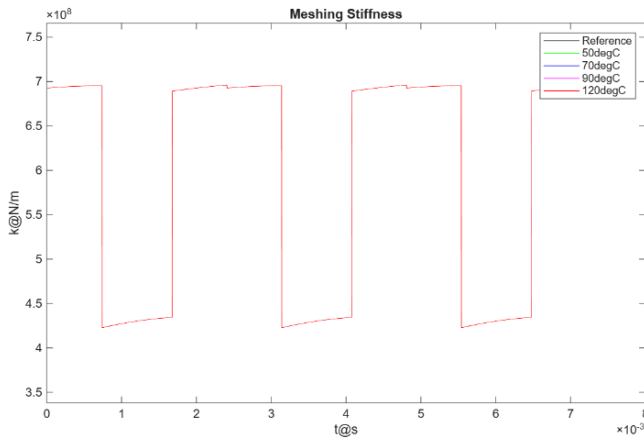


Fig. 70. Meshing Stiffness for different bath temperatures.

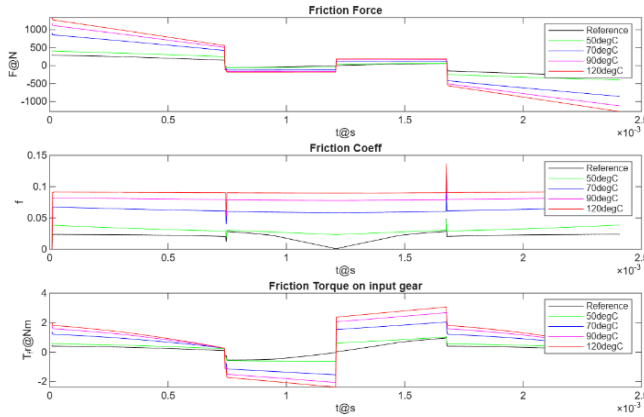


Fig. 71. Friction Force, Friction coefficient, and Friction Torque for different bath temperatures.

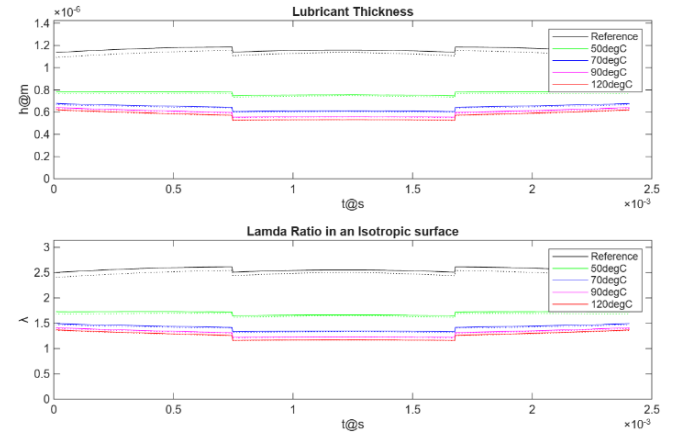


Fig. 72. Lubricant Thickness and Lambda Ratio for different bath temperatures.

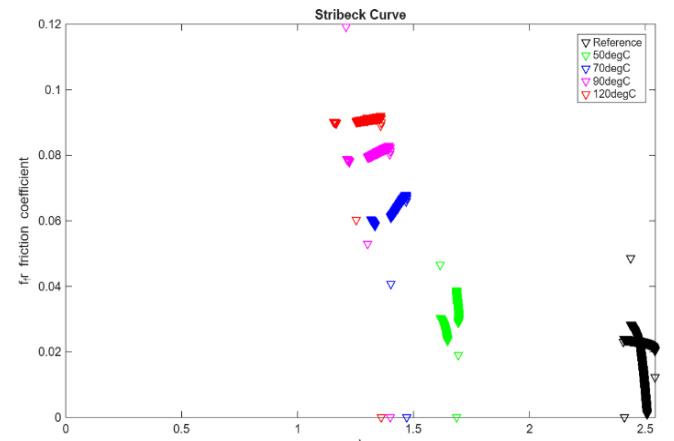


Fig. 73. Stribeck Curve for different bath temperatures.

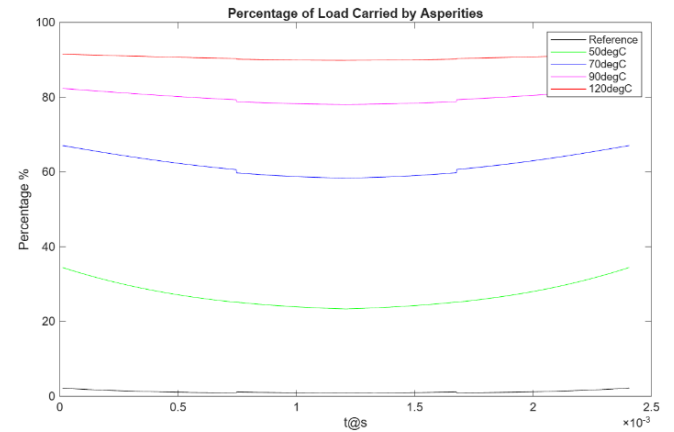


Fig. 74. Load and Pressure distribution for different bath temperatures.

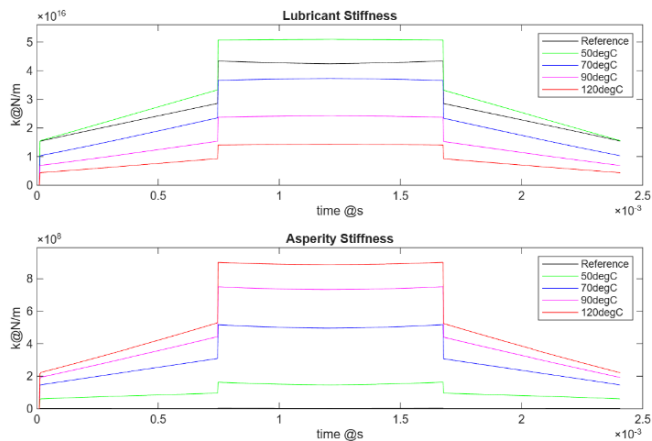


Fig. 75. Stiffness Comparison Between Lubricant and Asperities for different bath temperatures.

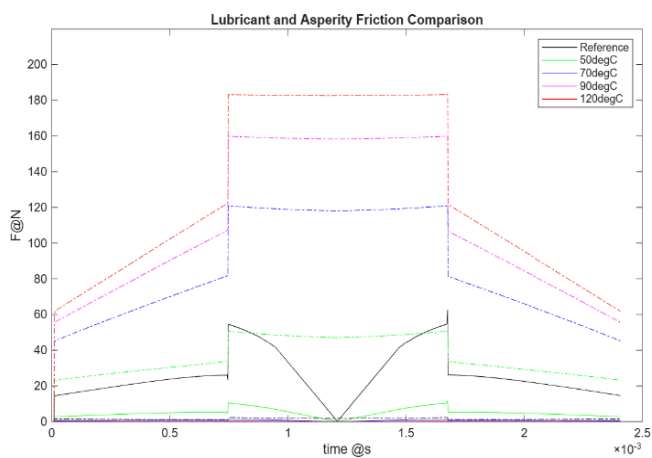


Fig. 76. Friction Comparison Between Lubricant and Asperities for different bath temperatures.

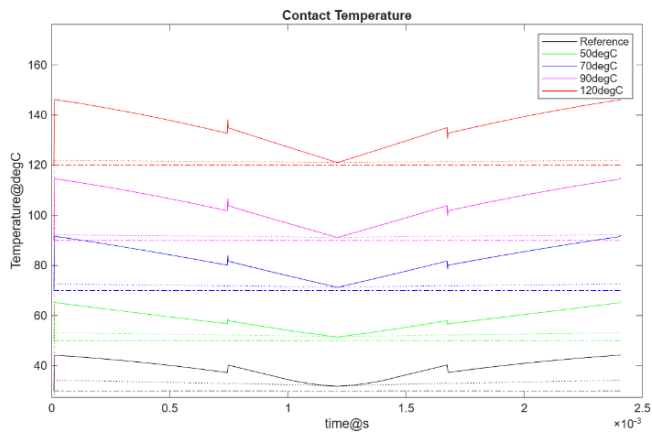


Fig. 77. Comparison Between Bath, Inlet and Contact Temperatures for different bath temperatures.

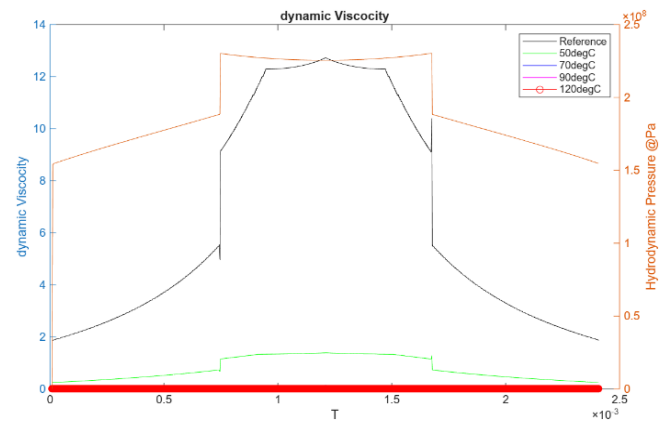


Fig. 78. Comparison between dynamic viscosities and Lubricant pressure for different bath temperatures.

The model is extremely sensitive to changes in bath temperature. Just a 20 °C increase from 30 °C to 50 °C is enough to increase the load carried by the asperities from 6 percent to 50. The system in essence goes from EHL lubrication straight to Boundary lubrication. It is observed that after 90 °C the lubricant film completely collapses. Also, the gap between inlet and central temperature widens with increasing temperature.

11 Conclusion

Following the model's results conclusions can be drawn regarding the dynamic behavior of the system. The transverse vibrations of the shafts are of equal magnitude but in opposite directions in 1:1 Gear-Ratio. Observing the decay of magnitude in vibrations after introducing damping to the system, gear meshing damping is necessary for accurate prediction of vibrational behavior. The FFT responses indicate excitations only of $nxGMF$ frequencies which is representative of healthy gears, excluding natural frequency excitation.

The dynamic response is completely unaffected by the contact parameters and primarily controlled by gear stiffness. The meshing stiffness shows a maximum variance of around 5% for the different contact parameters and bath temperature. The contact stiffness is two orders larger than gear stiffness and as these are connected in a series to synthesize the meshing stiffness, the former's effect is not significant. The difference in orders between gear and contact stiffness stems from the lubricant's stiffness. This is observed even in boundary lubrication. Asperity stiffness, the surface properties in essence, affects the total meshing stiffness only when lubrication is not used, otherwise can be omitted. It is because these two stiffnesses are connected in parallel. Neglecting the contact stiffness was tested in a secondary test code and proven correct. Total friction, however, is controlled by both asperities and lubricant, depending on the conditions. Torque generated by the frictional force is at the worst case of boundary lubrication, 2% of the input torque. Since the effect of the contact parameters is small, separating the contact and dynamic models proved correct. Knowing the parts of the model that can be omitted, further simplifications can be made without introducing any measurable errors, speeding up the algorithm even more.

Regarding the contact model, gears can operate in all lubrication schemes depending on the operating conditions, lubricant and surface characteristics. As expected, these variations remain bounded between two extremes, hydrodynamic lubrication and boundary lubrication. The responses in Elastohydrodynamic lubrication are in between the two extremes in both behavior and magnitude as it should. This agrees with previous works and theory. Friction is generated mostly by asperities in EHL and BL schemes and the upper ceiling of the friction is the value $f_c * F_{\text{mesh}}$ where all load is carried by the asperities. The only exception is found in the case of 500Nm loading torque. Even though the system operated in the hydrodynamic region viscosity was so high from the hydrodynamic pressure that friction exceeded the one produced in BL.

Changes in contact force in single/double pair regions can be seen in all outputs. It is more pronounced in the dynamic viscosity and the friction force diagrams. Lubricant thickness is less sensitive to the contact force. Friction from the lubricant is both controlled by the contact force, and the sliding speed. When the sliding speed drops below a certain threshold, lubricant friction drops dramatically reaching zero. Also, the temperature at the center of the contact is mostly the product of sliding speed. Secondary role the contact force plays. The equivalent Radius R_{eq} controls how much load asperities carry, with more curved surfaces carrying a lower amount.

All in all, the parameters which the contact model showed the greatest sensitivity are roughness, rotational speed and ambient temperature. All three affect directly the portion of the load carried by the asperities. Roughness controls the λ ratio. Rotational speed affects the thickness and viscosity, the second in an indirect manner. Viscosity is extremely sensitive to temperature. Even though contact force changes can be observed in the curves overall they do not affect to the same extent as the previous parameters. Gear geometry effects can be seen from the equivalent radius of curvature. This parameter affects the overall trend of the outputs as explained in section 10.4.

Even though it is proven that steady-state models the contact model can be omitted, the same cannot be said when operational life is considered. Wear affects vibrations greatly with different types of wear causing a plethora of issues [1]. The portion of the load carried by the asperities directly controls wear [76]. Researchers have already started utilizing analytical models to predict wear and how worn gears behave [77]. This model can be the basis of such work in the future.

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A unified theory for nonparallel axis gears conjugate to involute cylindrical gears including face gears (Consideration regarding gear ratio)

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Abstract. The axis arrangement of gears that mesh conjugately with involute cylindrical gears can be freely set, yielding infinitely many configurations and corresponding tooth-flank geometries. We investigate a unified theory for gears that mesh conjugately with involute cylindrical gears. In this theory, the conjugate gear meshes along a straight line of action, and the contact point travels at a constant velocity along the line of action. The continuous set of these lines of action forms a meshing contact surface of action. The intersection of this surface with the tooth surface of the conjugate gear defines a simultaneous meshing contact line. This geometric framework explains the meshing of all gears conjugate to involute cylindrical gears, including face gears. In our unified theory, the shaft angle and the offset crossed angle are parameters in a common coordinate system for conjugate gears. In this study, we focus on conjugate gears obtained using only the shaft angle, without the offset crossed angle. For parallel axes, the conjugate gear is an involute cylindrical gear. When a shaft angle is introduced between two shafts, the conjugate gear becomes an outer bevel-shaped gear; this is not a bevel gear, but a gear that meshes with an involute cylindrical gear at a prescribed shaft angle. When the shaft angle is tilted vertically, it becomes a face gear, which has been in practical use for many years. With further tilting, an inner bevel-shaped gear is formed, and when the shaft angle reaches 360° , the conjugate gear becomes an involute internal gear. This paper examines the effective tooth surface region of gears conjugate to involute cylindrical gears and presents the limiting values of the gear ratio and their practical range.

1 Introduction

Face gears are widely known as gears that mesh with involute cylindrical gears (hereafter referred to as pinion). In the past, it was used in the power transmission systems of helicopters. The most well-known example is the speed increase gear mechanism used in fishing spinning reels. [1][2] In recent years, it has also been used as a substitute for bevel gears and hypoid gears in orthogonal geared motors. Face gears are among the gears that always attracts attention.

Unlike bevel gears and hypoid gears, face gears have the advantage of not requiring axial positioning of the pinion. Furthermore, because they are conjugate to involute cylindrical gears, the meshing is line contact, allowing for constant velocity transmission of the rotational angle. Regarding face gear, several studies have been conducted in the past on theory, manufacturing, and practical application. Methods for determining the tooth surface geometry of face gears using mathematical techniques are known (Litvin, 2005; Gunbara, 2007)[3][4]. In these studies, a rotation angle is applied to the pinion in the simulation, and the tooth surface of the face gear is generated from the envelope

surface. These require advanced mathematical solutions and cannot be considered generalized versions. Regarding the design of face gears, there are no guidelines, such as simplified formulas or charts, for setting optimal parameters.

The authors have been conducting research on machining face gears.[5][6] In the process, the authors realized that the pinion and face gear have straight lines of action, and that meshing mechanism is established solely by the lines of action. Furthermore, we found that this line of action is continuous, and "a meshing contact surface of action" exists. Moreover, we found that by using this meshing contact surface of action, the meshing between the pinion and face gear can be clearly defined, and that the effective tooth surface coordinates and simultaneous contact lines can be easily obtained. [7] By using the method we propose, the coordinates of the effective tooth surfaces of the face gear can be determined relatively easily, without requiring complex mathematical processing. The authors are conducting research based on the idea that further developing this method will enable the creation of metrics that facilitate the design of face gear.

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Furthermore, the authors realized that, in addition to face gears, there are various other gears that mesh conjugately with involute cylindrical gears. These gear groups have not yet been widely put into practical use because a clear geometric design analysis method has not been clearly established. However, it appears they can exhibit sufficient performance as gears.

The authors reported that the meshing theory based on the line of action used for face gears is applicable to all gears conjugate to involute cylindrical gears. [8][9]

In this context, "conjugate gears" refers to gears that mesh with involute cylindrical gears through line contact. The conjugate gear has a tooth surface generated by the tooth surface of an involute cylindrical gear and meshes via line contact only with an involute cylindrical gear having a specific number of teeth.

We position this as a unified theory of gear groups conjugated to involute cylindrical gears. Face gears are a type of gear group that meshes conjugate with involute cylindrical gears.

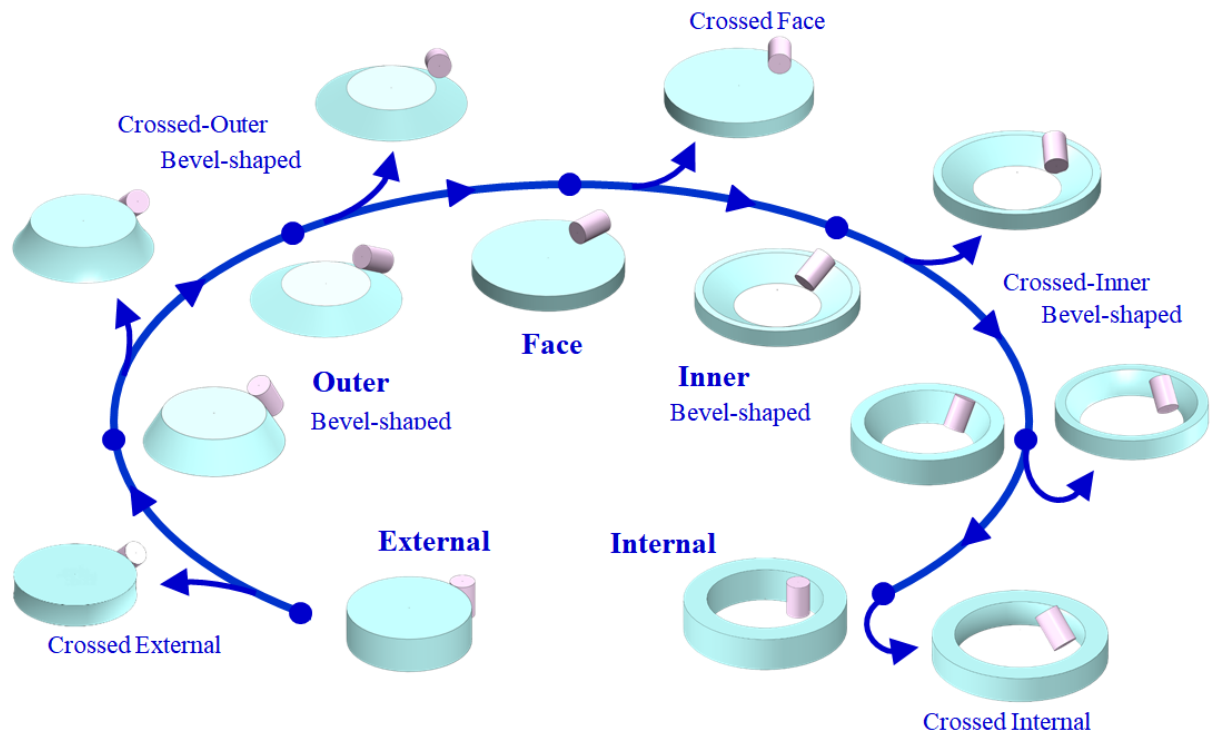


Fig. 1. Gears conjugate with involute cylindrical gear. The pink cylinders are image of the involute cylindrical gear. Light blue means conjugate gears.



Fig. 2. Conjugate gear prototype example

Figure . 1 shows a conceptual system diagram of a cylindrical gear and its conjugated gear.

For parallel axes, the conjugate gear is an involute cylindrical gear. When a shaft angle is introduced between two shafts, the conjugate gear becomes an outer bevel-shaped gear ; this is not a bevel gear, but a gear that meshes with an involute cylindrical gear at a prescribed shaft angle. When the shaft angle is tilted vertically, it becomes a face gear, which has been in practical use for many years. With further tilting, an inner bevel-shaped gear is formed, and when the shaft angle reaches 360° , the conjugate gear becomes an involute internal gear.

Figure 2 shows examples of various gears prototyped by the authors.

Based on the arrangement of the pinion's axis, gears are classified into two types ; orthogonal axis gears and crossed axis gears among face gears. In this study, we focus on a face gear with no crossed angle and investigate the effective tooth number ratio. Next, by varying the shaft angle, the correlation between the tooth number ratio and the effective tooth surface area is investigated for outer bevel-shaped and inner bevel-shaped gears.

2 Meshing contact surface of action

2.1 Coordinate system for Face Gear

The authors previously proposed a unified theory that, by using the straight line of action method, can clearly explain the meshing of all gears conjugated to an involute cylindrical gear. The coordinate system for the gear pairs used in the unified theory is given by Eq. (1).

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{O}_1 + \begin{bmatrix} \cos \Sigma & 0 & \sin \Sigma \\ \sin \Sigma \sin \sigma & \cos \sigma & -\cos \Sigma \sin \sigma \\ -\sin \Sigma \cos \sigma & \sin \sigma & \cos \Sigma \cos \sigma \end{bmatrix} \cdot \begin{bmatrix} U \\ V \\ W \end{bmatrix} \quad (1)$$

σ represents the shaft angle, and Σ represents the crossed angle.

In the coordinate system, the pinion has UVW coordinate axes, and W axis is the rotation axis of pinion. Conjugate gears can be analyzed in an XYZ coordinate system, where the Z axis is the axis of rotation.

When dealing with face gears in the coordinate system obtained from Eq. (1), the shaft angle is set to be $\sigma=90^\circ$. The crossed angle Σ is a parameter that determines the offset amount. Figure 3 shows the coordinate system used to analyze face gear in the unified theory.

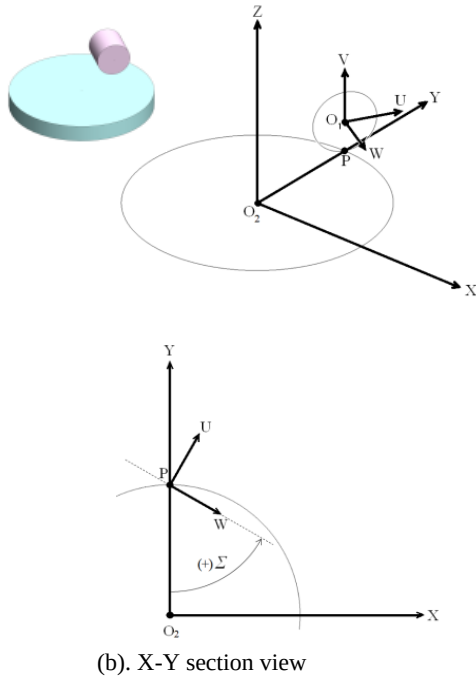


Fig. 3. Coordinate system for face gear

2.2 Mashing contact line of action

The face gear and pinion have straight mashing lines of action. The point of contact lies on the line of action of that straight line and moves at a constant speed. A face gear and a pinion are a pair of gears that mesh at a constant speed to transmit rotation.

Figure 4 shows the meshing point progressing along a straight line of action. The gray represents the involute tooth surface of the pinion, and the blue represents the tooth surface of the face gear. The red line represents the meshing line of action, and the contact point C moves at a constant velocity within this line of action.

The meshing of the face gear and pinion can be easily understood and represented by recognizing straight lines that exist in three-dimensional space.

Figure 5(a) shows a simulation of the meshing between the face gear and the pinion. The pink color represents the involute tooth surface of the pinion. The blue areas represent the tooth surfaces of the face gear, which were generated from the tooth surfaces of the pinion gear using numerical analysis. Figure 5(b) is a magnified view of one side of the tooth surface of the face gear shown in Fig. 5(a).

The collection of red lines represents the collection of meshing contact lines of action.

The meshing contact lines of action exist continuously in space and form a curved surface. We named this surface the meshing contact surface of action. The green curve in Fig. 5(c) represents the simultaneous contact line between the pinion and the face gear. This is also the boundary line at the moment when the tooth surface of the rotating blue face gear intersects with the red meshing contact surface of action.

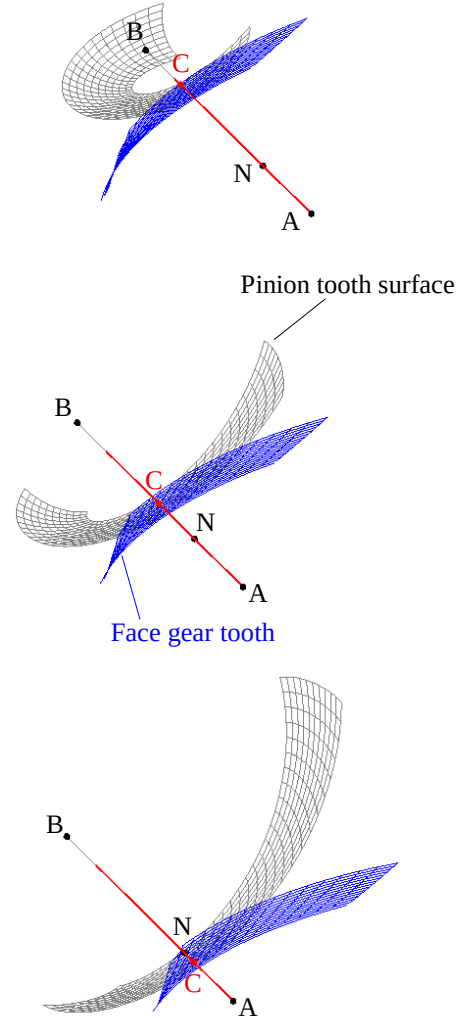
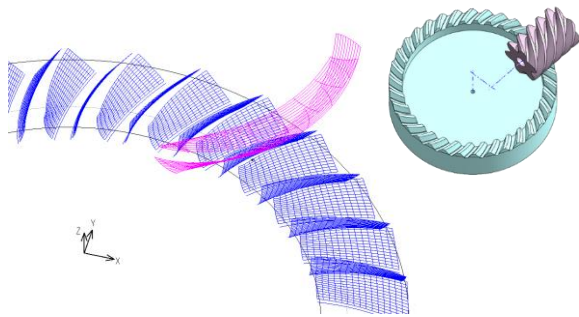


Fig. 4. Meshing contact line of action. The meshing contact line of action is directed in the normal direction of the tooth surface of the pinion.

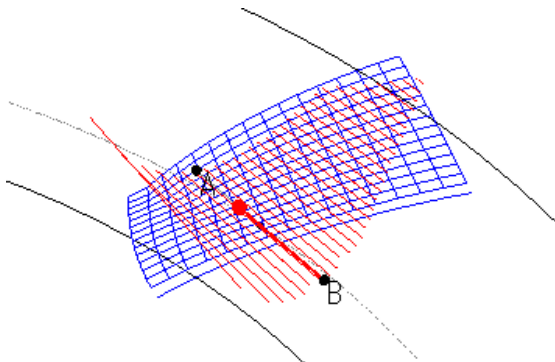
By using the meshing contact surface of action, the tooth surface coordinates of the face gear, the effective tooth surface area for constant-velocity meshing, the simultaneous contact line, the tooth surface slip, and the actual meshing ratio can be determined relatively easily, and the geometric meshing can be almost completely elucidated.

Figure 6 shows the tooth surface of the face gear obtained by numerical analysis, with the tooth surface region obtained from the meshing contact surface superimposed on it.

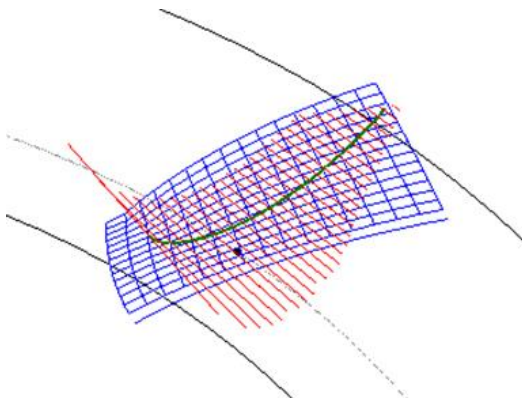
The brown tooth surface area obtained from the "meshing surface of action" is the effective tooth surface area where the involute engages correctly at a constant velocity. The area outside that region is an interference zone, and the gears will not mesh at a constant speed.



(a). Simulation of meshing face gear

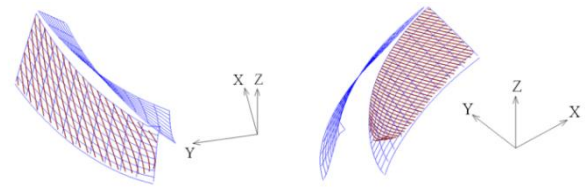


(b). Meshing contact surface of action



(c). Simultaneous contact line

Fig. 5. Coordinate system for face gear



(a) Convex side

(b) Concave side

Fig. 6. Tooth surface from the meshing contact surface of action. The blue surface represents the tooth surface of the face gear calculated by the conventional generation method. The brown curved surface is the tooth surface of the face gear obtained from meshing contact surface of action.

3 Effective meshing tooth surfaces

3.1 Tooth surface of face gears

A gear that meshes conjugately with an involute pinion at shaft angle of $\sigma = 90^\circ$ and crossed angle of $\Sigma = 0^\circ$ is an orthogonal face gear. In this study, focusing on orthogonal face gears and employing the line-of-action method from the unified theory, we investigate the correlation between the effective face gear tooth surface, derived from the meshing contact surface of action, and the gear ratio.

Table 1 is used for the specifications of the pinion and face gear to be analyzed. The number of teeth on the pinion (z_1) is varied across three levels—5, 25, and 100—and the number of teeth on the face gear (z_2) is similarly varied across the same three levels. A total of nine types of face gears are compared and evaluated based on combinations of number of teeth.

Figure 7 shows the tooth surface coordinates of the face gear, obtained from the meshing contact surface of action, plotted in three-dimensional space using the coordinate system of the unified theory. The blue curved surface represents the tooth surface of the face gear derived from the meshing contact surface of action. This tooth surface is an effective tooth surface capable of transmitting the angle of rotation at a constant velocity by meshing with the pinion's tooth profile in line contact along the normal direction.

Table 1. Specification

Axis position	Shaft angle	$\sigma [^\circ]$	90
	Crossed angle	$\Sigma [^\circ]$	0
Pinion	Module	m_n	1.0
	Number of teeth	z_1	5, 25, 100
	Pressure angle	$\alpha_n [^\circ]$	20
	Helical angle	$\beta [^\circ]$	0
	Tooth width	[mm]	10
Face gear	Number of teeth	z_2	5, 25, 100

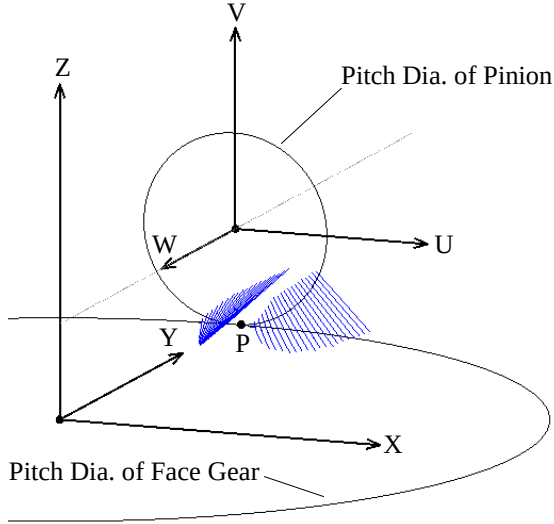


Fig. 7. Face gear tooth surface

Figure 8 shows the effective meshing tooth surfaces developed onto a two-dimensional plane to facilitate comparison. The pink rectangle represents the appearance of the pinion. The blue region represents the effective area of the face gear tooth surface and is plotted using only the face gear radius R and the coordinate Z in the tooth-height direction.

Figure 9 presents the plots from Fig. 8, calculated and arranged with different combinations of number of teeth.

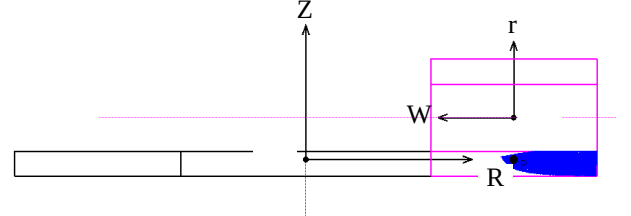


Fig. 8. 2-dimensional view of the effective meshing tooth surface of a face gear.

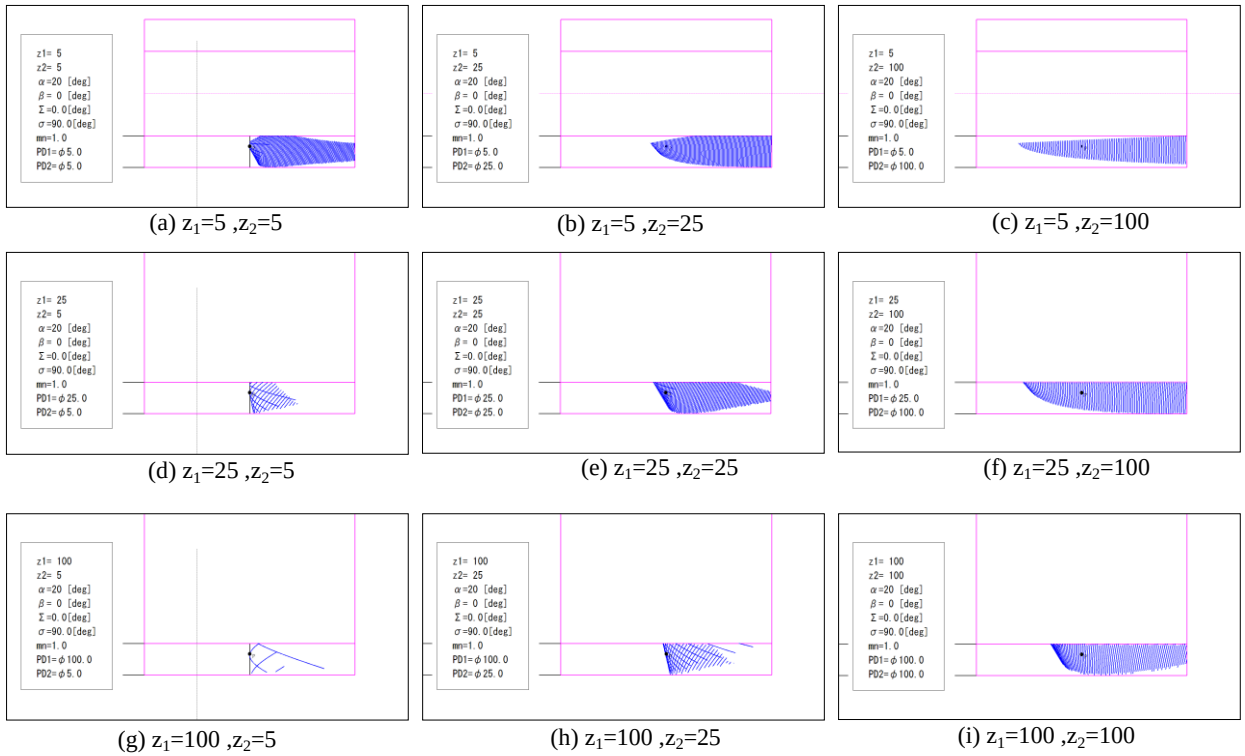


Fig. 9. 2-dimensional views of the effective meshing tooth surface of face gears with different combinations of number of teeth

In the configuration with a gear ratio of 1 shown in Fig. 9(a), it can be seen that no meshing occurs on the small-diameter side of the face gear, in the region below the pitch circle diameter. In that region, the pinion face width is not required. In Fig. 9(c), which shows a face gear with a large number of teeth and a high gear ratio, it can be seen that meshing occurs even on the small-diameter side of the face gear, in the region radially inward of the pitch circle diameter. For the combinations shown in Figs. 9(d), (g), and (h), where the number of teeth of pinion exceeds the number of teeth of face gear, the effective meshing region is narrow; therefore, using these specifications is not recommended.

3.2 Tooth surface of Outer bevel-shaped gear

In the range where the shaft angle σ satisfies $0 < \sigma < 90^\circ$, the gear meshing with the pinion is a bevel-shaped gear. It should be noted that this gear is completely different from a standard bevel gear. As in the case of face gears, the effective tooth surface area derived from the line-of-action method of the unified theory is plotted, and a comparison is made. Except for the shaft angle being set to $\sigma = 45^\circ$, the basic specifications are the same as those of the face gear shown in Fig. 9.

Figure 10 shows the positional relationship between the pinion and the outer bevel-shaped gear, as well as the effective tooth surfaces. The blue plot is a 2-dimensional view of the effective meshing tooth surface.

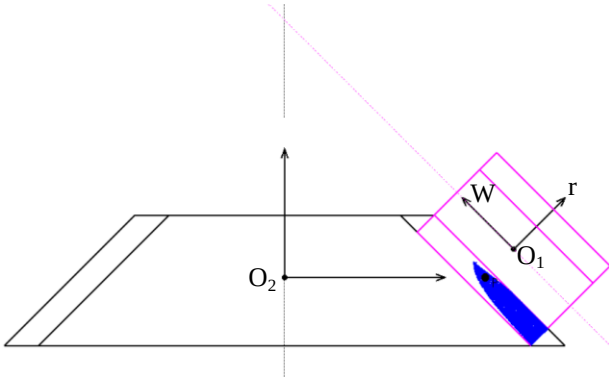


Fig. 10. 2-dimensional view of the effective meshing tooth surface of an outer bevel-shaped gear

Figure 11 shows the results for nine combinations of the number of teeth.

In Fig. 11(a), (d), and (g), effective meshing is not achieved at diameters smaller than the pitch circle diameter. In the regions with a large number of teeth on the outer bevel-shaped gears shown in Fig. 11(c), (f), and (i), the effective meshing tooth surface is wider than that of the face gear shown in Fig. 9. It was concluded that combinations involving a high number of pinion teeth are not practical for face gears. However, as an effective meshing region can be observed, as shown in Fig. 11(d),(g) and (h), there is a possibility that such combinations could be utilized for gears with an outer bevel-shaped gear configuration.

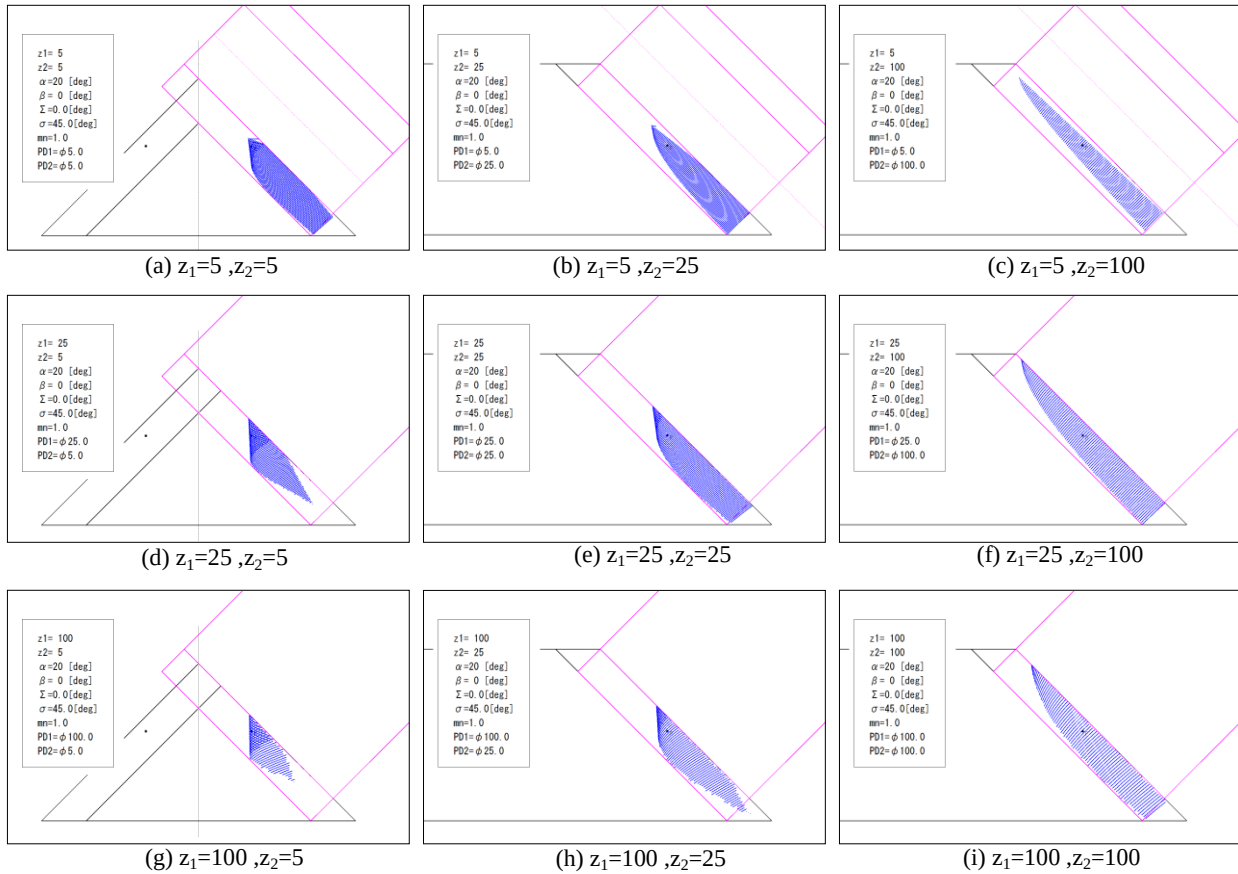


Fig.11. Two-dimensional views of the effective meshing tooth surface of outer bevel-shaped gears with different combinations of number of teeth.

3.3 Tooth surface of Outer bevel-shaped gear

We investigate the effective meshing tooth surfaces of inner bevel-shaped gears, in which the pinion meshes inside the conjugate gear, similar to face gears and outer bevel shaped gears.

Figure 12 shows the positional relationship between the pinion and the inner bevel-shaped gear, as well as the effective tooth contact surfaces. The blue plot is a two-dimensional view of the effective meshing tooth surface.

Figure 13 shows the results for nine combinations of number of teeth.

In Fig. 13(c) and (f), where the inner bevel-shaped gears have a large number of teeth, the effective meshing region is extensive, suggesting practical feasibility. In the

cases shown in Fig. 13(a), (e), and (i), where the pinion and the inner bevel-shaped gear have the same number of teeth, meshing does not occur at diameters smaller than the pitch diameter. Therefore, the face width of the pinion should be reduced. However, there is a possibility that teeth not contributing to meshing could interfere with other teeth, necessitating thorough verification during the design and manufacturing stages.

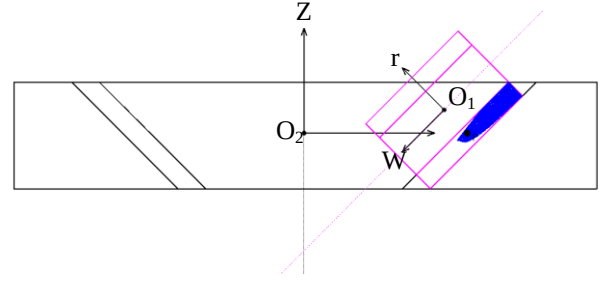


Fig. 12. 2-dimensional view of the effective meshing tooth surface of an inner bevel-shaped gear

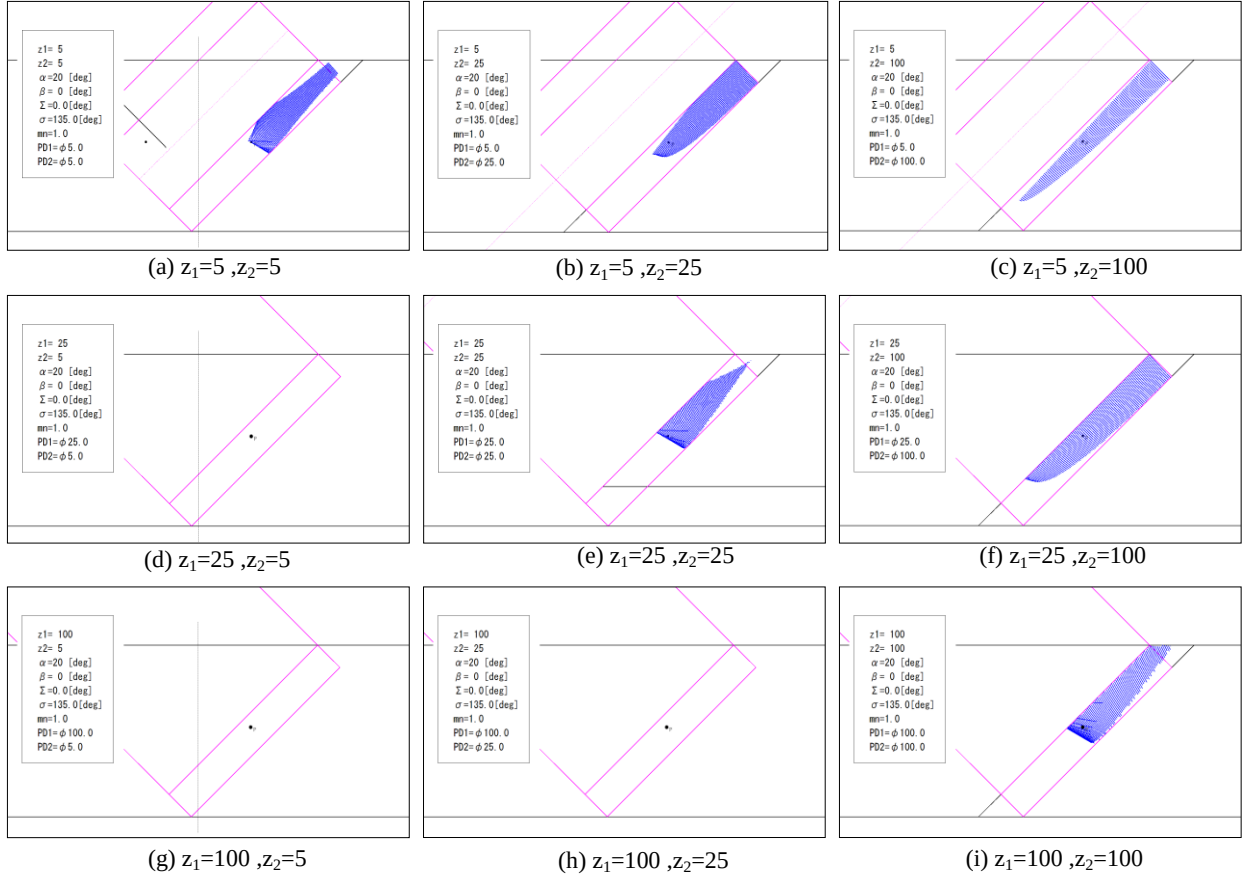


Fig. 13. Two-dimensional views of the effective meshing tooth surface of inner bevel-shaped gears with different combinations of number of teeth.

4 Conclusions

We investigated the relationship between the effective meshing tooth surfaces and the gear ratio for three types of gears, face gears, outer bevel-shaped gears, and inner bevel-shaped gears, designed to mesh conjugately with an involute cylindrical gear under conditions where the shaft angle $\Sigma=0$ and the helix angle $\beta=0$.

Face gears have a limited meshing zone in the region smaller than the pitch diameter.

Furthermore, it is not recommended to configure specifications where the number of pinion teeth exceeds the number of face gear teeth.

Outer bevel-shaped gears tend to offer a wider effective meshing tooth surface, suggesting potential viability even when the number of pinion teeth is high.

For inner bevel-shaped gears, it is recommended to use a configuration where the number of gear teeth exceeds the number of pinion teeth.

Regarding future prospects, while this study focused on the cases where $\Sigma=0$ and $\beta=0$, it is hypothesized that by utilizing these two parameters, it may be possible to design conjugate gears that mesh with involute cylindrical gears across a wide range of gear ratios.

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Development of modern technology for bevel gears machining

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Abstract. The purpose of the work is to increase the efficiency of machining high-precision bevel gears with circular teeth of gearboxes and to improve their operational performance by using a tool made of superhard materials.

To increase the operational performance of bevel gears, regular reliefs were formed on the internal base surfaces of the gears using a diamond ball with a radius of 1.5 mm. The tool performs a rotational motion at a speed of 50 rpm and a reciprocating motion for a length of 50 mm. Regular microrelief improves lubricating properties, reduces wear and increases the operational characteristics of gear parts.

The cutting of the teeth was performed on a 3-coordinate gear milling machine model "HAAS UMC 1000". The grinding of the teeth was carried out on a special CNC gear grinding machine mod. "Gleason 275". Abrasive ruby-corundum rings with a diameter of 177.8 mm (7 inches) were specially developed for the "Gleason 275" machine. Studies have shown that in some cases, synthetic diamond and cubic boron nitride grinding wheels can be used for grinding bevel gears.

The developed technology has made it possible to increase processing productivity by 1.5 times compared to existing technology, to improve the quality and accuracy of manufacturing engine gearboxes, and to ensure that they have the necessary technical and operational characteristics.

1 The actuality of the task

Growing requirements for the quality of gears – integral elements of gearboxes of most machines and mechanisms, are associated with the need to increase their accuracy and reliability, productivity and efficiency, which are largely determined by their operational properties. The solution of all these problems is inextricably linked with the development of new technologies and tools to increase the efficiency of machining high-precision (4–5 degree of accuracy) gears [1].

The main challenge is to improve the efficiency of making circular tooth bevel wheels with high precision. The production of such gears is very time-consuming and costly. Therefore, it is recommended to organize their production based on CNC machines and tools made of coated carbide and modern abrasive materials [2].

2 Problem statement

The main problem of cutting gears is the need for a systematic approach, both to the process of production preparation and to the technological process of

manufacturing, while maintaining the quality parameters and accuracy of their teeth.

A promising direction for the further development of methods for processing high-precision gears is largely the use of tools with superhard materials, in particular, cubic boron nitride (CBN).

The purpose of this work is to increase the processing efficiency of high-precision bevel gears with circular teeth of gearboxes and increase their performance using tools made of superhard materials.

3 Materials and methods

For the manufacture of gears, it was planned to use low-alloy cemented and hardened steel ISO 24CrMoV55. However, due to the need to work out the processing technology, it was decided to use round billets made of ISO 41Cr4 steel. The choice of these steels is due to the high physical and mechanical qualities of the manufactured precision and reference gears, a fairly large array of studies carried out by other researchers when processing with tools, including from superhard materials [3].

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The high labor intensity of the manufacture of hardened gears limits their use due to significant allowances that are prescribed for gear grinding operations to eliminate defects after heat treatment, the possibility of cauterization and microcracks, limitations in the size of gears (modules 2–5 mm) processed on gear grinding machines.

Gear processing technology consists of several stages [4, 5]:

- production of a blank – forging or round rolled products (ISO 41Cr4 steel);
- turning the gear wheel according to the developed drawing;
- heat treatment of the gear workpiece;
- grinding of base surfaces, holes and surfaces of the gear ring;
- cutting internal gear splines in gear holes;
- cutting and grinding the teeth on the gear ring.

4 Results and discussion

Turning of the gear workpiece was performed on a CNC machine of the SMEC SL2000BM (South Korea) (Fig. 1).



Fig. 1. Machining gears on SMEC SL2000BM CNC machine (Photographer: S. Riabchenko).

Turning modes: cutting speed $v = 250$ m/min; longitudinal feed $S = 0.1$ mm/rev.; cutting depth $t = 0.5$ – 1.5 mm. Cutting tools – cutters pass-through, cutting and bored from carbide and PKNB [6].

Heat treatment of the gear workpiece (hardening and tempering to hardness HRC 35–38) was performed in the Termolab thermal furnace of the SNOL 7,2/1100. Hardness was measured on hardness testers of the HBRV-2.

Grinding of the base surfaces, holes and surfaces of the gear ring was carried out on a CNC grinding machine of the RIBON RUR-H800 (Italy). Grinding modes: grinding speed $v = 25$ m/s; longitudinal feed $S = 0.5$ mm/rev; transverse feed $S = 0.05$ mm/double stroke; cutting depth $t = 0.5$ mm. Grinding tools – straight profile wheels of the T1 type with a diameter of 250 mm made of white alumina (25A) and cubic boron nitride (KV grade).

To increase the performance of bevel gears, regular reliefs were formed on the inner base surfaces of the gears, using a diamond ball with a radius of 1.5 mm [7]. The tool performs a rotational movement at a speed of 50 rpm and a reciprocating movement at a length of 50 mm. Regular microrelief improves lubricating properties, reduces wear and increases the performance of gear parts. The tool is a diamond-coated grinding tip mounted on the spindle of the machine. The pressure of the tool on the machined surface is provided by an adjustable clamp. During the machining process, each rotation of the tool leads to a movement in the longitudinal direction, which forms a periodic structure on the surface (Fig. 2).

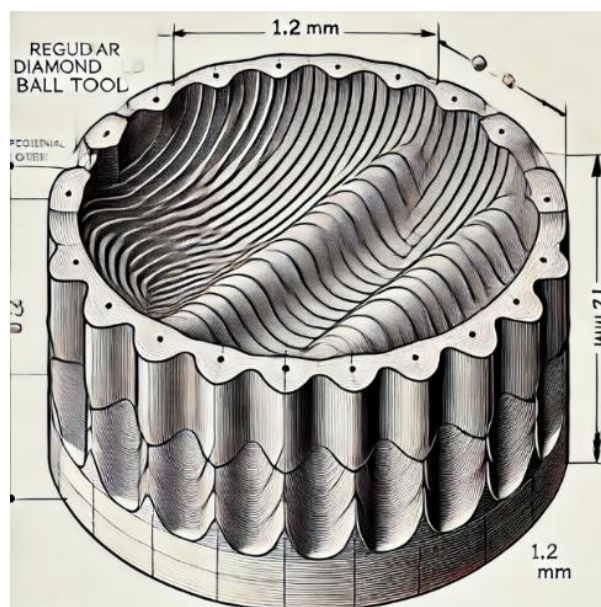


Fig. 2. Periodic structure on the surface.

With the specified processing parameters, a regular wavy relief is formed in increments of 1.2 mm. This relief can be used to improve lubricant retention and reduce friction. The depth of the resulting grooves depends on the clamping force of the tool and the hardness of the workpiece material.

The cutting of the internal gear splines in the holes of the gear blanks was carried out on a gear hammering machine model 5122. The choice of the cutting mode of the gear splines and the material of the cutting tool is due to the high hardness of the part [8]. Machining mode: frequency of double strokes $n = 200$ 1/min, feed $S_c = 0.2$ mm/double stroke (Fig. 3).

Quality control of the cut internal toothed splines is carried out by toothed gauges (passage – not a passage). Quality and accuracy control of internal gear splines in gear holes showed that the machined gear splines correspond to the gear drawing, namely the module 1 mm, the engagement angle is 30° , the maximum accumulated error of the circular pitch is 0.008 mm, the maximum profile error is 0.007 mm, the maximum deviation of the tooth direction is 0.006 mm. This allows in the future to perform a complex of technological operations for the processing of gears for aircraft engine gearboxes.

The teeth on the gear wheel were cut on a 3-coordinate gear milling machine model "HAAS UMC 1000" (Fig. 4), the tool is a spherical cutter with a diameter of 2.0 mm, a radius of 1.0 mm, a cutting part of 7.0 mm, a cutter length of 39.0 mm, a shank with a diameter of 3.0 mm. Cutting speed $n = 9500$ rpm ($v = 60$ m/min). After milling the crown, finishing was performed.



Fig. 3. Cut internal toothed splines and gauges (Photographer: S. Riabchenko).



Fig. 4. Machining of the gear ring on a machine model "HAAS UMC 1000" (Photographer: S. Riabchenko).

Grinding of bevel gears was carried out on a special CNC gear grinding machine "Gleason 275" (Germany). Grinding wheels for the processing process of bevel gears are a tool of complex shape. These are cup grinding wheels with conical surfaces connected at the

end along the inner and outer diameters of the wheel. Ruby-corundum ring-type abrasive wheels with a diameter of 177.8 mm (7 inches) were specially developed for the "Gleason 275" machine (Fig. 5).



Fig. 5. Ruby Corundum Cup Grinding Wheel (Photographer: S. Riabchenko).

Our studies have shown that in some cases grinding wheels made of synthetic diamond and cubic boron nitride can be used when grinding bevel gears.

Wheels with a diameter of 200 mm made of superhard materials (Fig. 6) were also tested when grinding bevel gears on Gleason machines: modulus $m = 5$ mm, number of teeth $z = 15$. The test results showed that when grinding bevel gears made of iron-carbon alloys with a hardness of 65 HRC, it is possible to use wheels made of CBN, and when grinding wheels made of titanium alloys – from synthetic diamond, the processing accuracy corresponds to 4 degree of accuracy [9].



Fig. 6. Cup grinding wheels made of super-hard materials for processing a bevel circular tooth (Photographer: S. Riabchenko).

Conclusions

The developed technology for processing bevel gears with a circular tooth (Fig. 7) made it possible to increase the processing productivity by 1.5 times compared to the existing technology, to improve the quality and accuracy of the manufacture of motor gearboxes, to ensure that they obtain the necessary technical and operational indicators [10].



Fig. 7. Bevel gear with a circular tooth after processing (Photographer: S. Riabchenko).

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Windage Loss Prediction in Jet-Lubricated High-Speed Gears: A Critical Assessment of Air–Oil Mixture Assumptions and Modeling Approaches

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Abstract. Windage power losses represent an increasingly important component of the load independent power losses in modern high-speed geared transmissions, particularly in aerospace applications and in electrified propulsion systems. Although numerous analytical, experimental and numerical approaches have been proposed over the last decades, significant uncertainties remain regarding the influence of the surrounding air–oil environment on windage-loss prediction. This work reviews the principal analytical formulations available for cylindrical and spiral bevel gears, from the pioneering models to recent developments. Attention is devoted to the treatment of the surrounding fluid, including the historical assumption of a 2.84% oil volume fraction which has been widely adopted despite the lack of a clear physical justification. The sensitivity of the equivalent fluid properties to the assumed oil concentration is analysed, showing that even small variations in oil volume fraction can substantially modify the mixture properties and consequently affect predicted windage losses. Eventually, recent CFD approaches, including single-phase, Volume of Fluid and Eulerian–Eulerian multiphase methods, are discussed and compared in terms of their capability to reproduce the complex flow conditions encountered in oil-jet-lubricated transmissions.

1 Introduction

The prediction of load-independent power losses has become an increasingly important aspect in the design of modern high-speed geared transmissions [1–3]. Although Windage Power Losses (WPL) have been investigated for several decades in aerospace applications [4–6], the increasing rotational speeds associated with electrified propulsion systems and high-speed reduction gearboxes have renewed interest in their accurate estimation [7–9]. At high rotational speeds, the aerodynamic interaction between rotating gears and the surrounding fluid may generate significant losses, directly affecting transmission efficiency and thermal requirements [10–12].

In oil-jet lubricated transmissions, the fluid surrounding the gears is generally neither pure air nor a fully flooded lubricant, but rather a complex air–oil environment characterized by highly transient multiphase flow phenomena [13–15]. Consequently, the prediction of WPL remains particularly challenging, especially when simplified analytical formulations are employed [5,16]. A substantial amount of research has been devoted to the development of analytical, empirical and numerical approaches for estimating WPL in cylindrical gears [1–3]. Among the earliest contributions, Ariura et al. investigated the influence of lubricant and operating conditions on no-load losses in high-speed spur gears

[4]. Subsequently, Anderson and Loewenthal proposed one of the first formulations specifically intended for aerospace gearboxes operating under oil-jet lubrication conditions [5,6]. Their work introduced the concept of equivalent air–oil mixture properties, which remains widely adopted in modern analytical models [6,11,12]. Additional experimental investigations were conducted by Dawson, who analyzed the effects of gear geometry, enclosure conditions and lubrication environment on WPL in large high-speed gears [7,8]. More physically based formulations were later introduced by Diab et al., who proposed a quasi-analytical approach accounting separately for losses generated on gear side surfaces and tooth regions [10]. This formulation subsequently became the basis for several recent developments, including the model proposed by Zhu et al. [11] for load-independent losses under shroud-approaching conditions and the generalized windage-churning framework developed by Dai et al. [12]. Experimental investigations have also played a crucial role in understanding the underlying aerodynamic mechanisms. Massini et al. developed dedicated experimental and numerical procedures for high-speed gears [16], whereas Ruzek et al. investigated WPL in meshing pinion-gear pairs [17]. Quiban et al. experimentally analyzed the transition between churning and windage regimes [18], while Handschuh et al. [19] provided additional evidence regarding the effects of

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gear geometry and lubrication conditions on no-load losses.

Compared to cylindrical gears, significantly fewer analytical models are available for spiral bevel gears [20,21]. One of the first dedicated formulations was proposed by Zhu et al., who developed a quasi-analytical model based on the decomposition of losses into toe, heel and tooth contributions [20]. The model was later refined and assessed through Computational Fluid Dynamics (CFD) investigations [21]. Li and Wang combined experiments and numerical simulations to investigate WPL under oil-jet lubrication conditions and proposed refined analytical formulations for aerospace spiral bevel gears [22,23]. More recently, Dai et al. extended unified windage-churning approaches to spiral bevel transmissions, while Kai et al. derived an analytical model directly from the Navier–Stokes equations [24,25].

Alongside analytical developments, CFD has become an increasingly important tool for investigating gearbox aerodynamics and lubrication phenomena [26–28]. Early CFD studies demonstrated the potential of numerical simulations for enclosed spur gears and shrouded aerospace transmissions [26,27]. Later investigations showed that CFD can capture complex flow structures, lubricant distribution, enclosure effects and no-load loss mechanisms that are difficult to represent through analytical methods alone [28–30]. Recent review studies have further highlighted the increasing relevance of CFD for the analysis and design of lubricated mechanical components and gear jet lubrication systems [31,32]. Despite the increasing availability of CFD methods, analytical and empirical formulations remain essential engineering tools [1–3]. Their negligible computational cost makes them particularly attractive during preliminary design activities, where rapid assessments of multiple gearbox configurations are required [5,10,20]. Furthermore, they provide valuable reference values for assessing whether CFD predictions remain within physically plausible ranges and for identifying possible modelling inconsistencies before undertaking computationally expensive numerical campaigns [31,32]. A common feature of most analytical formulations is the need to define the density and viscosity of the surrounding fluid [5,6,10]. These properties are generally obtained by assuming a homogeneous air–oil mixture characterized by prescribed oil fractions [6,11,24]. However, the origin, physical meaning and applicability of these assumptions are often not discussed in detail, despite their potentially dominant influence on the predicted WPL [1,5,12]. Different interpretations of the oil concentration, particularly when expressed as volume fraction or mass fraction, may lead to predicted losses differing by more than one order of magnitude [5,6,24].

The objective of the present work is therefore to collect and critically compare the main analytical and empirical methodologies currently available for WPL prediction in high-speed gears.

2 Materials and Methods

2.1 Literature review and model selection

A systematic literature review was conducted to identify analytical, empirical and semi-analytical formulations for the prediction of WPLs in high-speed gears. The search was performed using the Scopus database and included all publications containing the term windage in the title, together with additional references identified through citation tracking and cross-referencing of the most relevant studies. Each publication was individually reviewed and classified according to the gear type (cylindrical, helical, double-helical or spiral bevel), modelling approach (empirical, analytical, semi-analytical, numerical or experimental), fluid environment (air or air–oil mixture), consideration of shrouding effects, availability of experimental validation. Only studies providing original analytical formulations, experimental measurements, or validated predictive methodologies were retained for the present investigation. Pure CFD studies without analytical developments were excluded from the analytical comparison but were considered in the discussion of numerical approaches.

The resulting database includes the major contributions currently available in the literature and represents, to the authors' knowledge, the most comprehensive collection of analytical WPL formulations for both cylindrical and spiral bevel gears.

2.2 Classification of analytical windage models

2.2.1 Cylindrical gears

The earliest analytical formulations were developed for spur gears operating in air. Anderson and Loewenthal (1980, 1982) [5,6] proposed empirical correlations derived from experimental investigations performed on high-speed gears. These models constitute the foundation of most subsequent WPL formulations and assume an air environment without oil injection or shrouding effects.

Dawson (1984) [7] extended the available experimental database and proposed improved empirical correlations accounting for gear dimensions, rotational speed and enclosure effects. The model also introduced correction factors to describe partially shrouded configurations and therefore represents one of the first attempts to quantify the influence of aerodynamic confinement on WPL.

Handsuh and Kilmain (2003) [33] revisited the problem in the context of aerospace transmissions and proposed updated formulations applicable to helicopter gearbox applications.

Diab et al. (2004) [10] introduced a more physics-based methodology by decomposing the total WPL into multiple aerodynamic contributions associated with tooth surfaces and gear side faces. Their approach represented an important transition from purely empirical formulations towards semi-analytical models derived from aerodynamic drag theory.

More recently, Zhu et al. (2023) [11], Dai et al. (2023) [12] and Yang and Lin (2023) [34] proposed advanced formulations capable of accounting for modern high-speed transmissions. In particular, Yang and Lin developed a dedicated analytical model for double-helical gears, explicitly considering the influence of the herringbone geometry on the resulting aerodynamic losses.

2.2.2 Spiral bevel gears

Compared with cylindrical gears, analytical studies devoted to spiral bevel gears are significantly more recent. The first dedicated formulations were proposed by Zhu et al. (2020, 2021) [20,21], who developed analytical models specifically for spiral bevel gears operating in air. These formulations decompose the total WPL into contributions associated with the various gear surfaces and were subsequently validated through CFD simulations. Li and Wang (2023) [22] extended the analytical framework to aviation spiral bevel gears operating under oil-jet lubrication conditions. Their model includes tooth-surface losses, toe and heel surface contributions, conical surface losses and tooth-circumference effects. The formulation explicitly considers the presence of an oil–air environment and was experimentally validated through a dedicated test rig. Dai et al. (2024) [24] proposed a unified formulation capable of describing both windage and churning losses, thereby addressing operating conditions characterized by the simultaneous presence of air and lubricant. Kai et al. (2024) [25] further refined the analytical description of spiral bevel gear aerodynamics by introducing detailed surface-based formulations suitable for aerospace gearbox applications.

2.2.3 Air, air–oil mixture and shrouding assumptions

One of the main differences among the available formulations concerns the representation of the surrounding fluid. Most early models assume that the gear rotates in quiescent air. Under these conditions, the fluid density and viscosity correspond to atmospheric values and WPL arise solely from aerodynamic drag mechanisms. However, transmissions are typically lubricated through oil jets, resulting in a highly aerated environment composed of air, dispersed oil droplets and suspended lubricant. Under these conditions, the effective density and viscosity experienced by the rotating gears may differ substantially from those of pure air.

Another important distinction concerns the treatment of shrouding effects. Anderson and Loewenthal neglected shrouding and focused on free-air configurations. Dawson introduced correction factors to account for shrouded arrangements and demonstrated that aerodynamic confinement may significantly reduce WPL. Subsequent experimental studies, including those performed by Johnson et al. on spiral bevel gears,

confirmed that appropriately designed shrouds can reduce WPL by more than 70%.

As a consequence, the applicability of a given formulation strongly depends on the actual gearbox configuration, particularly with respect to lubricant presence and enclosure geometry.

2.2.4 Air–oil mixture properties

Whenever a selected analytical model required the specification of fluid properties, the surrounding medium was represented as an equivalent homogeneous air–oil mixture. The equivalent density can be calculated as

$$\rho_{\text{mix}} = \alpha_{\text{oil}} \cdot \rho_{\text{oil}} + (1 - \alpha_{\text{oil}}) \cdot \rho_{\text{air}} \quad (1)$$

where:

ρ_{mix} is the equivalent density [$\text{kg} \cdot \text{m}^{-3}$];

ρ_{oil} is the oil density [$\text{kg} \cdot \text{m}^{-3}$];

ρ_{air} is the air density [$\text{kg} \cdot \text{m}^{-3}$];

α_{oil} is the oil volume fraction [-] e.g. $\alpha_{\text{oil}} = 0.05$ means that the air–oil mixture contains 5% of the volume of oil.

For viscosity, two alternative formulations were considered. The first corresponds to a linear volumetric mixing law:

$$\mu_{\text{mix}} = \alpha_{\text{oil}} \cdot \mu_{\text{oil}} + (1 - \alpha_{\text{oil}}) \cdot \mu_{\text{air}} \quad (2)$$

where:

μ_{mix} is the equivalent dynamic viscosity [$\text{Pa} \cdot \text{s}$];

μ_{oil} is the oil dynamic viscosity [$\text{Pa} \cdot \text{s}$];

μ_{air} is the air dynamic viscosity [$\text{Pa} \cdot \text{s}$];

The second adopts a logarithmic mixing rule frequently used for dispersed multiphase systems:

$$\mu_{\text{mix}} = \mu_{\text{oil}}^{\alpha_{\text{oil}}} \cdot \mu_{\text{air}}^{(1 - \alpha_{\text{oil}})} \quad (3)$$

Particular attention was devoted to the interpretation of the oil concentration because identical nominal percentages may correspond to significantly different fluid properties depending on whether they are defined on a mass or volume basis. This aspect becomes especially important when comparing formulations originally developed for pure-air conditions with more recent models intended for oil-jet lubricated transmissions.

2.3 Influence of the assumed oil volume fraction on the equivalent mixture properties

2.3.1 Possible ranges of volume fraction

Figures 1 and 2 illustrate the variation of the equivalent density and dynamic viscosity assigned to the air–oil mixture as a function of the dispersed-oil volume fraction, α_{oil} , over the range 0% – 5%. The calculations consider air properties of $\rho_{\text{air}} = 1 \text{ kg} \cdot \text{m}^{-3}$ and $\mu_{\text{air}} = 2.09 \cdot 10^{-5} \text{ Pa} \cdot \text{s}$, together with broad oil-property

intervals representative of lubricants potentially encountered in geared transmissions: $800 \leq \rho_{oil} \leq 1000 \text{ kg} \cdot \text{m}^{-3}$ and $2.4 \cdot 10^{-3} \leq \mu_{oil} \leq 1 \cdot 10^{-2} \text{ Pa} \cdot \text{s}$. The results demonstrate that even a small volumetric amount of lubricant can strongly alter the properties assigned to the equivalent homogeneous mixture. At $\alpha_{oil} = 5\%$, the linear density law gives $40.95 \leq \rho_{mix} \leq 50.95 \text{ kg} \cdot \text{m}^{-3}$, which corresponds to approximately 41–51 times the assumed air density. The viscosity is even more sensitive to the selected homogenization rule. The logarithmic law predicts only a moderate increase at low oil fractions, and the mixture remains comparatively close to the pure-air condition throughout the investigated range.

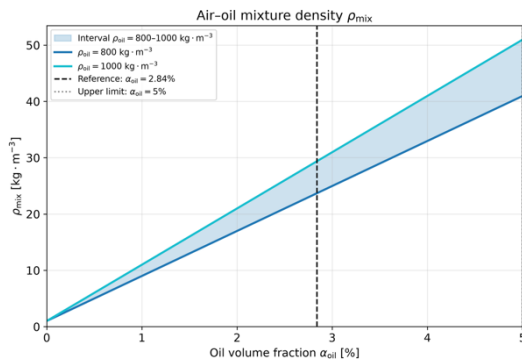


Fig. 1. Air-oil mixture density for different oil density and oil volume fraction.

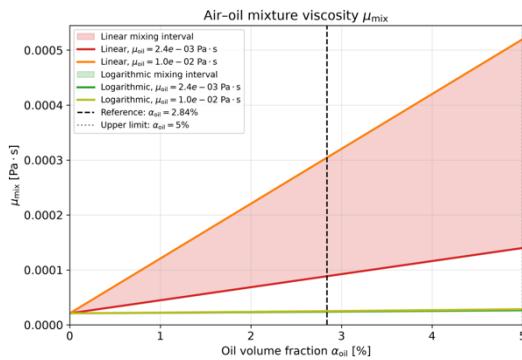


Fig. 2. Air-oil mixture dynamic viscosity for different oil viscosity, mixture models, and oil volume fraction.

2.3.2 Historical origin and critical assessment of the 2.84% assumption

A concentration of approximately 3% oil by volume, or more precisely 2.84%, is commonly used as a reference in analytical windage calculations for jet-lubricated gearboxes. The earliest source identified in the reviewed primary documents is the 1969 report [35] by Bowen, Braddock and Walker, *Installation of a High-Reduction-Ratio Transmission in the UH-1 Helicopter*, USA Army Aviation Material Laboratories, Technical Report 68-57. In the WPL calculation, the conventional air-based expression is corrected to represent the dense oil-laden atmosphere inside the gearbox. On page 200 the authors

state that the approximate mixture for a dense, oily atmosphere consists of 34.25 parts air to 1.0 part oil i.e. volume fraction of 0.02837.

However, the report does not explain how the value 34.25 was obtained. No experimental sampling procedure, local oil-concentration measurement, mass balance, droplet-size distribution or spatial averaging method is provided. The ratio is introduced directly as an approximate engineering representation of the gearbox atmosphere. It should therefore be regarded as an assumed design value for that specific transmission, rather than as a generally established concentration for oil-jet lubricated gearboxes.

The same ratio was subsequently adopted by Anderson and Loewenthal [5] in NASA Technical Paper 1622, *Spur-Gear-System Efficiency at Part and Full Load*. They state that the environment inside a jet-lubricated gearbox is an air–oil mixture and explicitly refer to the earlier helicopter-transmission report when defining a nominal composition of 34.25 parts air to one part oil, described as an oil-to-air ratio of approximately 3%.

The same assumption was retained in their subsequent 1982 journal paper [6], *Design of Spur Gears for Improved Efficiency*. The primary-document review therefore indicates that Bowen et al. [35] introduced the ratio first, while Anderson and Loewenthal transferred it into the windage correlation that later became widely cited. The assumption was later reproduced in other studies. For example, Chaari et al. [36] adopted the Anderson–Loewenthal expressions for a nominal 3% air–oil condition in their investigation of spur-gear windage. Their numerical results show the practical significance of this choice: at 20 000 rpm, the predicted loss for one investigated gear increased from approximately 50 W in air to about 700 W when the equivalent air–oil properties were used. Thus, replacing pure-air properties with those obtained from the inherited 3% assumption changed the result by roughly a factor of fourteen.

This historical propagation is problematic because the 2.84% concentration has often been reused without being independently measured or critically reassessed.

The sensitivity plots emphasize why this issue cannot be considered secondary. Increasing the assumed oil fraction from 3% to 4% represents a relative increase of one third.

Accordingly, $\alpha_{oil} = 2.84\%$ should be retained only as a historical reference condition, useful for reproducing the Bowen and Anderson–Loewenthal calculations and comparing later studies based on the same convention. It should not be treated as a universally validated oil concentration. A more robust procedure is to evaluate the analytical models over a plausible concentration interval, such as 0% – 5%, and to report explicitly the sensitivity to both the oil volume fraction and the selected viscosity-mixing rule. Ideally, the local oil fraction should instead be obtained from dedicated measurements or from a multiphase numerical model capable of predicting the spatial distribution and suspension of the dispersed lubricant.

3 Analytical Approaches

The analytical and semi-analytical approaches collected in the literature differ substantially in their level of physical detail. Early formulations are mainly empirical correlations expressed as functions of rotational speed, characteristic gear dimensions and fluid properties. Later approaches decompose the losses into contributions generated by the gear side faces, tooth spaces, peripheral surfaces and, when relevant, the meshing region. The models considered below are presented in their generic form, without reference to a specific gear pair. Unless otherwise stated, ρ_{mix} and μ_{mix} denote the density and dynamic viscosity assigned to the surrounding fluid. When a formulation originally employs the kinematic viscosity ν , it is rewritten here using

$$\nu_{\text{mix}} = \frac{\mu_{\text{mix}}}{\rho_{\text{mix}}} \quad (4)$$

For a transmission composed of a pinion and a gear, each single-gear formulation must generally be evaluated separately using the corresponding geometry and rotational speed. The total WPL is then obtained as the linear sum of the WPL of the gear and the pinion.

3.1 Analytical approaches for cylindrical gears

In the present section the variables are:

P_W [kW]: windage power loss of the considered gear;
 m_n [mm]: normal module;
 z [—]: actual number of teeth;
 x [—]: profile-shift coefficient;
 β [deg]: helix angle;
 α_p [deg]: reference pressure angle;
 α_t [deg]: pressure angle at the tooth tip;
 b [mm]: gear face width;
 r_p [mm]: pitch-circle radius;
 d_f [mm]: root-circle diameter;
 n [rpm]: rotational speed in rotations per minute;
 ω [rad · s⁻¹]: rotational speed in radians per second;
 ρ_{mix} [kg · m⁻³]: equivalent density of the surrounding fluid;
 μ_{mix} [Pa · s]: equivalent dynamic viscosity of the surrounding fluid.

3.1.1 Anderson and Loewenthal

Anderson and Loewenthal [5] proposed one of the earliest empirical correlations specifically applied to the windage losses of high-speed spur gears operating inside an aerospace gearbox. The formulation combines the influence of rotational speed, pitch radius, face width, fluid density and fluid viscosity:

$$P_W = 3.233 \cdot 10^{-22} \left(1 + 2.3 \frac{b}{r_p}\right) \rho_{\text{mix}}^{0.8} n^{2.8} r_p^{4.6} \mu_{\text{mix}}^{0.2} \quad (5)$$

The model does not explicitly describe tooth shape, module, helix angle or the detailed geometry of the surrounding shroud. The experimental configuration from which the correlation was derived had a specific

axial distance between the rotating gear and the casing, approximately equal to $0.217 r_p$. Consequently, the equation should not be interpreted as a general shroud-clearance model.

3.1.2 Dawson

Dawson [7] proposed an empirical equation for large high-speed gears in which the total WPL is represented through the combined contribution of the gear side surfaces and the toothed periphery:

$$P_W = 10^{-20} n^{2.9} [0.16 d_f^{3.9} + d_f^{2.9} b^{0.75} m_n^{1.15}] \Phi \lambda \varphi \quad (6)$$

Where:

Φ : correction associated with the effective density of an oil-laden atmosphere. For an oil-free atmosphere, $\Phi = 1$;

λ : enclosure or gear-case correction;

φ : helix-angle correction.

Dawson did not provide a general closed expression for Φ . Therefore, when an oil-laden atmosphere is considered, this coefficient must be selected from the assumptions or recommendations associated with the original work rather than calculated directly from ρ_{mix} and μ_{mix} .

The enclosure coefficient is approximately $\lambda = 1$ for a gear rotating in free space, $\lambda \approx 0.6 - 0.7$ for a gear operating in a relatively large enclosure, and $\lambda \approx 0.5 - 0.6$ for a close-fitting gear case or shroud.

The helix-angle coefficient is generally taken as $\varphi = 1$ for spur gears, while a value close to $\varphi \approx 0.7$ was suggested for helical gears with a helix angle of approximately 30°. These values are empirical and should not be extrapolated indiscriminately to substantially different helix angles.

3.1.3 Handschuh and Kilmain

Handschuh and Kilmain [33] adopted and applied a correlation derived from Dawson's later work to high-speed aerospace helical gears. In its generic form, the windage power is

$$P_W = 8.896 \cdot 10^{-23} C' \rho_{\text{mix}}^{0.85} \mu_{\text{mix}}^{0.15} n^{2.85} d_f^{4.7} \lambda \quad (7)$$

The shape factor C' is not given by a single closed equation. It must be obtained from the diagrams proposed by Dawson and reproduced or referenced by Handschuh and Kilmain. The diagram expresses C' as a function of the equivalent number of teeth and the face-width-to-diameter ratio in Figure 3.

For a helical gear, the equivalent number of teeth is generally evaluated as

$$z_e = \frac{z}{\cos \beta} \quad (8)$$

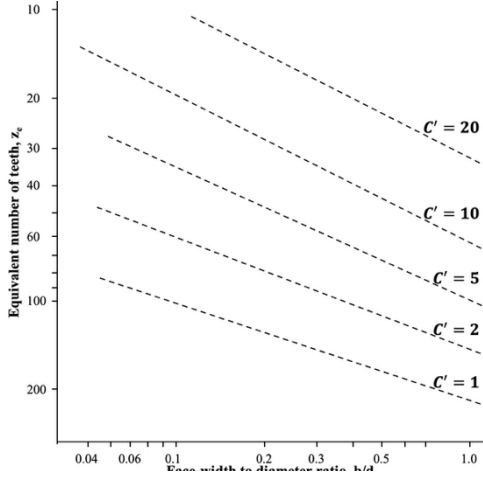


Fig. 3. Shape factor as a function of number of teeth, helix angle, face-width and gear diameter.

3.1.4 Diab et al.

Diab et al. [10] developed a quasi-analytical method in which windage losses are decomposed into the contributions produced by the gear side faces and by the tooth spaces. The total loss is written as

$$P_W = 5.7419 \cdot 10^{-22} (C_f + C_l) \rho_{\text{mix}} n^3 r_p^5 \quad (9)$$

The two moment coefficients are:

C_f : coefficient associated with the two side faces;

C_l : coefficient associated with the tooth spaces and peripheral region.

Considering the Reynolds number as

$$Re = \frac{\rho_{\text{mix}} \omega r_p^2}{\mu_{\text{mix}}} \cdot 10^{-6} \quad (10)$$

The transition radius between the laminar and turbulent regions is

$$R^* = \left(\frac{\mu_{\text{mix}} Re^*}{\rho_{\text{mix}} \omega} \right)^{1/2} \quad (11)$$

where Re^* is the critical Reynolds number, generally taken as $Re^* \approx 3 \times 10^5$. The side-face coefficient can be written as

$$C_f = \frac{2n_1\pi}{5-2m_1} (Re^*)^{-m_1} \left(\frac{R^*}{r_p} \right)^5 + \frac{2n_2\pi}{5-2m_2} \left[Re^{-m_2} - (Re^*)^{-m_2} \left(\frac{R^*}{r_p} \right)^5 \right] \quad (12)$$

The coefficients adopted for the two flow regimes are $n_1 = 1.293, m_1 = 0.5$ for laminar flow, and $n_2 = 0.074, m_2 = 0.2$ for turbulent flow. The tooth-region coefficient is

$$C_l = \xi \frac{z}{4} \frac{b}{r_p} \left[1 + \frac{2(1+x)}{z} \right]^4 (1 - \cos(\frac{\pi}{2}\phi)) (1 + \cos(\frac{\pi}{2}\phi))^3 \quad (13)$$

Where, the angular tooth-space parameter is

$$\phi = \frac{\pi}{z} - 2[\text{inv}(\alpha_p) - \text{inv}(\alpha_t)] \quad (14)$$

where the involute function is

$$\text{inv}(\alpha) = \tan \alpha - \alpha \quad (15)$$

and the addendum pressure angle is

$$\alpha_t = \arcsin \left(\frac{r_p}{r_t} \cdot \sin(\alpha_p) \right) \quad (16)$$

The shroud factor ξ is evaluated from the left- and right-side clearances:

$$\xi = \left[\frac{\sqrt{b_{0L}^2 + (r_t - r_{0L})^2}}{r_p} \right]^{0.56} + \left[\frac{\sqrt{b_{0R}^2 + (r_t - r_{0R})^2}}{r_p} \right]^{0.56} \quad (17)$$

Here: b_{0L}, b_{0R} : axial distances (in mm) between the gear side faces and the left and right shrouds;

r_{0L}, r_{0R} : radial dimensions of the left and right surrounding obstacles or shrouds.

For an unconfined gear, the two contributions forming ξ are generally set equal to 0.5, giving $\xi = 1$.

Unlike correlations based on a single enclosure coefficient, the Diab formulation directly incorporates the actual axial and radial distances between the gear and its surroundings.

3.1.5 Zhu et al.

Zhu et al. extended the Diab framework to meshing spur gears operating near a shroud. The total load-independent power loss is decomposed as

$$P_{\text{tot}} = P_{W,1} + P_{W,2} + P_{\text{pump}} \quad (18)$$

where:

$P_{W,1}$ [W]: windage loss of the first gear;

$P_{W,2}$ [W]: windage loss of the second gear;

P_{pump} [W]: pumping or squeezing loss generated in the meshing region.

The windage loss of each gear retains the generic Diab form:

$$P_W = 10^{-15} \cdot C_W \rho_{\text{mix}} \omega^3 r_p^5 \quad (19)$$

$$C_W = C_f + C_l \quad (20)$$

The C_f can be calculated as in Equation 12. The tooth coefficient is modified to account for the number of tooth spaces actually exposed to the surrounding fluid:

$$C_l = \xi \frac{z-\varepsilon}{4} \frac{b}{r_p} \left[1 + \frac{2(1+x)}{z} \right]^4 (1 - \cos(\frac{\pi}{2}\phi)) (1 + \cos(\frac{\pi}{2}\phi))^3 \quad (21)$$

where ε is the total contact ratio.

The pumping model uses mass conservation inside the tooth-space volume. In generic form,

$$\rho' V = \rho_{\text{mix}} V_0 + \Delta m \quad (22)$$

where the additional mass term is evaluated from the local change in the tooth-space opening during mesh progression. Compressibility is included through an isentropic correction:

$$\frac{\rho''}{\rho} = \left[1 + \frac{\gamma-1}{2} Ma^2 \right]^{1/(\gamma-1)} \quad (23)$$

where:

ρ' : density obtained from the preliminary mass balance;

ρ'' : density after compressibility correction;

γ : ratio of specific heats;

Ma : Mach number.

The flow through the tooth-space opening is evaluated with an orifice-type equation involving a discharge coefficient C_d . The value of C_d is obtained from the Reynolds- and geometry-dependent correlation reported in the original paper rather than from a universal constant or table. The pumping power is then obtained from the volume flow rate and the pressure difference:

$$P_{\text{pump}} = Q \Delta p \quad (24)$$

The model therefore requires the numerical evaluation of the tooth-space volume variation, pressure evolution and local flow area during meshing.

3.2 Analytical approaches for spiral bevel gears

In the present section the variables are:

P_W [kW]: windage power loss of the considered gear;

R [m]: external or reference gear radius;

B [m]: total face width;

b [m]: local coordinate along the face width;

z [-]: number of teeth;

δ [rad]: pitch-cone angle;

δ_b [rad]: base-cone angle;

δ_a [rad]: addendum-cone angle;

h_a^* [-]: normalized addendum height;

x_1 [-]: profile-shift coefficient;

α_p [deg]: reference pressure angle;

α_a [deg]: pressure angle at the addendum;

β_m [rad]: mean spiral angle;

β' [rad]: local flow-deviation angle;

R_e [m]: external cone distance;

R_{em} [m]: mean external-cone radius;

D_0 [m]: nominal cutter diameter;

χ [-]: shroud or casing shielding coefficient;

γ [rad]: cone angle;

n [rpm]: rotational speed in rotations per minute;

ω [rad · s⁻¹]: rotational speed in radians per second;

ρ_{mix} [kg · m⁻³]: equivalent density of the surrounding fluid;

μ_{mix} [Pa · s]: equivalent dynamic viscosity of the surrounding fluid.

3.2.1 Zhu, Dai and Ma

Zhu, Dai and Ma [20,21] developed a quasi-analytical model specifically for spiral bevel gears. The total windage power loss of one gear is expressed as

$$P_W = C_t \rho_{\text{mix}} \omega^3 R^5 \quad (25)$$

where the total moment coefficient is

$$C_t = C_f + C_{td} \quad (26)$$

Here:

C_f : contribution of the toe and heel side surfaces;

C_{td} : contribution of the toothed region;

The side-surface coefficient is

$$C_f = \frac{2n_2\pi}{5-2m_2} \left[Re^{-m_2} - (Re^*)^{-m_2} \left(\frac{R^*}{R} \right)^5 \right] + \frac{2n_1\pi}{5-2m_1} (Re^*)^{-m_1} \left(\frac{R^*}{R} \right)^5 \quad (27)$$

The Reynolds number and transition radius are

$$Re = \frac{\rho_{\text{mix}} \omega R^2}{\mu_{\text{mix}}} \quad (28)$$

$$R^* = \left(\frac{\mu_{\text{mix}} Re^*}{\rho_{\text{mix}} \omega} \right)^{1/2} \quad (29)$$

As in the Diab model, n_1, m_1, n_2, m_2 are coefficients associated with laminar and turbulent rotating-surface flow regimes. The lateral conical-surface coefficient is

$$C_l = \frac{4\pi\mu_{\text{mix}}}{\rho_{\text{mix}} \omega \sin(\gamma/2)} \left\{ \left[\frac{1}{R - b \sin(\gamma/2)} \right]^2 - \frac{1}{R^2} \right\} \quad (30)$$

The toothed-region coefficient is evaluated by integration along the face width:

$$C_{td} \simeq \frac{\chi}{R^5} \int_0^B \frac{z}{4} [(R_e - b) \sin \delta]^4 \left[1 + \frac{\cos \delta (h_a^* + x_1)}{z} \right]^4 \cdot (1 - \cos \phi) (1 + \cos \phi)^3 (1 - \sin^2 \beta') db \quad (31)$$

The flow-deviation angle is evaluated as

$$\sin \beta' = \frac{1}{D_0} \left[R_e' + \frac{R_{em}}{R_e} (D_0 \sin \beta_m - R_{em}) \right] \quad (32)$$

with

$$R_e' = R_e - b \quad (33)$$

The projected radius is

$$r' = (R_e - b) \sin \delta \quad (34)$$

The active tooth length is

$$x = r_a' (1 - \cos \phi) \quad (35)$$

$$\phi = \frac{\pi}{z} - 2(\text{inv} \alpha_p - \text{inv} \alpha_A) \quad (36)$$

The spherical involute functions are

$$\text{inv} \alpha_p = \frac{1}{\sin \delta_b} \arccos \left(\frac{\cos \delta}{\cos \delta_b} \right) - \arccos \left(\frac{\tan \delta_b}{\tan \delta} \right) \quad (37)$$

$$\text{inv} \alpha_A = \frac{1}{\sin \delta_b} \arccos \left(\frac{\cos \delta_a}{\cos \delta_b} \right) - \arccos \left(\frac{\tan \delta_b}{\tan \delta_a} \right) \quad (38)$$

The shielding coefficient χ is not obtained from a universal analytical expression. It must be selected according to the shrouding condition considered in the original model or calibrated from experimental or numerical data. Because C_{td} involves an integral along the face width, the method remains quasi-analytical and requires numerical integration.

3.2.2 Li and Wang

Li and Wang [22, 23] decomposed the WPL of an aviation spiral bevel gear into contributions associated with the principal external surfaces:

$$P_W = P_t + P_s + P_c + P_p \quad (39)$$

where:

P_t : contribution of the tooth surfaces;

P_s : contribution of the toe and heel side surfaces;

P_c : contribution of the conical surfaces;

P_p : peripheral or circumferential contribution.

Each surface contribution is obtained from the local aerodynamic force or shear stress and the corresponding velocity. In generic form,

$$P_i = \int_{A_i} \tau_i \cdot \mathbf{v}_i \, dA \quad (40)$$

for shear-dominated surfaces, or

$$P_i = \omega \int_{A_i} r_i \, dF_i \quad (41)$$

when the contribution is expressed through a local drag force.

A commonly used local drag representation is

$$dF_i = \frac{1}{2} C_{D,i} \rho_{\text{mix}} v_i^2 \, dA_i \quad (42)$$

which gives

$$P_i = \frac{1}{2} C_{D,i} \rho_{\text{mix}} \int_{A_i} v_i^3 \, dA_i \quad (43)$$

The coefficients $C_{D,i}$, effective surface factors and local flow correction factors are obtained from the correlations and calibration procedure reported by Li and Wang. Some of these quantities are derived from the experimental and numerical results of the original study

rather than from universally applicable closed expressions.

The method should therefore be regarded as a surface-decomposition framework whose final implementation requires the detailed spiral bevel geometry and the coefficient values reported in the paper.

3.2.3 Dai et al.

Dai et al. [24] proposed a unified framework for conditions in which windage and churning coexist. The total no-load loss is written as

$$P_{NL} = P_W + P_C \quad (44)$$

where:

P_W : windage contribution generated on surfaces exposed to the gaseous or aerated phase;

P_C : churning contribution generated on surfaces interacting with the continuous liquid phase.

The generic windage contribution follows the moment-coefficient structure as in Equation 25. Whereas the churning contribution is written as a function of lubricant properties, immersion geometry and the wetted gear surface. The transition between the two regimes is introduced by dividing the gear surface into wetted and non-wetted regions or by using regime-dependent weighting functions. Therefore, the complete method requires the transition functions or charts reported by Dai et al., especially near the coexistence region.

3.2.4 Kai et al.

Kai et al. [25] developed an analytical model for predicting the WPL of aerospace spiral bevel gears operating in a single-phase air environment. The approach is based on the Navier–Stokes equations and decomposes the total windage loss into three physically distinct contributions:

losses generated on the large and small end faces;

losses generated on the conical circumferential surfaces;

losses generated by the fluid motion within the tooth spaces.

The total windage loss is therefore expressed as

$$P_W = P_m + P_{w1} + P_{w2} + P_{c1} + P_{c2} \quad (45)$$

where

P_m = tooth-space contribution;

P_{w1} = large end-face contribution;

P_{w2} = small end-face contribution;

P_{c1} = large circumferential-surface contribution;

P_{c2} = small circumferential-surface contribution.

For the rotating end surfaces, Kai et al. adopted classical rotating-disk correlations.

For laminar flow:

$$P_w^{(L)} = 0.9675 R^4 \rho_{\text{mix}} v_{\text{mix}}^{0.5} \omega^{2.5} \quad (46)$$

For turbulent flow:

$$P_w^{(T)} = 0.03647 R^5 \rho_{mix} \omega^3 Re^{-0.2} \quad (47)$$

Where the Reynolds number can be calculated according to Equation 28. The equations are applied separately to the large and small end faces using the corresponding radii R_1 (small-end radius) and R_2 (large-end radius).

The conical circumferential surfaces are treated as rotating cones obtained by integrating the drag generated on infinitesimal cylindrical elements.

The resulting power loss is

$$P_c = \pi \mu_{mix} \omega^2 b (R_1 + R_2)^2 \quad (48)$$

Where b is the cone width; For a spiral bevel gear, the two circumferential surfaces are evaluated independently:

$$P_{c1} = \pi \mu_{mix} \omega^2 b_1 (R_1 + R_{a1})^2 \quad (49)$$

$$P_{c2} = \pi \mu_{mix} \omega^2 b_2 (R_2 + R_{a2})^2 \quad (50)$$

where R_{a1} and R_{a2} are the outer radii of the corresponding circumferential surfaces.

The most original contribution of the model concerns the losses generated within the tooth spaces.

By solving a simplified Navier–Stokes formulation and applying a global momentum balance, the average pressure acting on the fluid trapped within a tooth space is obtained as

$$p = \frac{\mu_{mix}^2}{8\rho_{mix}} \left(\frac{1}{R_m^2} - \frac{1}{R_{a1}^2} \right) + \frac{1}{2} \rho_{mix} \omega^2 (R_m^2 - R_{a1}^2) \quad (51)$$

Where:

$$R_m = \frac{R_{a1} + R_{a2}}{2} \quad (52)$$

is the mean tooth-space radius. The resulting tooth-space windage loss is

$$P_m = -\beta z A_2 R_m \omega \cdot \left[\frac{\mu_{mix}^2}{8\rho_{mix}} \left(\frac{1}{R_m^2} - \frac{1}{R_{a1}^2} \right) + \frac{1}{2} \rho_{mix} \omega^2 (R_m^2 - R_{a1}^2) \right] \quad (53)$$

Where:

z = number of teeth;

A_2 = effective tooth-surface area;

β = effective-area coefficient;

Following Kai et al. [25] adopted $\beta = 0.4$ for all calculations. This coefficient is not derived analytically but introduced to account for the fact that only a fraction of the geometrical tooth surface effectively contributes to aerodynamic loading. The complete analytical model is

$$P_W^{(L)} = P_m + P_{w1}^{(L)} + P_{w2}^{(L)} + P_{c1} + P_{c2} \quad (54)$$

for laminar conditions, and

$$P_W^{(T)} = P_m + P_{w1}^{(T)} + P_{w2}^{(T)} + P_{c1} + P_{c2} \quad (55)$$

for turbulent conditions.

3.3 General remarks on model implementation

The correlations described above employ different characteristic dimensions, unit systems and definitions of the surrounding fluid. Anderson–Loewenthal, Dawson and Handschuh–Kilmain are predominantly empirical and can be evaluated rapidly, but their shroud and geometry corrections are either fixed by the original test configuration or obtained from diagrams and approximate coefficients. The Diab and Zhu approaches require more geometrical information but provide a clearer decomposition of the mechanisms producing windage losses. Eventually, it is essential to stress that replacing the fluid properties of air with ρ_{mix} and μ_{mix} does not automatically make a formulation valid for an oil-suspension environment. Such a substitution constitutes a homogeneous-mixture approximation and should be accompanied by a sensitivity analysis with respect to the assumed oil concentration and the selected viscosity mixing law.

4 CFD approaches for WPL prediction

CFD has become an important tool for estimating WPL because it allows the aerodynamic torque to be obtained directly from the pressure and viscous stresses acting on the rotating gear surfaces. Early and still widely used approaches treat the gearbox environment as a single-phase air flow. Examples include the CFD studies of Rapley [37] on spiral bevel gears, Concli et al. [3,38] on cylindrical gears, Marchesse et al. [39] on spur gears, Zhu et al. [21] on aeronautical spiral bevel gears, and Dai et al. [14] using an overset-mesh formulation. These models are particularly suitable for studying the effects of rotational speed, tooth geometry, gear interaction, shrouds and casing clearances under nominally dry-air conditions.

When lubricant is explicitly included, most numerical studies adopt a multiphase formulation [40]. The Volume of Fluid (VOF) method treats air and oil as immiscible separated phases and solves one mixture velocity field together with a transport equation for the oil volume fraction. It is therefore well suited to flows characterised by a recognisable free surface, such as oil-bath lubrication, churning, liquid accumulation and relatively coherent oil jets. Imai et al. [27], for example, employed a VOF-type interface-tracking method together with a porous representation of the meshing region. However, the method becomes less appropriate when high-speed oil injection generates droplets much smaller than the computational cells, because a sharp

oil–air interface can no longer be resolved accurately [41].

This limitation is particularly relevant for high-speed gears, where the injected lubricant breaks into droplets and forms an aerosol-like cloud around the rotating components. Concli [41] therefore proposed a Eulerian–Eulerian dispersed-phase model, in which air and oil are treated as interpenetrating continua with separate velocity fields. Interphase momentum exchange is represented through drag, lift and virtual-mass models, while the droplet diameter is introduced as an input parameter. This formulation is better aligned with the physical nature of oil suspension because it does not require the droplets to be individually resolved by the mesh.

The comparison reported by Concli is particularly significant. For an average mesh size of approximately $250\ \mu\text{m}$ and an assumed droplet diameter of $100\ \mu\text{m}$, the VOF solver was unable to preserve the oil distribution correctly and strongly underpredicted the wet-windage losses. By contrast, the Eulerian–Eulerian model reproduced oil-jet penetration into the meshing zone, axial and tangential oil expulsion, the formation of a dispersed droplet cloud and power losses much closer to the experimental measurements.

Accordingly, VOF remains the most appropriate option when the liquid phase is continuous or forms structures comparable with the mesh size, whereas Eulerian–Eulerian methods are more suitable for high-speed oil-jet lubrication dominated by finely dispersed droplets. For WPL prediction, the latter approach is therefore physically more representative because it can predict the local oil volume fraction and the resulting mass loading of the air, which directly control the magnitude of the so-called wet-windage losses. Its main limitation is the need for additional closure information, particularly the droplet-size distribution and the interphase-force correlations.

5 Conclusions

This paper examined the evolution of WPL prediction methods for high-speed geared transmissions, with particular emphasis on cylindrical and spiral bevel gears. The available analytical formulations range from simple empirical correlations to more advanced semi-analytical models that separately account for the contributions of gear side surfaces, tooth spaces and meshing regions. While these models remain attractive because of their simplicity and low computational cost, their predictions are strongly influenced by the assumed properties of the surrounding fluid.

A detailed analysis of the air–oil mixture assumptions revealed that the widely adopted oil volume fraction of approximately 2.84% originates from the report by Bowen et al. [35], where a ratio of 34.25 parts air to 1 part oil was introduced without a documented derivation. This value was subsequently adopted by Anderson and Loewenthal and propagated through numerous later studies. Despite its widespread use, the assumption has rarely been critically reassessed.

The sensitivity analysis demonstrated that relatively small variations in oil volume fraction can significantly alter the equivalent density and viscosity of the surrounding medium. Depending on the selected mixing law, the resulting fluid properties may differ by more than one order of magnitude, leading to substantial variations in predicted WPL. Consequently, the use of a single fixed oil concentration should be regarded as a historical convention rather than a universally valid physical condition.

Recent CFD developments provide valuable insight into the complex multiphase flow phenomena occurring in oil-jet-lubricated transmissions. While single-phase simulations remain useful for analysing purely aerodynamic effects and enclosure interactions, multiphase methods are required to capture the influence of lubricant dispersion. Among these approaches, Eulerian–Eulerian formulations appear particularly promising because they can represent the suspended oil-droplet cloud surrounding the gears more realistically than interface-capturing VOF methods, thereby providing a more physically consistent basis for wet-windage prediction.

Future developments should therefore combine advanced multiphase CFD simulations with experimentally validated analytical formulations and sensitivity analyses over realistic oil-concentration ranges. Such an approach would improve both the physical fidelity and the predictive capability of windage-loss models for next-generation high-speed transmissions.

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Comparative evaluation of gears with different finishing processes using a shape deviation network

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Abstract. In general, the accuracy grade of manufactured gears is classified based on standards such as ISO 1328-1. However, tooth profile and helix deviations are usually evaluated individually using measurement data from each tooth flank, and methods for evaluating the relative relationships among tooth flanks remain insufficiently established. Therefore, in our previous studies, we proposed a shape deviation network based on the correlation coefficients of tooth profile or helix deviations among tooth flanks. In this study, the shape deviation network and a cross-sectional shape evaluation method based on tooth helix deviations were applied to four types of steel gears: a hard-hobbed gear, a gear finished by polish honing after hard hobbing, a ground gear, and a gear finished by polish honing after gear grinding. In the tooth profile deviation network, the gear finished by polish honing after gear grinding showed the highest second-smallest eigenvalue, indicating higher geometric consistency and accuracy when the whole gear was evaluated based on tooth-flank correlations. In the tooth helix deviation network, a three-tooth-periodic structure was observed in the hard-hobbed gears, indicating that cutting-tool characteristics can be reflected in the proposed evaluation.

1 Introduction

Steel gears are manufactured through gear cutting processes such as hobbing, followed by heat treatment such as carburizing and quenching, and finally by finishing of the tooth flanks. In general, heat-treated gears may undergo heat-treatment distortion due to thermal expansion and contraction as well as phase transformation, which can reduce tooth flank accuracy [1]. Since an increase in tooth flank deviation leads to increases in transmission error, meshing vibration, and noise in gear pairs, it is important to improve the tooth flank geometry by finishing after heat treatment. Gear grinding is widely used as a representative finishing process. However, it requires a dressing mechanism for the grinding wheel and a highly rigid machine structure, resulting in high equipment and machining costs. Therefore, finishing methods applicable to heat-treated gears, such as hard hobbing and polish honing, have been proposed, and dedicated machine tools have been developed.

In general, the accuracy of manufactured gears is evaluated based on standards such as ISO 1328-1 [2]. However, tooth profile deviations and tooth helix deviations, which are shape deviations of tooth flanks, are usually evaluated based on measurement data obtained from individual tooth flanks. Therefore, it is not easy to comprehensively evaluate the distribution of shape deviations over the entire gear or the relative relationships among tooth flanks.

In our previous studies, we constructed shape deviation networks based on the correlation coefficients of tooth profile deviations and tooth helix deviations among tooth flanks, and proposed a method for comprehensively evaluating shape deviations over the entire gear [3–7]. In addition, we proposed a method for quantitatively evaluating the strength of the connections among tooth flanks by applying eigenvalue analysis to the graph Laplacian of the shape deviation network [8]. Furthermore, we have evaluated the characteristics of shape deviations distributed over the entire gear using a cross-sectional shape evaluation method based on the deviation curves of the left and right tooth flanks [9].

The proposed methods have mainly been applied to hobbed steel gears and injection-molded plastic gears [10, 11]. However, no studies have applied both the shape deviation network and the cross-sectional shape evaluation method to actual gears manufactured using different finishing processes and compared the obtained evaluation results. Therefore, in this study, the previously proposed methods were applied to four types of gears: a hard-hobbed gear, a gear finished by polish honing after hard hobbing, a ground gear, and a gear finished by polish honing after gear grinding. The shapes of these gears were then compared. The obtained pixel images, eigenvalues of the graph Laplacian, and cross-sectional shape evaluation results were compared to investigate the influence of different finishing processes on the distribution of shape deviations over the entire gear and on the relative relationships among tooth flanks. In addition, the

influence of the mounting condition during measurement on the tooth helix deviation network was also discussed.

2 Gear specifications and evaluation methods

2.1 Gear specifications

In this study, four types of steel spur gears with different tooth flank finishing processes were used. The gear finished by hard hobbing after heat treatment is referred to as Gear H. The gear finished by polish honing after hard hobbing is referred to as Gear HP. The gear finished by gear grinding is referred to as Gear G. The gear finished by polish honing after gear grinding is referred to as Gear GP. All the gears had the same specifications and were made of SCM415. After carburizing and quenching, different tooth flank finishing processes were applied to each gear. The gear specifications are shown in Table 1, and photographs of the tooth flank appearances are shown in Fig. 1.

On the tooth flank of Gear H shown in Fig. 1(a), a scale-like pattern, which is considered to be a machining mark caused by hard hobbing, was observed. In Gear HP shown in Fig. 1(b), the scale-like pattern became less distinct than that of Gear H, suggesting that the machining marks on the tooth flank were smoothed by polish honing. On the other hand, in Gear G and Gear GP shown in Figs. 1(c) and 1(d), respectively, no distinct scale-like pattern such as that observed in Gear H was found. These results indicate that the tooth flank appearances of the gears differed depending on the applied finishing process.

2.2 Measurement of tooth flank deviations

In this study, tooth profile deviations and tooth helix deviations on the left and right tooth flanks were measured for all 43 teeth of the four test gears using a gear measuring instrument, CPL-35S, manufactured by TPR OSAKA SEIMITSU KIKAI Co., Ltd. The measurement positions are shown in Fig. 2. The tooth profile deviations were measured at the section located at the center of the face width, whereas the tooth helix deviations were measured at the mid-tooth-height position.

The measured tooth profile deviation data include the effects of local end geometries, such as the trochoidal shape near the tooth root, tooth tip modification, and chamfering. Since these geometrical factors differ from the finishing condition of the central region of the tooth flank, they were excluded from the analysis in this study. Specifically, a range of 2.5 mm in roll length on the tooth root side and 1.0 mm in roll length on the tooth tip side were excluded from the measured tooth profile deviation data. In addition, for the tooth helix deviation data, a range of 1.0 mm from each end in the face width direction was excluded to avoid the influence of chamfering and edge geometries at both ends of the tooth flank.

In this study, the proposed methods were applied to gears with different finishing processes, and the relative relationships among tooth flanks were comparatively

evaluated using the tooth profile deviation curves and tooth helix deviation curves after removal of the DC component. In this paper, the inclination component, which was removed as a preprocessing step in previous studies, was not removed. Therefore, the influence of the inclination component caused by the mounting condition during measurement was also included in the evaluation.

Table 1. Gear specifications.

Parameter	Value
Normal module [mm]	2.5
Normal pressure angle [deg.]	20
Helix angle [deg.]	0
Number of teeth [-]	43
Face width [mm]	20
Profile shift coefficient [-]	0.102
Tip diameter [mm]	113
Base circle diameter [mm]	101.017
Material	SCM415

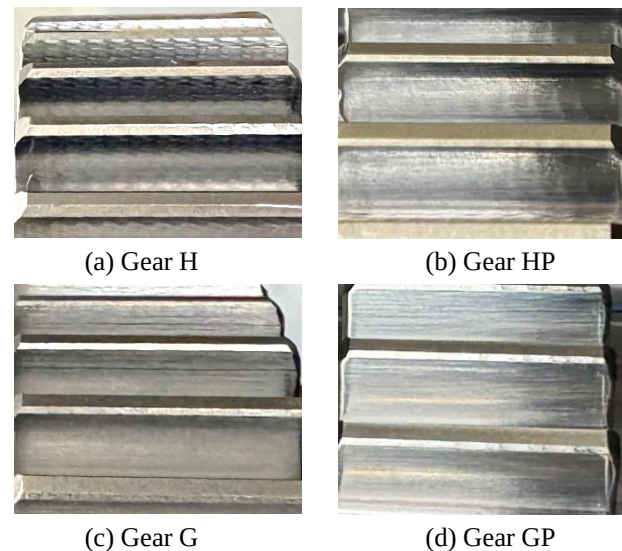


Fig. 1. Tooth flank appearances of each gear.

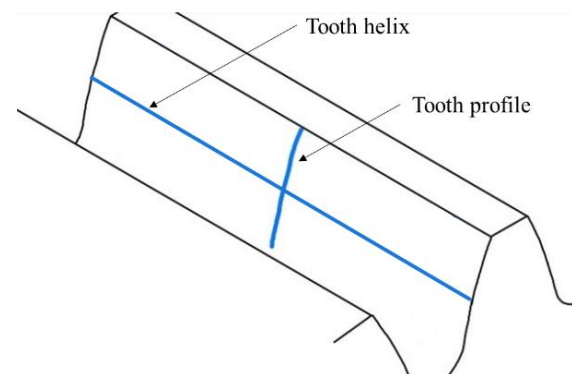


Fig. 2. Measurement positions of tooth profile and tooth helix deviations.

2.3 Preprocessing before evaluation

In this study, the DC component was removed from the measured tooth profile deviation curves and tooth helix deviation curves in the same manner as in previous studies. Therefore, the deviation curves after removal of the DC component were used for the subsequent shape deviation network analysis and cross-sectional shape evaluation.

2.3.1 Preprocessing and evaluation method for tooth profile deviations

The DC component was removed from the measured tooth profile deviation curves. The tooth profile deviation curves of the right and left tooth flanks of the i -th tooth are denoted as $\hat{f}_{\alpha Ri}(\kappa)$ and $\hat{f}_{\alpha Li}(\kappa)$, respectively. Here, κ_α represents the position in the roll length direction within the tooth profile evaluation range. When the DC components of these curves are denoted as $f_{\alpha DC Ri}(\kappa)$ and $f_{\alpha DC Li}(\kappa)$, respectively, the tooth profile deviation curves after removal of the DC component, $f_{\alpha Ri}(\kappa)$ and $f_{\alpha Li}(\kappa)$, are expressed as follows:

$$f_{\alpha Ri}(\kappa) = \hat{f}_{\alpha Ri}(\kappa) - f_{\alpha DC Ri}(\kappa) \quad (1)$$

$$f_{\alpha Li}(\kappa) = \hat{f}_{\alpha Li}(\kappa) - f_{\alpha DC Li}(\kappa) \quad (2)$$

Here, $f_{\alpha DC Ri}(\kappa)$ and $f_{\alpha DC Li}(\kappa)$ are the DC components corresponding to the mean values of the respective tooth profile deviation curves. In this study, the tooth profile deviation network was constructed using the tooth profile deviation curves after removal of the DC component according to Eqs. (1) and (2).

2.3.2 Preprocessing and evaluation method for tooth helix deviations

The DC component was removed from the measured tooth helix deviation curves in the same manner as for the tooth profile deviations. The tooth helix deviation curves of the right and left tooth flanks of the i -th tooth are denoted as $\hat{f}_{\beta Ri}(\chi)$ and $\hat{f}_{\beta Li}(\chi)$, respectively. Here, χ_β represents the position in the axial direction of the cylindrical gear, namely, the face width direction. When the DC components of these curves are denoted as $f_{\beta DC Ri}(\chi)$ and $f_{\beta DC Li}(\chi)$, respectively, the tooth helix deviation curves after removal of the DC component, $f_{\beta Ri}(\chi)$ and $f_{\beta Li}(\chi)$, are expressed as follows:

$$f_{\beta Ri}(\chi) = \hat{f}_{\beta Ri}(\chi) - f_{\beta DC Ri}(\chi) \quad (3)$$

$$f_{\beta Li}(\chi) = \hat{f}_{\beta Li}(\chi) - f_{\beta DC Li}(\chi) \quad (4)$$

Here, $f_{\beta DC Ri}(\chi)$ and $f_{\beta DC Li}(\chi)$ are the DC components corresponding to the mean values of the respective tooth helix deviation curves. In this study, the tooth helix deviation network and the cross-sectional shape evaluation were performed using the tooth helix deviation curves after removal of the DC component according to Eqs. (3) and (4).

2.4 Evaluation method using the shape deviation network

In this study, the similarity of shape deviations among tooth flanks in each gear was evaluated using the shape deviation network proposed in our previous studies [3–8]. In this method, the deviation curve obtained from each tooth flank is regarded as one signal, and an adjacency matrix is constructed based on the correlation coefficients among tooth flanks. In this study, the left and right tooth flanks were analyzed independently for both the tooth profile deviation curves and the tooth helix deviation curves.

Using the deviation curves after removal of the DC component, the correlation coefficient r_{ij} between the i -th tooth flank and the j -th tooth flank was calculated. The correlation coefficient is an index representing the similarity between two deviation curves, and a value closer to 1 indicates that the two waveforms are more similar. The obtained correlation coefficients were stored in a number-of-teeth-by-number-of-teeth matrix, and this matrix was visualized as a pixel image by representing the values using colors. This enabled a comprehensive evaluation of the similarity among tooth flanks and periodic structures over the entire gear.

Furthermore, eigenvalue analysis was applied to the shape deviation network. Since the correlation coefficients can include negative values, they were converted into link weights w_{ij} ranging from 0 to 1 using the following equation, as in previous studies.

$$w_{ij} = \frac{r_{ij} + 1}{2} \quad (5)$$

The adjacency matrix whose elements are the link weights was denoted as $\mathbf{A}$, and the degree matrix whose diagonal elements are the sums of the link weights connected to each node was denoted as $\mathbf{D}$. The graph Laplacian $\mathbf{L}$ was then defined as follows:

$$\mathbf{L} = \mathbf{D} - \mathbf{A} \quad (6)$$

In this study, the second smallest eigenvalue of the graph Laplacian was used to evaluate the overall strength of the connections in the shape deviation network. A larger second smallest eigenvalue indicates that the deviation curves among tooth flanks are more similar and that the overall network is more strongly connected. However, the second smallest eigenvalue does not directly represent the magnitude of the deviations or gear accuracy itself. Rather, it is an index representing the similarity of the shapes among tooth flanks.

2.5 Cross-sectional shape evaluation method based on tooth helix deviations

In this study, in addition to the evaluation using the shape deviation network, the characteristics of the cross-sectional shape composed of the tooth helix deviations of the left and right tooth flanks were evaluated using the cross-sectional shape evaluation method proposed in our previous study [9]. In this method, characteristic

deformation tendencies appearing in the gear cross section are evaluated by adding or subtracting the left and right tooth helix deviation curves.

By adding the left and right tooth helix deviation curves, an evaluation curve corresponding to a deformation tendency in which the left and right tooth flanks incline in the same direction was obtained. In this study, this evaluation is referred to as the Trapezium-shape evaluation. On the other hand, by subtracting the left and right tooth helix deviation curves, an evaluation curve corresponding to a deformation tendency in which the left and right tooth flanks incline in different directions was obtained. In this study, this evaluation is referred to as the Parallelogram-shape evaluation.

A linear approximation was then applied to each evaluation curve, and its slope was calculated. By calculating this slope for each tooth, the distributions of the Trapezium-shape evaluation and the Parallelogram-shape evaluation over the entire gear were evaluated. In this study, the cross-sectional shape evaluation was used as a supplementary evaluation to support the results obtained from the tooth helix deviation network.

3 Results and discussion

This chapter presents the results obtained by applying the shape deviation network and the cross-sectional shape evaluation to the four types of gears with different finishing processes. First, the pixel images and the second smallest eigenvalues of the shape deviation networks based on tooth profile deviations are compared. Next, the shape deviation networks based on tooth helix deviations are compared, and the effects of the finishing process and the mounting condition during measurement on the evaluation results are discussed. Finally, the cross-sectional shape evaluation results composed of the left and right tooth helix deviations are presented, and their relationship with the characteristics obtained from the tooth helix deviation network is examined.

3.1 Evaluation using tooth profile deviation networks

Table 2 shows the pixel images of the shape deviation networks obtained from the tooth profile deviations. In this table, the different finishing processes are arranged in rows, and the pixel images obtained from the left and right tooth flanks are shown in columns. In the pixel images, the rows and columns correspond to the tooth numbers, and the color of each pixel represents the magnitude of the correlation coefficient between tooth flanks. The color scale of each pixel image was adjusted individually to clearly show the periodic structures in each gear.

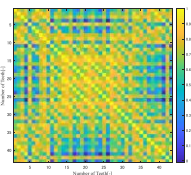
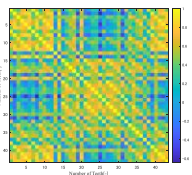
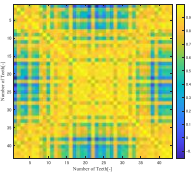
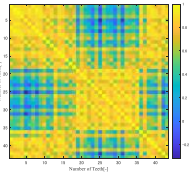
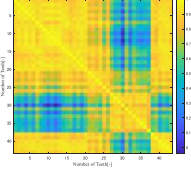
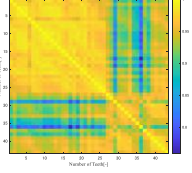
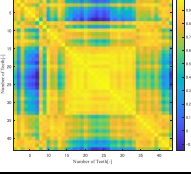
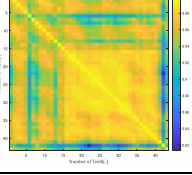
In the tooth profile deviation networks shown in Table 2, different pixel images were obtained depending on the finishing process. In Gear H and Gear HP, which were finished by hard hobbing, a narrow grid-like pattern varying at intervals of three teeth and a wide grid-like pattern varying once over the entire gear were observed. In contrast, in Gear G and Gear GP, which were finished by gear grinding, regions with high correlation were more

widely observed than in the other gears, and the wide grid-like pattern was relatively distinct.

Fig. 3 shows the second smallest eigenvalues of the graph Laplacian obtained from the tooth profile deviation networks. The values shown in this figure are the mean values of the second smallest eigenvalues calculated independently from the left and right tooth flanks. Among the gears measured in this study, Gear GP showed the highest second smallest eigenvalue. The second smallest eigenvalue does not directly represent the magnitude of the deviations or the superiority of gear accuracy. Rather, it is an index representing the similarity of the deviation curves among tooth flanks, namely, the overall strength of the connections in the network. Therefore, when the entire gear is evaluated comprehensively based on the correlation relationships among tooth flanks, Gear GP can be judged to have the highest geometric consistency, with similar tooth profile deviation curves formed over the entire gear.

The periodic structures observed in the tooth profile deviation networks of the hard-hobbed gears may be related to the fact that these gears were machined using a three-start hob. This suggests that grid-like patterns appearing at intervals of three or six teeth were formed due to the influence of the hard hobbing process.

Table 2. Pixel images of tooth profile deviation networks.

	Left tooth profile deviation pixel image	Right tooth profile deviation pixel image
Gear H		
Gear HP		
Gear G		
Gear GP		

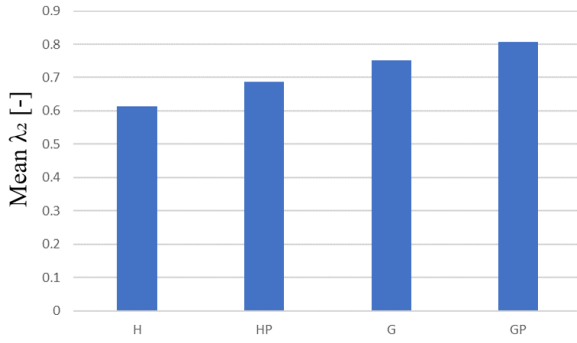


Fig. 3. Mean second smallest eigenvalues of tooth profile deviation networks.

3.2 Evaluation using tooth helix deviation networks

Table 3 shows the pixel images of the shape deviation networks obtained from the tooth helix deviations. As in the tooth profile deviation networks, the different finishing processes are arranged in rows, and the pixel images of the left and right tooth flanks are shown in columns. The color scale of each pixel image was adjusted individually, as in the pixel images of the tooth profile deviation networks, to clearly show the periodic structures in each gear.

As shown in Table 3, clear periodic structures with a three-tooth interval were observed in the pixel images of Gear H and Gear HP, which were finished by hard hobbing, as was also observed in the tooth profile deviation networks. As mentioned above, these gears were machined using a three-start hob; therefore, this periodicity is considered to originate from the machining marks formed during hard hobbing. In addition, in Gear HP, regions with high correlation were more widely observed than in Gear H. This indicates that polish honing increased the similarity of the tooth helix deviation curves among tooth flanks.

Fig. 4 shows the second smallest eigenvalues of the graph Laplacian obtained from the tooth helix deviation networks. The values shown in this figure are the mean values of the second smallest eigenvalues calculated independently from the left and right tooth flanks. Gear HP showed a higher second smallest eigenvalue than Gear H, and Gear GP also showed a higher second smallest eigenvalue than Gear G. These results indicate that the gears finished by polish honing tended to have higher similarity among tooth helix deviation curves than the corresponding gears before polish honing.

On the other hand, Gear G and Gear GP showed lower second smallest eigenvalues than Gear H and Gear HP. This may be because the tooth helix deviation curves of the gear-ground gears were strongly affected by axial runout caused by the mounting condition during measurement. A slight inclination may have occurred when the gear was mounted on the measuring instrument, and this influence may have appeared as a large inclination component in the tooth helix deviation curves. Although this influence was also slightly observed in the tooth profile deviations, it was more clearly observed in the tooth helix deviations. As a result, the inclination component caused by the axial runout of the gear was strongly reflected in the tooth helix deviation network and the second smallest eigenvalue. These results indicate that the shape deviation network is sensitive not only to differences in the relationships among tooth flanks caused by finishing processes, but also to axial runout and mounting conditions during measurement. This also suggests that the mounting condition during measurement should be carefully controlled when applying the proposed method.

Table 3. Pixel images of tooth helix deviation networks.

	Left tooth helix deviation pixel image	Right tooth helix deviation pixel image
Gear H		
Gear HP		
Gear G		
Gear GP		

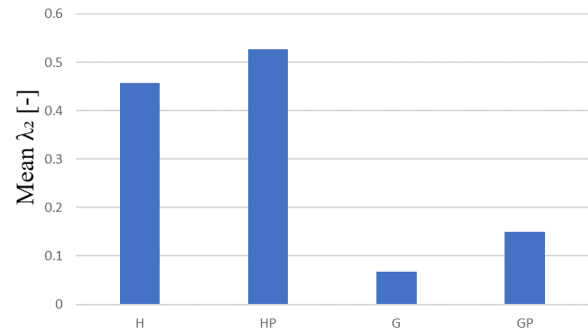


Fig. 4. Mean second smallest eigenvalues of tooth helix deviation networks.

3.3 Cross-sectional shape evaluation based on tooth helix deviations

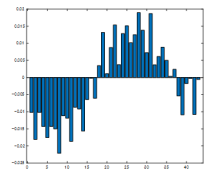
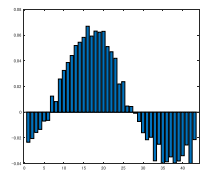
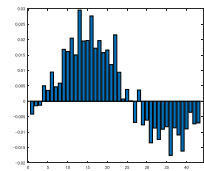
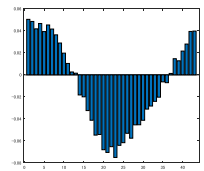
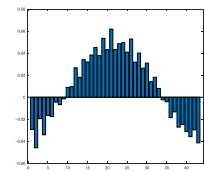
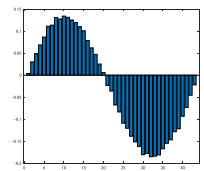
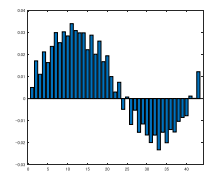
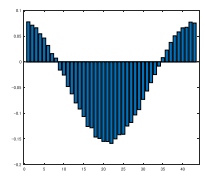
Table 4 shows the cross-sectional shape evaluation results obtained using the left and right tooth helix deviation curves. In this evaluation, an evaluation curve corresponding to the Trapezium-shape evaluation was obtained by adding the left and right tooth helix deviation curves, whereas an evaluation curve corresponding to the Parallelogram-shape evaluation was obtained by subtracting them. A linear approximation was applied to each evaluation curve, and the obtained slopes were arranged according to the tooth number.

The evaluation results show that the positions of the antinodes in the Trapezium-shape evaluation tended to correspond to the positions of the nodes in the Parallelogram-shape evaluation. This indicates that, for teeth in which the Trapezium-shape evaluation component appears strongly, the Parallelogram-shape evaluation component becomes small. Conversely, for teeth in which the Parallelogram-shape evaluation component appears strongly, the Trapezium-shape evaluation component becomes small. In other words, in the cross-sectional shape in the tooth helix direction, the two deformation components do not appear strongly at the same time; rather, one of the two components tends to become dominant.

This behavior differs from the tendency observed in the injection-molded plastic gears investigated in our previous study. In injection-molded plastic gears, characteristic periodic deformation distributions appear over the entire gear due to the gate positions and weld lines. On the other hand, in the steel gears investigated in this study, in addition to the similarity among tooth flanks caused by the finishing process, the inclination component caused by the mounting condition during measurement is considered to have been strongly reflected in the cross-sectional shape evaluation results.

Furthermore, the periodic structures and differences in the second smallest eigenvalues observed in the tooth helix deviation networks may correspond to the slope distributions obtained from the cross-sectional shape evaluation. Therefore, by using the shape deviation network and the cross-sectional shape evaluation together, it was shown that not only the relative similarity among tooth flanks but also changes in the cross-sectional shape composed of the left and right tooth flanks can be evaluated.

Table 4. Cross-sectional shape evaluation results based on tooth helix deviations.

	Trapezium-shape evaluation	Parallelogram-shape evaluation
Gear H		
Gear HP		
Gear G		
Gear GP		

4 Conclusions

In this study, the shape deviation network and the cross-sectional shape evaluation were applied to four types of steel gears: a hard-hobbed gear, a gear finished by polish honing after hard hobbing, a ground gear, and a gear finished by polish honing after gear grinding. The obtained results were compared and evaluated. The findings obtained in this study are summarized as follows.

1. In the tooth profile deviation network, Gear GP showed the highest second smallest eigenvalue. This result indicates that polish honing after gear grinding tends to increase the similarity of tooth profile deviation curves among tooth flanks.
2. In the tooth helix deviation network, a periodic structure with a three-tooth interval was observed in the hard-hobbed gears. This periodicity may be attributed to machining using a three-start hob.
3. In the tooth helix deviation network, Gear HP and Gear GP, which were finished by polish honing, showed higher second smallest eigenvalues than Gear H and Gear G, respectively, which were the corresponding gears before polish honing. This result indicates that polish honing tends to increase the similarity of tooth helix deviation curves among tooth flanks.
4. In the tooth helix deviation networks of the gear-ground gears, the influence of the inclination component caused by axial runout during measurement was considered to have appeared. This result indicates that the proposed method is sensitive not only to differences in finishing processes but also to the mounting condition during measurement.
5. In the cross-sectional shape evaluation based on tooth helix deviations, the positions of the antinodes in the Trapezium-shape evaluation tended to correspond to the positions of the nodes in the Parallelogram-shape evaluation. This result suggests that, in steel gears, one of the deformation components may become dominant in the cross-sectional shape.

These results demonstrate that the combined use of the shape deviation network and the cross-sectional shape evaluation enables the evaluation of the relative relationships among tooth flanks and the characteristics of the cross-sectional shape in gears with different finishing processes.

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Challenge of Finishing Hardened Gears by Ultra-High-Speed Hobbing

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Abstract. Gears in EV transmissions have attracted attention as a source of vibration and noise, and the production of higher-quality gears is required. However, producing high-precision gears will increase manufacturing costs, so efforts must be made to reduce costs. The goal of this research is to use ultra-high-speed hobbing to finish high-hardness materials, which is normally done by grinding. However, even if ultra-high-speed hobbing solves problems in terms of environment and processing efficiency, the value of use decreases if the processing accuracy is significantly inferior to that of grinding. There are various causes of errors in hobbing gears, but polygon errors and feed mark errors inevitably occur in hobbing, which is an intermittent cutting process. This has a large impact on the accuracy of the tooth surfaces and is thought to cause the gear pairs to not mesh smoothly, leading to vibration and noise during gear operation. The accuracy and surface quality of gears is actually investigated in detail under ultra-high cutting speeds exceeding 1 000 m/min, and the negative effects on the product were discussed as well as the impact on noise during driving.

1 Introduction

In recent years, there has been a demand for affordable machinery—such as automobiles—that combines energy efficiency, quiet operation, and durability. Gears are widely used in machinery due to their high transmission efficiency and the ease with which speed can be increased or decreased by varying the sizes of meshing gears. Consequently, factors such as transmission efficiency, noise, productivity, and service life directly impact the performance and cost of the machinery. Gear manufacturing requires high-precision machining of hardened, high-hardness materials at low cost and high efficiency. Hobbing is the most common method for cutting gear teeth. It involves cutting while rotating both the workpiece and the hob—a tool featuring multiple cutting edges arranged in a helix. While hobbing is an efficient process, it entails specific types of errors. Because the process relies on generating the tooth profile using straight cutting edges, a discrepancy known as "polygonal error" arises between the theoretical and actual tooth profiles. Additionally, the hob is fed along the tooth trace after each full rotation, creating ridges at the boundaries between the sections cut before and after the feed; these are known as "feed marks." Although reducing the number of hob threads or the feed rate is effective in mitigating these errors, doing so compromises machining efficiency.

This study proposes increasing the cutting speed to enhance machining precision without sacrificing efficiency [1]. While typical hobbing is performed at

speeds of 300 m/min or less, this method employs ultra-high speeds exceeding 500 m/min. A common gear manufacturing sequence involves hobbing, followed by heat treatment (quenching), and finally finishing via tooth flank grinding. This research aims to achieve the finishing process through ultra-high-speed hobbing. While this approach could eliminate the need for grinding machines—potentially reducing costs and improving efficiency—the effects on machining performance and hob wear due to heat remain unknown. In this study, gears are machined using ultra-high-speed hobbing while varying the peripheral speed; the practicality of the process and optimal parameters are then evaluated through measurements of machining accuracy and surface roughness of hobbled gears.

2 Gear specifications and machining conditions

The specifications for the gears machined in this study were as follows: module of 3.0, the number of teeth of 31, tip circle diameter of 105 mm, normal pressure angle of 20°, helix angle of 20° (right-hand), and face width of 40 mm; the workpiece material was SCM415. All gears were cut using the hobbing process and subsequently subjected to carburizing and quenching. Post-quenching finishing was performed via ultra-high-speed hard hobbing. A Nidec GE15HS hobbing machine was used, equipped with a carbide hob featuring a single thread, an outer diameter of 75 mm, and 18 gashes. Hard hobbing

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was conducted using the climb cutting method; the axial generating feed rate was kept constant at 0.3 mm/t-rev, while the hob peripheral speed was varied for comparative analysis. Six hob peripheral speeds were set: 100, 200, 300, 500, 750, and 1000 m/min. The same hob was used for all tests, with machining performed sequentially from the lowest speed to the highest. After completing the run at 750 m/min, the hob was shifted, and machining at 1000 m/min was performed using a different section of the hob's cutting edges.

3 Heat generated during hard hobbing

Increasing the hob cutting speed leads to a higher rate of chip formation and increased frictional heat, resulting in greater heat generation. This chapter examines these conditions. Figure 1 shows still images extracted from video footage of the machining process. Table 1 presents the surface temperatures of the workpiece gear and the hob measured with a contact thermometer after machining, while Figure 2 shows microscopic images of the chips produced at each cutting speed.

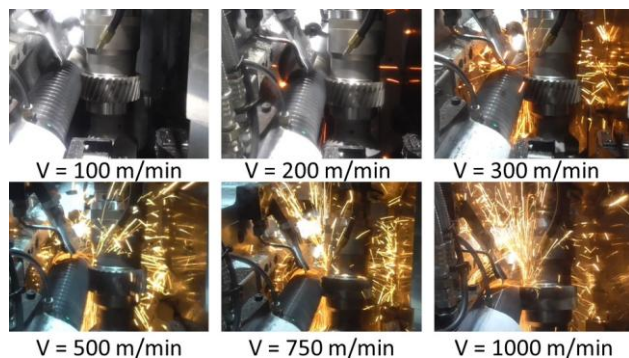


Fig. 1. Still images extracted from video at each cutting speed.

Table 1. Surface temperature after processing.

Processing speed	m/min	100	200	300	500	750	1000
Work Temperature	°C	24.0	28.5	34.6	37.9	40.0	37.0
Tool Temperature	°C	19.0	20.1	23.8	25.8	28.1	24.1

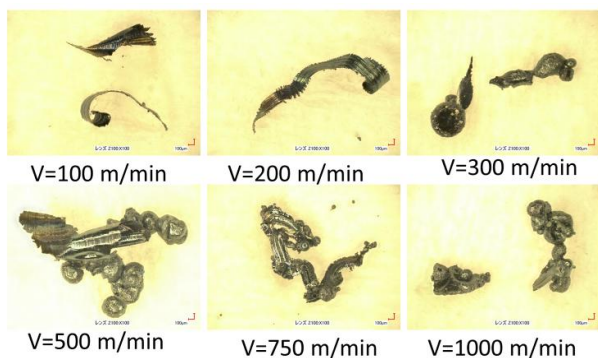


Fig. 2. Microscopic images of the chips produced at each cutting speed.

Sparks were generated at a peripheral speed of 200 m/min or higher; the sparking intensified and became brighter as the speed increased. Furthermore, Figure 2 shows signs of partial melting of the chips at speeds of

500 m/min and above, with the chips becoming more rounded as the speed increased. These observations imply a rise in chip temperature. Meanwhile, the gear tooth surface temperature after machining peaked at 40.0°C at a peripheral speed of 750 m/min, whereas at 1000 m/min, it was 37.0°C—lower than at 750 m/min. This is attributed to the shorter machining time, which resulted in minimal heat diffusion into the gear. Since the post-machining tooth surface temperature remained low under all conditions, the likelihood of thermal deformation or phase changes in the gear is minimal. Like the tooth surface, the hob temperature also peaked at 750 m/min. Given the smaller temperature fluctuation compared to the gear surface and the material's low thermal conductivity, it can be concluded that there was limited heat transfer to the hob. While these findings suggest that the majority of the generated heat was dissipated through the chips, the potential for localized high temperatures during machining—combined with observations of chips wrapping around the hob—raises the possibility that high-temperature chips adhering to the tool could have an adverse effect on it.

4 Hob damage

Hard hobbing was performed using the same position on the hob at cutting speeds of 100, 200, 300, 500, and 750 m/min, in that order. Between each machining run, the wear on the hob's rake and flank faces was examined. While no visible hob damage was observed after machining at speeds of 100, 200, 300, or 500 m/min, localized damage became visible after machining at 750 m/min; consequently, this damage was examined. Figure 3 shows the cutting edge where the damage was most pronounced. Subsequently, the hob was shifted axially to use a fresh cutting edge region for machining at 1,000 m/min; as damage was observed in this instance as well, the resulting damage was examined. The captured damage at 1,000m/min are shown in Figure 4. Cutting edge numbers are designated based on the generating center tooth.

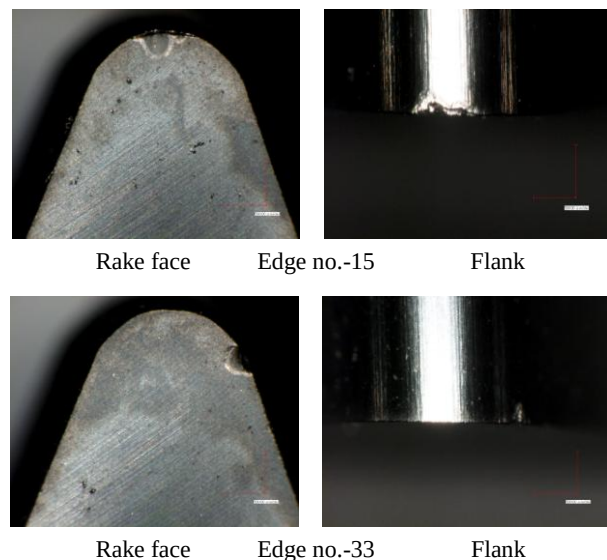


Fig. 3. Damage in cutting edges after hobbing at 750 m/min.

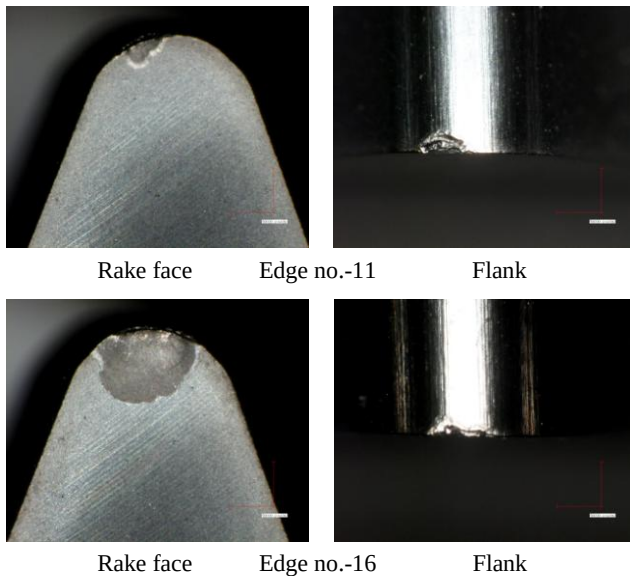


Fig. 4. Damage in cutting edges after hobbing at 1,000 m/min.

After hobbing at a cutting speed of 750 m/min, wear was observed on the rake face near the corner of the cutting edge; damage was particularly pronounced in the rough-machining zone. For the cutting edge used at 1000 m/min, in addition to the same type of wear seen at 750 m/min, significant damage resembling flaking was observed on the rake face as shown in the edge number of -16. A comparison between the generation diagram and the worn cutting edge suggested that damage to the cutting edge responsible for generating the area near the base circle of the right tooth flank would likely have a significant impact on the tooth flank itself; consequently, an investigation into the effect on the gear tooth flank accuracy was conducted.

5 Tooth flank accuracy

A CLP35 manufactured by TRP-Osaka Seimitsu Kikai was used to measure tooth flank topography. Measurements were taken at three different locations along the tooth profile of a single tooth and deviations from an involute helicoid were plotted on a single graph. As the variations due to peripheral speed and the differences between the right and left tooth flanks were minimal, Figure 5 presents the measurement results for the right tooth flank at peripheral speeds of 500, 750, and 1000 m/min.

No significant effects of hob cutting-edge damage were observed in the tooth profile deviation curves. However, periodic undulations in the tooth profile deviation were observed at all peripheral speeds. The period of these undulations was approximately from 9 to 10 mm, which corresponds to the normal pitch (9.35 mm). Therefore, it can be concluded that the undulations were caused by hob runout. To reduce these periodic undulations, it is necessary to improve the hob support and reduce eccentricity.

To investigate the influence of hob cutting-edge damage on the tooth surface in greater detail, the number of measured tooth profiles was increased and the measurement range was expanded for a more comprehensive evaluation. The measurement range was set to 38 mm—excluding 1 mm at each end of the 40 mm face width—and tooth profile measurements were taken at 24 equally spaced intervals along the tooth trace. The spacing between measurement points along the tooth trace was approximately 1.65 mm. Furthermore, a moving average method was used with a 1 mm averaging interval to remove waviness caused by hob runout and then displayed the tooth profile deviation components on an enlarged scale. The results are shown

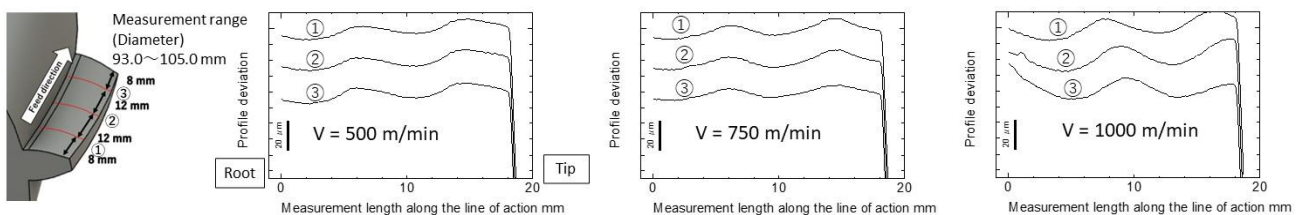


Fig. 5. Measured profile deviations in the right representative flanks.

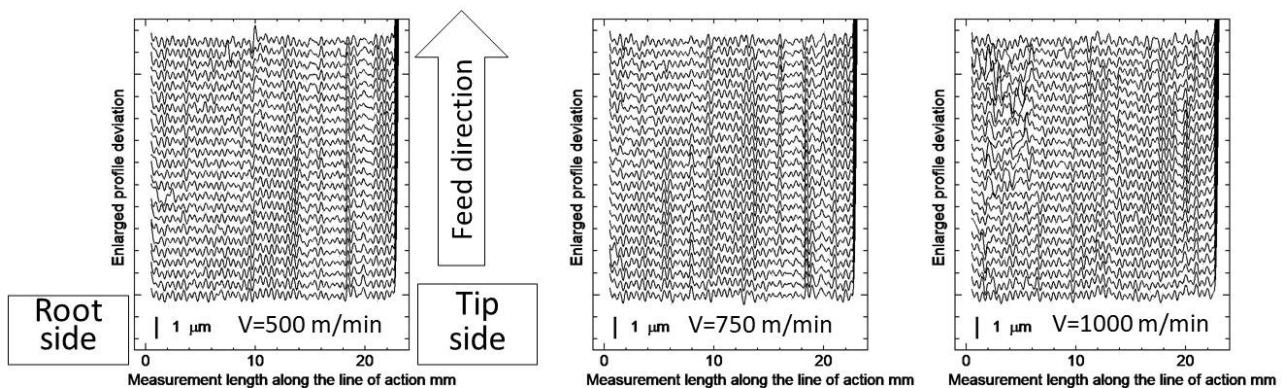


Fig. 6. Measured profile deviations in the right flanks.

in Figure 6.

At cutting speeds of 500 m/min and 750 m/min, no significant differences were observed based on the measurement position. After removing the waviness, the tooth profile deviation components remained below 1 μm across the entire working surface, indicating a high-quality tooth surface. In contrast, at a cutting speed of 1000 m/min, slight irregularities in some profile deviation curves were observed near the tooth root. As the location of these irregularities corresponded to the area generated by the worn cutting edge, they are attributed to hob wear. However, the magnitude of these irregularities was approximately 1 μm —far smaller than the waviness caused by hob runout—and because they were confined to the vicinity of the tooth root, they are not considered to affect the gear's operational performance.

6 Influence of hob damage around tooth root

It has become evident that the hob damage had no adverse effect on the working tooth flanks. However, given the severe damage observed on the hob cutting edges—specifically extending from the corners to the rounded sections—a more detailed investigation was conducted on the gear surface near the tooth root, where these edges are presumed to engage. To examine the surface roughness near the tooth root, surface roughness measurements were performed using a contact-type surface roughness tester (SURFCOM NEX, manufactured by Tokyo Seimitsu Co., Ltd.). Measurements were taken at three locations along the face width: the middle and upper and lower positions 5 mm from each end. The results are shown in Figure 7. Roughness profiles are presented for positions 5 mm from the bottom end (1), the middle of the face width (2), and 5 mm from the top end (3). To focus on the replication of the cutting edge geometry, a band-pass filter was used to remove waviness components with wavelengths exceeding 3 mm and short-wavelength components under 80 μm . In the figures, the left side corresponds to the tooth root and the right side to the tooth tip; alignment was based on the start of the tooth tip chamfer, and a vertical line indicates the diameter of the base circle.

At a cutting speed of 500 m/min, where no hob damage was observed, there were no differences attributable to the measurement position; in contrast, at speeds of 750 and 1000 m/min—where hob damage did occur—surface irregularities increased in magnitude from the bottom end to the top end in the region closer to the tooth root than the base circle. Since no significant irregularity was observed at measurement position (1), it is inferred that wear progressed during the hard hobbing process.

7 Conclusions

Hard Finishing was performed via ultra-high-speed hobbing on carburized and quenched gears with a

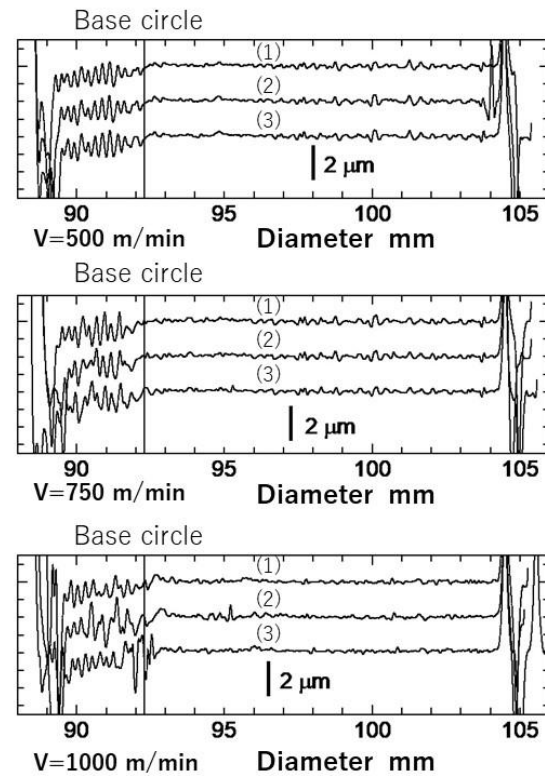


Fig. 7. Measured surface roughness including working flank and tooth root region below base circular diameter

module of 3. No damage to the hob cutting edges was observed at cutting speeds of 500 m/min or lower. However, damage occurred at speeds of 750 m/min and 1000 m/min, with flaking observed on the rake face at 1000 m/min. Potential causes include the impact and cutting forces resulting from high-speed contact between the cutting edge and the hard gear material, as well as localized heating of the tool due to cutting heat; a combination of these factors is also likely at play.

Nevertheless, even at the cutting speeds where tool damage was observed, the impact of such damage on the gear's surface integrity was limited, and the working tooth surfaces were not significantly affected. Future work will require addressing hob runout and investigating tool cooling methods to prevent excessive temperature rise.

References

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Analysis of the maneuverability and stress-strain state of the drawbar of a transport and technological machine for vacuum sealing and transportation of succulent feeds

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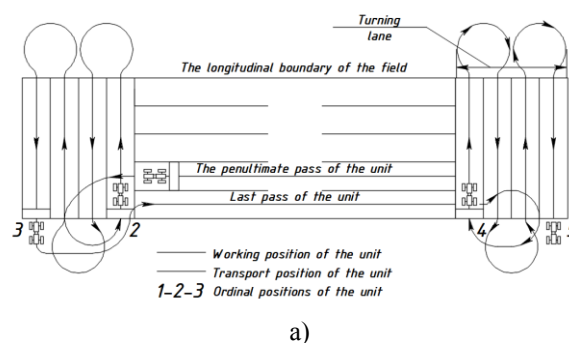
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Abstract The article presents the results of theoretical, experimental and computational studies of the parameters of the drawbar of a transport and technological unit designed for transporting succulent feed in containers. The purpose of the study was to substantiate the rational length of the drawbar, taking into account the indicators of maneuverability and structural strength. In the course of experimental studies, three drawbar length options were considered: 1,6, 1,8, and 2,0 m. To determine the motion parameters, visual fixation of the trajectory of the unit using an unmanned aerial vehicle was used. It was found that the turning radius of the transport and technological unit was 7,55 m, and the displacement of the single-axle trailer relative to the center of rotation was 1,65 m. The results showed the possibility of performing a turning maneuver with the unit fitting into a given transport corridor. A finite element analysis was performed to assess the strength of the selected structure. The maximum equivalent Mises stress was 122,7 MPa, which corresponds to 59,3% of the yield strength of St3sp steel (207 MPa). The maximum displacement was 2,074 mm, and the safety margin varied from 1,25 to 15. The results confirm the sufficient strength, rigidity and operability of the structure. The rationality of using a 2.0 m long drawbar is substantiated. The data obtained allows us to recommend the selected design parameters for further design and improvement of the unit. Studies have shown that increasing the drawbar length to 2,0 m improves maneuvering conditions and reduces the likelihood of tractor wheels coming into contact with trailer elements when turning. The proposed approach can be used in the development of similar transport vehicles during the operation of the unit.

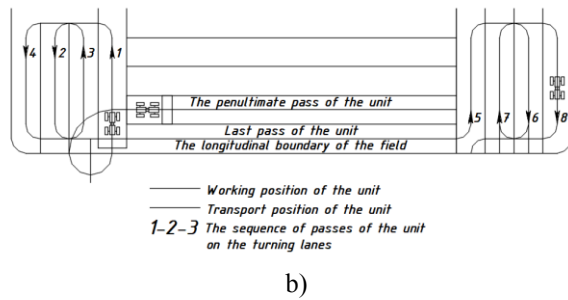
1 Introduction

Harvesting is the most intense technological process of agricultural production, and in order to ensure the maximum possible harvest of the grown crop with the highest quality of grain and with the minimum allowable labour and cost, it is necessary to fulfil technological and organizational requirements [1, 2]. The main mode of transport in agriculture is by road, accounting for up to 85% of cargo transportation. The use of a transport and technological machine (TTM) is one of the effective methods of increasing the productivity of TTM when used in agriculture [1, 2]. When performing agricultural work, the TTM cyclically repeats working and idle moves, and the order of cyclically repeating movement elements is a method of movement [1, 2]. The TTM performs many maneuvers, the maneuverability of the unit is characterized by agility, stability of movement and controllability. All these three indicators are interrelated and complement each other, and this is complemented by maneuverability [1, 2]. The movement of the corn harvesters is coordinated with the sowing methods. The work of silage harvesters is organized by the group method. The movement of combine harvesters for harvesting silage crops is carried out in a circular or

rolling manner. The corners of the pens are sheathed along a radius of 30-50 meters. Depending on the type of rotation, there are: loop-free, loop-free, reverse-running of the unit and needle-shaped turns (Figure 1). The method of movement of the units is chosen taking into account the size and configuration of the field [3, 4].



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a) loop mode of movement; b) loop-free mode of movement.

Fig.1. Schematics of the units during sowing and harvesting corn for grain and silage.

With all modes of movement, turns and arrivals make up a significant proportion of the unit's path. The shorter the race, the greater the percentage of the way is made up of turns and races. On average, they make up 10-12% of the total path of the unit, and in short sections – 40% or more. Incorrectly executed turns increase the idle distance of the unit to tens of kilometers per season, increase the turning lanes, and often reduce the quality of work. TTM perform 90, 180° turns and other angular turns [3, 4]. Analytically, the turning radius of a wheeled unit is usually determined based on the geometric construction

$$\rho = L \cot \alpha = \cot(\omega) \quad (1)$$

where, L is the distance between the axes of the driving and guide wheels, m ; α is the average angle of rotation of the guide wheels relative to the axis of the driving wheels; ω is the average angular velocity of rotation of the guide wheels over time t .

The speed of the TTM along the way is set depending on the condition of the roads, visibility, and the presence of descents, ascents, and turns. It should not exceed 10 km/hour on access roads and driveways.

90° turns are made with the circular method of movement of the tractor-transport unit, 180° turns with idle runs at the ends of the paddocks, when the TTM operates in a horizontal and diagonal manner [3, 4]. In this case, the main method of movement of silage harvesting units is circular. If, for any reason, the rolling method is adopted, the field is cut (by machines with front-mounted cutting units) into narrow and long paddocks, the size of which corresponds approximately to the daily productivity of the unit or group of units [3, 4]. In small flat areas, the width of which differs slightly from the length, it is recommended to use a circular motion method for harvesters with a continuous-cut header. With a significant rutting length and the presence of ridges on the field surface, racing methods of movement are advisable. With the group cleaning method, each unit drives into a separate pen. At the same time, during the execution of transport operations, the main deterrent is the increase in the width of the transport corridor when turning the TTM, which reduces the safety of use.

2 Materials and methods

To assess the strength of the drawbar of a transport and technological machine, a theoretical calculation of the force effects arising from its movement and braking was performed. The calculation was carried out for the total weight of the trailer $m=4350$ kg. The weight of the transport and technological machine is determined by the expression:

$$G = mg \quad (2)$$

where m is the total mass of the trailer, kg; $g=9,81$ m/s² is the acceleration of gravity.

At the accepted total weight, it is obtained:

$$\begin{aligned} G &= m \cdot g = 4350 \cdot 9,81 = 42673,5 \text{ N} \\ G &\approx 42,67 \text{ kN} \end{aligned} \quad (3)$$

When braking a transport and technological machine, the longitudinal force is determined taking into account the braking coefficient $a = 0,8g$:

$$\begin{aligned} F &= 4350 \cdot 9,81 \cdot 0,8 \\ F &= 34138,8 \text{ N} \\ F &\approx 34,14 \text{ kN} \end{aligned} \quad (4)$$

For a V-shaped drawbar, the longitudinal force is distributed between its two branches. With the length of the branch $l = 1950$ mm and half the distance between the attachment points $b/2 = 372,5$ mm, the angle of inclination of the branch is determined by:

$$a = \arcsin(372,5/1950) = 11^\circ \quad (5)$$

Taking into account the geometry of the V-shaped drawbar, the longitudinal force in one branch is:

$$\begin{aligned} \cos 11^\circ &= 0,9816 \\ F_l &= 34138,8 / (2 \cdot 0,9816), \\ F_l &= 17389 \text{ N} \\ F_l &\approx 17,39 \text{ kN} \end{aligned} \quad (6)$$

For a 16P channel with a cross-sectional area of $A = 20,2 \text{ cm}^2 = 2020 \text{ mm}^2$, the normal stress from the longitudinal force is determined by:

$$\begin{aligned} A &= 20,2 \text{ cm}^2 = 2020 \text{ mm}^2 \\ \sigma &= 17389 / 2020 \\ \sigma &= 8,61 \text{ MPa} \end{aligned} \quad (7)$$

In addition to the longitudinal force, the vertical load on the coupling device was taken into account during the operation of the structure. It is assumed to be equal to 10% of the weight of the trailer:

$$\begin{aligned} P &= 0,1G \\ P &= 0,1 \cdot 42673,5, \\ P &= 4267 \text{ N} \end{aligned} \quad (8)$$

With the estimated length of the drawbar branch $l = 1950$ mm, the bending moment is determined by:

$$\begin{aligned} M &= P \cdot L \\ M &= 4267 \cdot 1950, \\ M &= 8,32 \cdot 10^6 \text{ N/mm.} \end{aligned} \quad (9)$$

At the moment of resistance of the cross section $W = 109000 \text{ mm}^3$ The bending stress is:

$$\begin{aligned} W &= 109000 \text{ mm}^3 \\ \sigma_b &= (8,32 \cdot 10^6) / 109000 \\ \sigma_b &= 76,3 \text{ MPa} \end{aligned} \quad (10)$$

The total normal stress under the combined action of longitudinal and bending loads is defined as:

$$\sigma_{equ} = 76,3 + 8,6 = 84,9 \text{ MPa} \quad (11)$$

The value obtained is compared with the allowable stress of the drawbar material. With a yield strength of 235 MPa, the calculated safety margin is determined by the ratio of the yield strength to the maximum design stress:

$$\begin{aligned} \sigma_r &= 235 \text{ MPa} \\ n &= 235 / 84,9 = 2,77 \end{aligned} \quad (12)$$

Thus, the calculated total stress in the considered drawbar section does not exceed the yield strength of the material, and the safety margin coefficient is about 2,77, which indicates sufficient structural strength under the assumed design loads.

At the first stage, a constructive scheme of a transport and technological machine was developed in the Autodesk AutoCAD 2026 (Figures 2 and 3) software environment. Based on it, Autodesk Inventor 2026 has created a three-dimensional model of the structure, taking into account the basic geometric and layout parameters. The resulting model is used as a computational geometry for the subsequent analysis of the stress-strain state of the structure by the finite element method [9, 10].

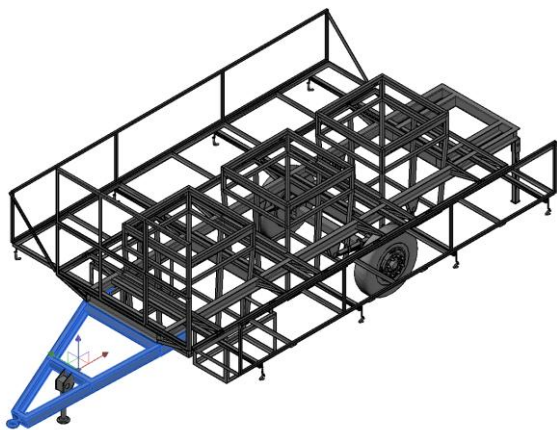


Fig. 2. General view of a three-dimensional model of a transport and technological machine.

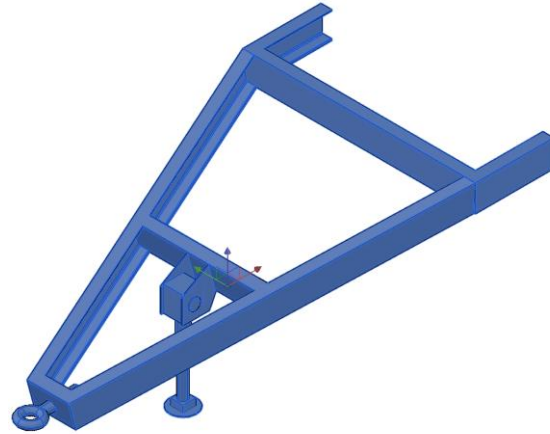
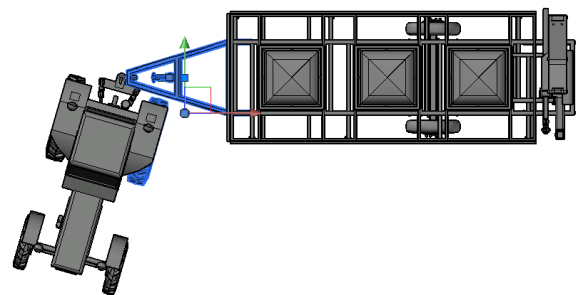
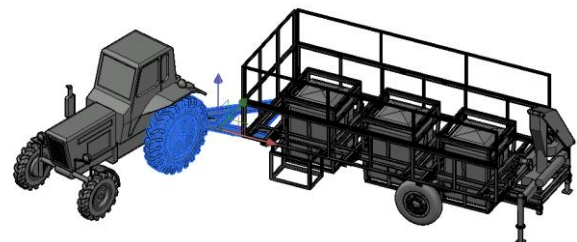


Fig. 3. General view of a three-dimensional model of the drawbar of a transport and technological machine.

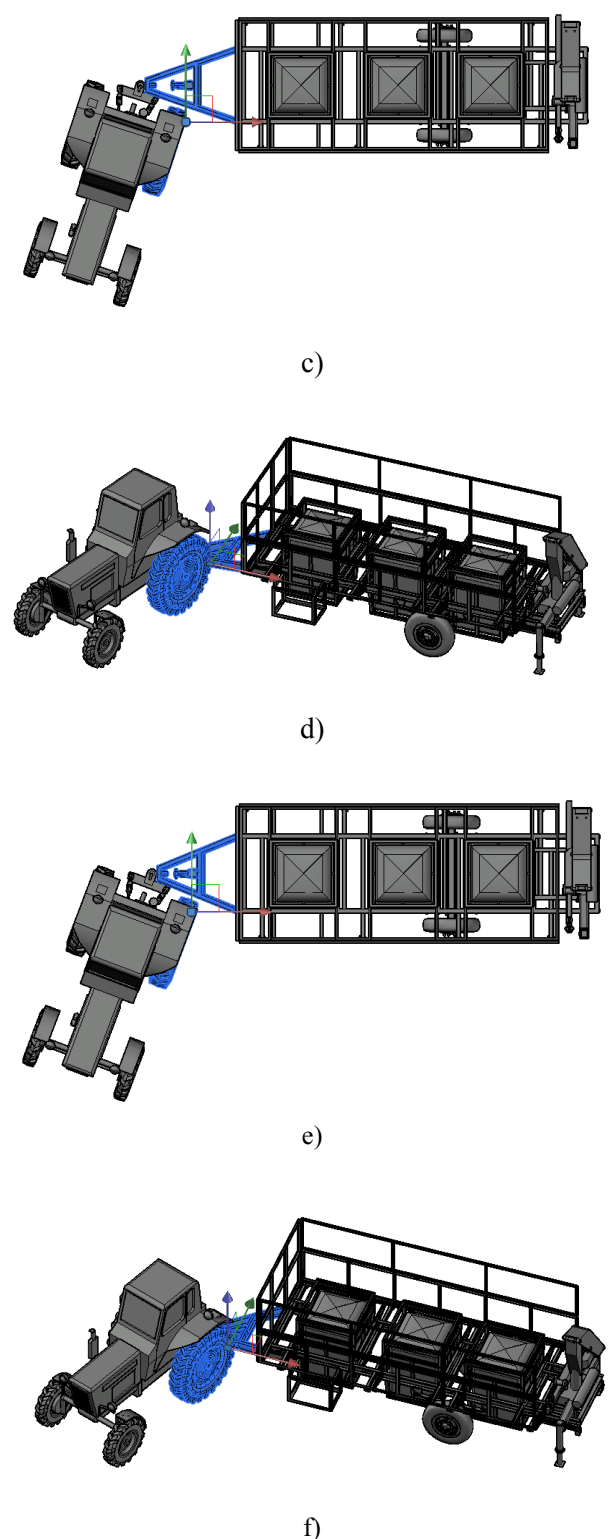
A three-dimensional model of a transport and technological machine has been created in AutoCAD, designed to visually simulate its movement when turning. Based on the top and side views, the relative position of the tractor wheels and drawbar was estimated when the angle of rotation and the geometric dimensions of the drawbar changed. During the simulation, the possibility of wheels contacting the drawbar elements was checked, which made it possible to justify its geometric parameters and ensure the necessary clearance between the structural elements during the maneuver (Figures 4 a-f).



a)



b)



a) drawbar size 1,6 meters (view from above); b) drawbar size 1,6 meters (side view); c) drawbar size 1,8 meters (view from above); d) drawbar size 1,8 meters (side view); e) drawbar size 2 meters (view from above); f) drawbar size 2 meters (side view);

Fig. 4. A three-dimensional model of a transport and technological machine for simulating movement and forming a transport corridor with a different coupling device dimensions in accordance with the specified position.

To determine the turning radius, during field tests, to fix motion parameters, and the position of the main structural elements, visual surveillance equipment was used in the form of an unmanned aerial vehicle, the DJI Mavic 3 Fly More Combo RB Unit and GOST standard 28307-2013 (Tractor trailers and semi-trailers. Test methods). A fragment of experimental studies on determining the turning radius and width of a transport corridor is shown in Figure 5 [5-8].



a)



b)

1 – Unmanned aerial vehicle quadcopter DJI Mavic 3 Fly More Combo RB Unit; 2 – TTM; a) motion detection, visual surveillance UAV quadcopter DJI Mavic 3 Fly More Combo RB Unit; b) TTM tests

Fig. 5. Fragment of experimental studies on determining the turning radius and width of a transport corridor.

When developing TTM, it is necessary to solve the problem of their fitting into a corridor of a given width while driving on turns. The condition for the correct rotation of the TTM is movement without damage to the mechanisms. These damages can occur if the wheels of the unit are sliding sideways or if the parts of the vehicle come dangerously close to each other. The parts of the unit should not come dangerously close, and a certain warranty interval must be observed between the parts approaching each other on a turn, the value of which depends on the design of the vehicle, speed and other driving conditions [5, 6].

To test the effectiveness of using TTM, experimental studies have been conducted, which allow us to draw a number of conclusions. In this regard, three different drawbar sizes of 1,6, 1,8, and 2 meters were chosen. The track width of the tractor on the front wheels is 1,54 m, the track width of the tractor on the rear wheels is 1,56 m, the track width of the trailer is 1,84 m. And to fix the motion parameters and the position of the main structural elements, visual surveillance tools were used in the work in the form of an unmanned aerial vehicle, the DJI Mavic 3 Fly More Combo RB Unit quadcopter (Figures 5 and 6). To determine the optimal conditions for the turning radius, a multifactorial experimental plan was implemented in accordance with the theory of experimental planning, which demonstrates a change in the angle of rotation of the controlled wheels of the TTM depending on the speed of the tractor and transport unit per turning radius.

The plan of the experiment is set out in Table 1.

Table 1. The plan of the experiment is set out.

Factor Level	Coded value	Factors		
		Angle of rotation of the driving wheels, α	The speed of movement, m/s	Trailer drawbar length, meters
Base level	0	25	1,245	1,8
Variation interval	ϵ	10	0,415	0,2
Upper level	+1	35	1,66	2,0
Lower level	-1	15	0,83	1,6
High Point	+1,68	41,8	1,94	2,14
Low Point	-1,68	8,2	0,55	1,46
Code designation	x_i	X_i	X_i	X_i

Levels and intervals of independent factors for determining optimal turning radius conditions Table 2.

Table 2. Levels and intervals of independent factors for determining optimal turning radius conditions.

An experiment, N	X_1	Angle of rotation of the driving wheels, α	X_2	Move ment speed, m/s	X_3	Trailer drawbar length, m	Turnin g radius of the unit, m
1	-1	15	-1	0,83	-1	1,6	6,6
2	+1	35	-1	0,83	-1	1,6	2,5
3	-1	15	+1	1,66	-1	1,6	7,1
4	+1	35	+1	1,66	-1	1,6	2,7
5	-1	15	-1	0,83	+1	2,0	7,3
6	+1	35	-1	0,83	+1	2,0	2,8
7	-1	15	+1	1,66	+1	2,0	7,55
8	+1	35	+1	1,66	+1	2,0	3,1
9	-1,68	8,2	0	1,245	0	1,8	12,11
10	+1,68	41,8	0	1,245	0	1,8	2,1
11	0	25	-1,68	0,55	0	1,8	4,1
12	0	25	+1,68	1,94	0	1,8	4,6
13	0	25	0	1,245	-1,68	1,46	3,6
14	0	25	0	1,245	+1,68	2,14	5,1
15	0	25	0	1,245	0	1,8	4,4
16	0	25	0	1,245	0	1,8	4,2
17	0	25	0	1,245	0	1,8	4,5
18	0	25	0	1,245	0	1,8	4,3
19	0	25	0	1,245	0	1,8	4,4
20	0	25	0	1,245	0	1,8	4,3

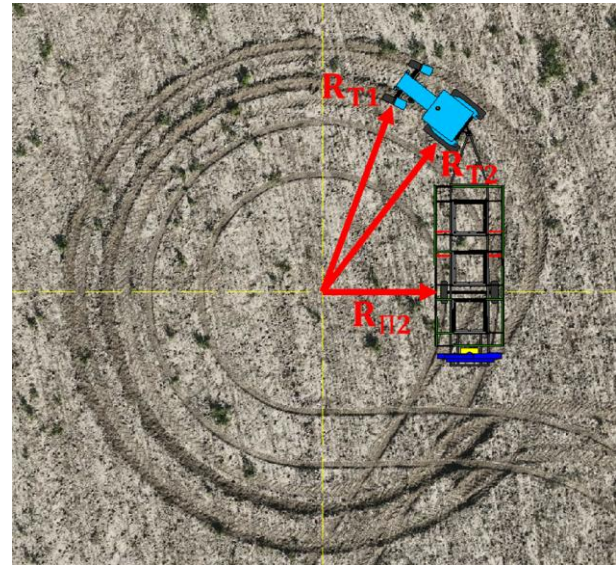
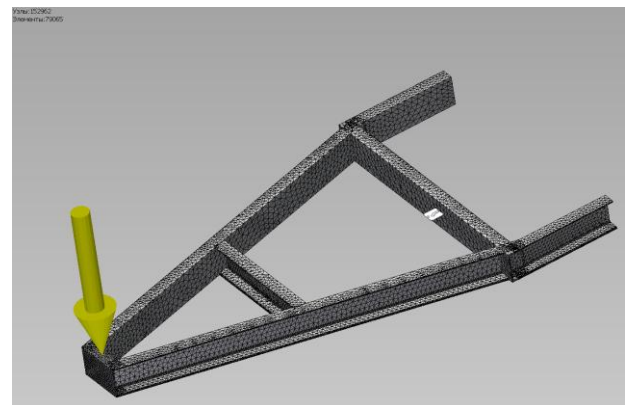


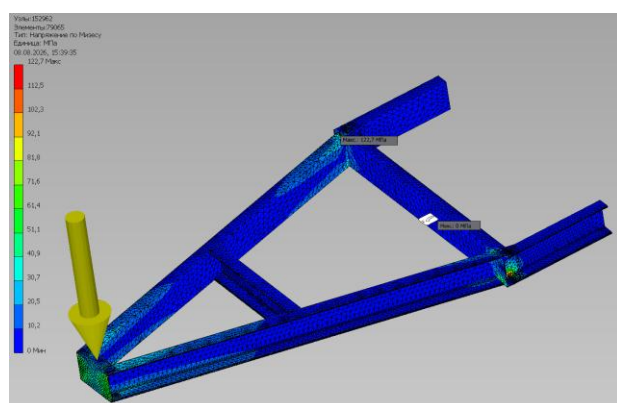
Fig. 6. A fragment of an experimental study to determine the turning radius and width of a transport corridor using a DJI Mavic 3 Fly More Combo RB Unit quadcopter UAV.L is the width of the tractor, m.

3 Results and discussion

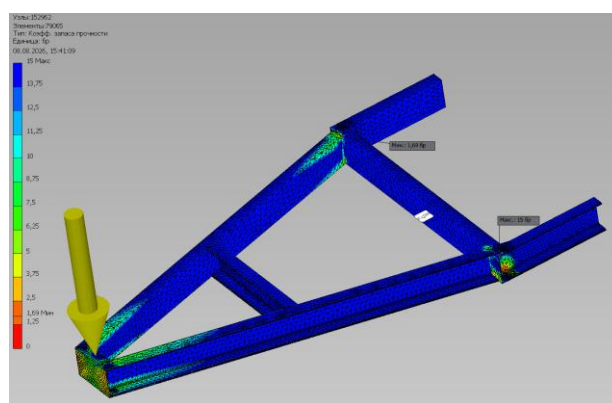
Thus, with the help of numerical modeling, a study was conducted of the stress and strain state of hot-rolled steel structures of the frame of a transport and technological machine under specified loading conditions. During the calculations, a vertical external force of 4367 N was applied to the frame structure, and its mechanical response was determined in the Autodesk Inventor 2026 software environment. Based on the results obtained, a visual analysis of the stress and strain conditions of the structures was carried out. The results of the finite element beam analysis performed in the Autodesk Inventor 2026 software environment are shown in (Figures 7 a-f).



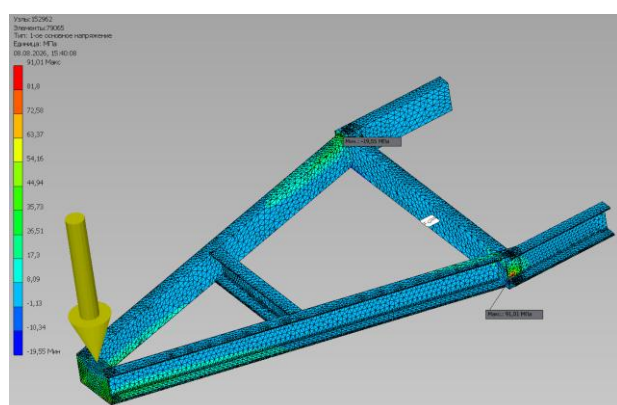
a)



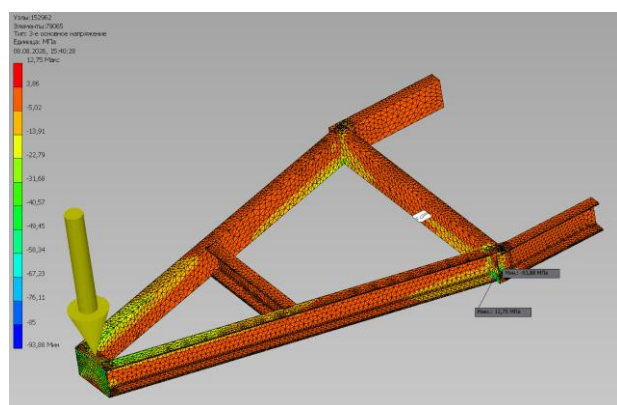
b)



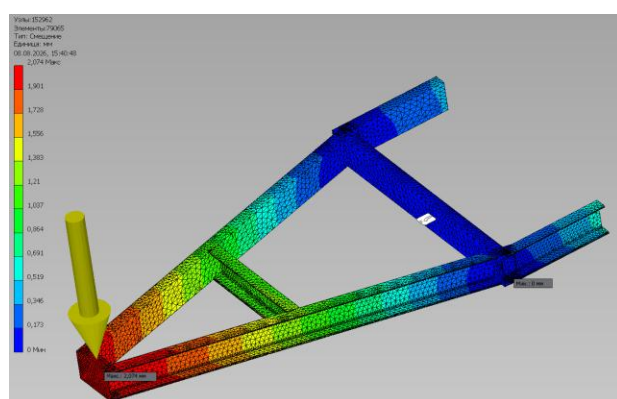
f)



c)



d)



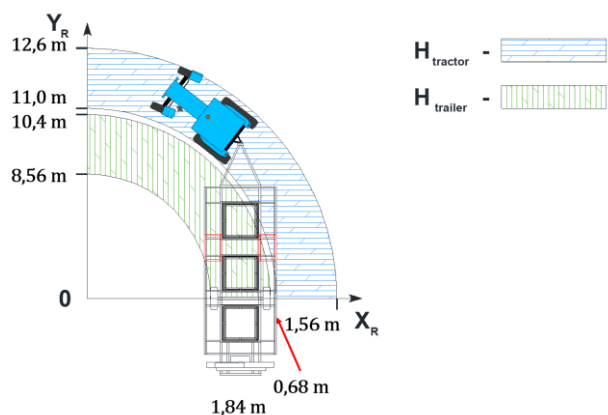
e)

a) the grid of the frame model; b) the mises stress; c) the 1st main stress; d) the 3rd main stress; e) displacement; f) safety margin coefficient.

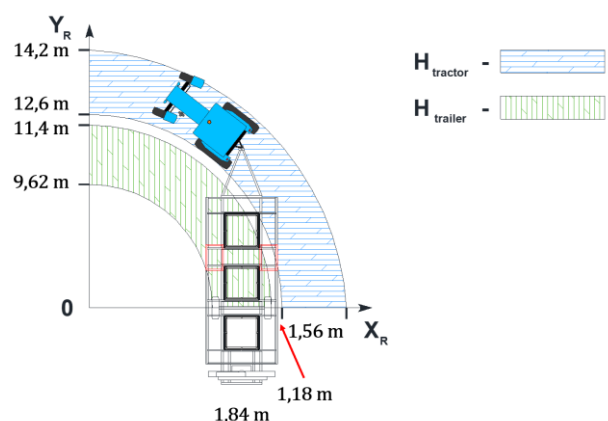
Fig. 7. The results of the analysis of the stress-strain state of the drawbar of the transport and technological machine.

The grid of the calculated model of the drawbar of a transport and technological machine and the applied loads are shown in Figure 7, a. Figure 6, b-f shows the results of finite element analysis: the distribution of equivalent stresses according to Mises, main stresses, total displacements and the safety margin coefficient. The maximum equivalent stress was 122,7 MPa, which corresponds to 59,3% of the yield strength of St3sp steel (207 MPa). The first and third main stresses were 91,01 MPa and 12,75 MPa, respectively. The maximum displacement was 2,074 mm, and the safety margin varied from 1,25 to 15. The greatest stresses and displacements were observed in the most stressed areas of the drawbar and the places where it connects to the frame. The results obtained confirm the sufficient strength and rigidity of the drawbar under the specified loading conditions.

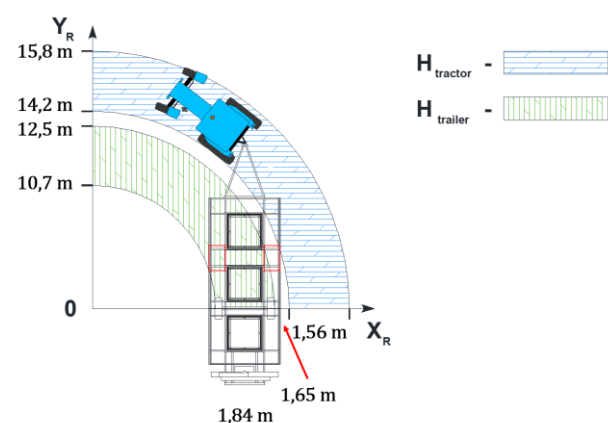
As a result of the conducted research, it was found that the turning radius of the TTM was 7,55 m. As can be seen from the results obtained, the trailer was shifted from the center of the turn to the curb by 1,65 m, due to the fact that this trailer is single-axle without a swivel frame. However, this does not prevent the maneuver from being performed, which makes it possible to ensure the safety of oncoming traffic and guarantees the entry of routes into the transport corridor (Figure 8). The radius of the trajectory of the rear trailer (semi-trailer) when turning is always less than the turning radius of the tractor, which is due to the kinematics of the movement of the unit links and the presence of a joint between the tractor and the trailer.



a)



b)



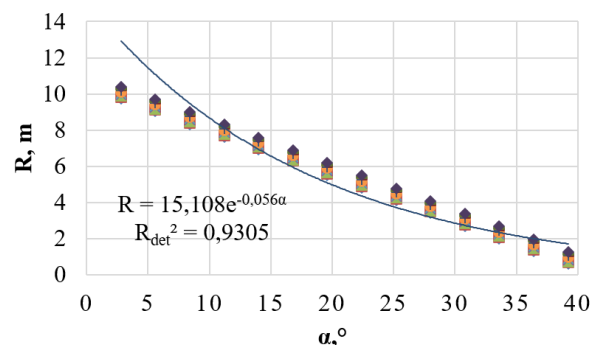
c)

H_{tractor} is the width of the tractor's transport corridor, m,
 H_{trailer} is the width of the trailer's transport corridor, m. a)
 1,6 meters; b) 1,8 meters; c) 2 meters.

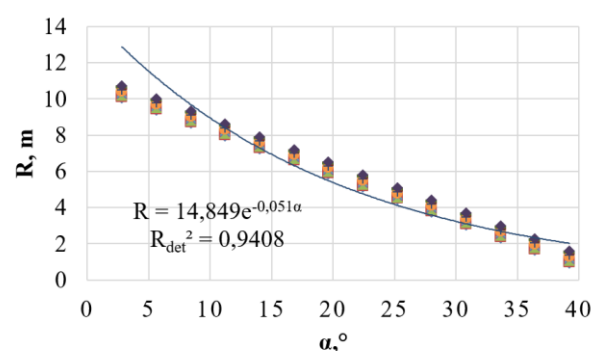
Fig. 8. Graph of the movement and formation of the transport corridor a transport and technological machine with different drawbar sizes, according to the specified position.

The obtained dependence is shown on a two-dimensional graph that demonstrates the change in the angle of rotation of the steered wheels of the transport and technological machine depending on the speed of the

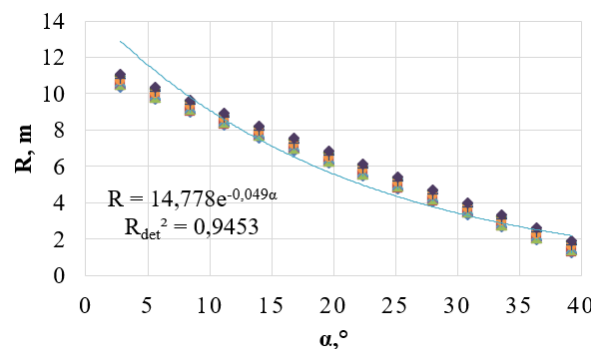
transport and technological machine per turning radius (Figure 9).



a)



b)



c)

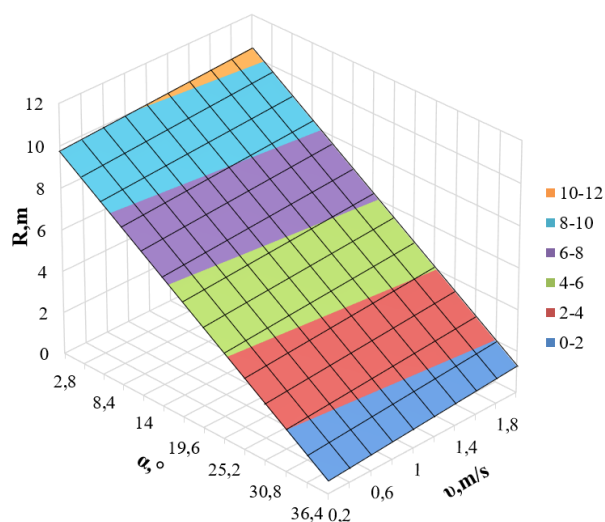
a) 1,6 meters; b) 1,8 meters and c) 2 meters

Fig. 9. Graph of the dependence of the turning radius of a tractor-transport unit on the angle of rotation of the driving wheels at different lengths of the drawbar.

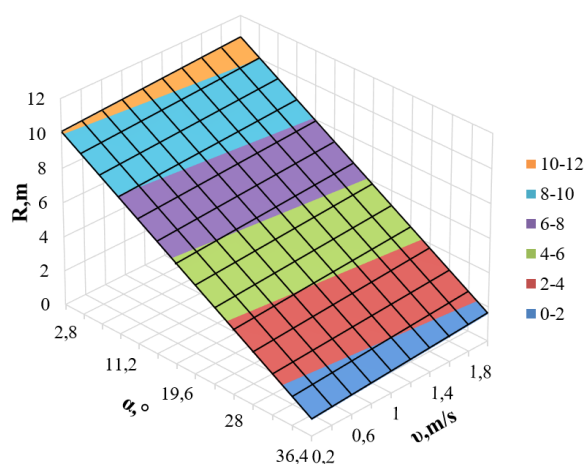
To determine the optimal conditions for the turning radius, a multifactorial experimental plan was implemented in accordance with the theory of experimental planning, which demonstrates a change in the angle of rotation of the controlled wheels of the TTM depending on the speed of the tractor and transport unit per turning radius. Based on the results of experimental studies, the following regression equation was obtained:

$$Y = 7,812 - 0,251 X_1 + 0,37 X_2 + 1,597 X_3. \quad (13)$$

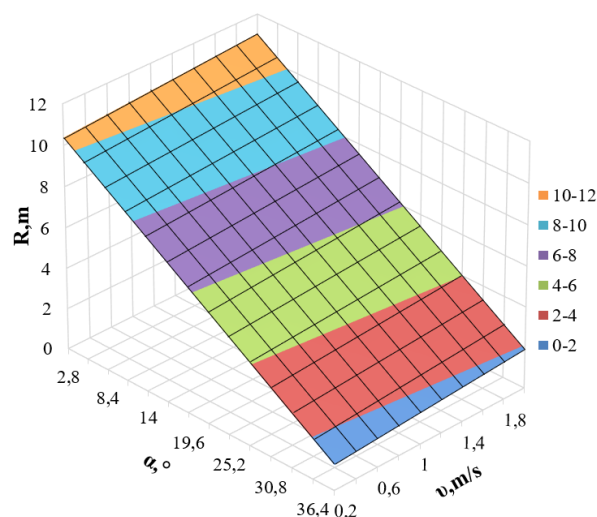
A graphical interpretation of the value of the function (turning radius) in the regression equations is shown in Figure 10 in three-dimensional space, which demonstrates the change in the angle of rotation of the controlled wheels of the TTM depending on the speed of movement of the TTM to the turning radius with different sizes of the drawbar of the TTM 1,6 m, 1,8 m and 2,0 m.



a)



b)



c)

Fig. 10. Change in the angle of rotation of the steered wheels of the TTM depending on the speed of movement of the TTM to the turning radius with different drawbar sizes: (a) 1,6 m; (b) 1,8 m; (c) 2,0 m.

Changing the angle of rotation of the steered wheels of the transport technology machine depending on the speed of movement of the wheels of the transport technology machine by the turning radius with different sizes of the drawbar of the wheels of the transport technology machine 1,6 m, 1,8 m, and 2,0 m. The most optimal drawbar is 2 meters in size.

4 Conclusions

The conducted theoretical, experimental and computational studies allowed us to substantiate the parameters of the drawbar of the transport and technological unit. In the course of experimental studies, drawbar variants with lengths of 1,6, 1,8, and 2,0 m were considered. It was found that with the selected parameters of the unit, the turning radius is 7,55 m, and the displacement of the single—axle trailer relative to the center of rotation is 1,65 m. The results showed the possibility of performing a turning maneuver with the unit fitting into a given transport corridor. According to the results of the finite element analysis of the drawbar, the maximum equivalent stress according to Mises was 122,7 MPa, which corresponds to 59,3% of the yield strength of St3sp steel (207 MPa). The maximum displacement was 2,074 mm, and the safety margin varied from 1,25 to 15. The obtained values indicate sufficient strength and rigidity of the structure under the specified loading conditions. Thus, the use of a 2,0 m long drawbar is reasonable for the considered transport and technological unit and provides the required indicators of maneuverability and operability of the structure.

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Influence of Spur-Gear Tooth-Profile Geometry on Mesh Efficiency and Transmission Error: A Common-Benchmark Multi-Objective Comparison

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Abstract

Reducing gear-mesh loss without compromising static and dynamic transmission-error performance and load capacity is a central challenge in high-performance transmission design. Non-involute tooth-flank geometries provide additional freedom to redistribute sliding, contact conformity and mesh compliance; however, published comparisons rarely preserve transmission ratio and macrogeometry together with material, load, speed, lubrication and numerical treatment. Reported gains can therefore reflect the benchmark or model as much as the tooth geometry. This study establishes a common, traceable comparison and asks which spur-gear family is preferred when mesh loss and transmission-error excitation are optimized separately and jointly on an invariant 19/49 steel-gear benchmark.

The compared tooth-flank geometries comprise the standard and pressure-angle-optimized involutes, Cubic-CP, S-gear, involute-cycloid composite and a circular-contact-path construction within a common event-resolved conjugate-contact formulation. For every admissible profile, the framework resolves the complete contact sequence, signed local curvature, rolling and sliding velocities, phase-dependent normal-force moment arms, unilateral multi-pair load sharing and tooth compliance. The resulting identity-preserving contact-force history is used consistently by both the loaded static transmission-error solver and a pressure-resolved thermal partial-EHL model, thereby coupling geometry, contact mechanics and tribology without introducing family-specific load-sharing assumptions. Geometry-specific nonlinear phase-load torque surfaces subsequently provide the interface to a common torsional DTE model. Constrained multi-fidelity optimization and dense final qualification are combined with independent continuum finite-element strength assessment of the retained designs.

No profile is uniformly superior. Relative to the standard involute, the selected S-gear joint design reduces mesh loss, loaded STE_{pp} and DTE_{pp} by 57.1%, 19.5% and 14.8%, respectively, while its common-card strength audit gives 18.9% lower root stress and 52.6% lower peak surface pressure. This defines the efficiency-strength extreme, although the dynamic response remains sensitive to overlap, elevated-error dwell and second-order mesh content. Cubic-CP provides the most gradual compromise: its joint selection reduces mesh loss, loaded STE_{pp} and DTE_{pp} by 37.3%, 17.8% and 27.9%, respectively. The composite and circular-path geometries occupy intermediate regions shaped by tribological gain, curvature regularity and load-transfer timing. Similar peak-to-peak STE values need not produce similar DTE or resonance exposure, demonstrating why loss, STE, DTE and strength must remain distinct objectives. By holding the benchmark, mechanics and numerical qualification constant, the framework replaces isolated family claims with an evidence-traceable, objective-dependent basis for selecting non-involute spur gears.

Keywords: non-involute spur gears; multi-objective optimization; thermal EHL; static transmission error; dynamic transmission error; finite-element strength

1 Introduction

The mechanical efficiency of a single cylindrical-gear mesh is generally high. Nevertheless, even modest reductions in gear-mesh power loss can provide meaningful energy and thermal benefits in continuously operated, high-power-density, or electrified drivetrains. Concurrently, the high rotational speeds and reduced masking by combustion-related noise in electric drivetrains increase the prominence of gear-mesh-order excitation and tonal vibration. Gear design for such systems cannot therefore be adequately based on a single load-carrying-capacity or efficiency criterion. Geometry, tribology, and dynamics must instead be evaluated under a common set of design and operating conditions, because changes in tooth-flank geometry that redistribute sliding and contact pressure also alter mesh compliance and the amplitude and harmonic content of transmission error [1], [2].

The involute remains the dominant tooth form because it combines exact conjugate action with a straight line of action, compatibility with standardized rack-type generating tools, interchangeability within standard tooth systems, and preservation of a constant angular velocity ratio under moderate centre-distance variations, provided that geometrically admissible contact is maintained [3]. These properties are decisive for industrial manufacture and operation. They do not, however, imply that the resulting distributions of relative profile curvature, sliding velocity, or mesh stiffness are optimal for a specified operating

objective. In a conventional involute mesh, the sliding velocity vanishes only at the pitch point and increases in magnitude towards the start and end of contact. During a mesh period, load is transferred among successive tooth pairs and, for a non-integer transverse contact ratio, the number of simultaneously contacting pairs alternates. Tooth-pair load sharing and mesh stiffness are therefore inherently periodic. Gear-mesh frictional loss and loaded static transmission error (STE) consequently arise from the same evolving contact state, but depend on different combinations of the associated local quantities.

Non-involute tooth synthesis provides additional freedom to reshape these coupled kinematic, tribological, and elastic fields. Modern synthesis methods may be classified broadly according to the geometric entity prescribed: the path of contact, a generating rack, or the flank of one gear followed by the conjugate generation of its mating flank [4], [5]. Contact-path synthesis is particularly attractive because the pressure-angle history, local profile curvatures, and sliding kinematics can be evaluated from the prescribed path before the conjugate tooth flanks are recovered. Sheng et al. prescribed an odd cubic contact-path function using this approach. Their example designs maintained continuous conjugate action while reducing the sliding ratios, and their numerical evaluations predicted higher mesh efficiency and lower wear than for the corresponding involute pair [6]. The Cubic-CP family thus provides a compact means of

redistributing off-pitch sliding and curvature while retaining direct control of the pitch-point pressure angle.

Rack-generated S-gears originated from the search for more durable tooth flanks for heavy-duty rolling-mill transmissions. Hlebanja's analytically defined curved generating rack produces a conjugate gear pair with an S-shaped path of contact, favourable curvature evolution, and, for suitable parameter combinations, comparatively thick tooth-root sections. Published analytical and numerical studies, industrial experience, and controlled tests have associated selected S-gear designs with reduced sliding and lower local contact pressures near mesh entry and exit, more favourable lubrication and scuffing behaviour, and improved tooth-root load capacity and flank durability relative to the involute reference gears examined in those studies [7], [8], [9]. These findings motivate the inclusion of S-gears in the present comparison without assuming that the reported advantages transfer unchanged to a different transmission design and operating specification.

The involute-cycloid composite developed by Terauchi and co-workers represents a different compromise. Starting from a cycloidal tooth geometry, the region around the pitch point is replaced by an involute segment and smoothly joined to cycloidal root- and tip-side segments. This construction preserves familiar pitch-region behaviour while exploiting favourable convex-concave conformity away from the pitch point. Experimental and analytical studies have reported improvements in tooth-root bending strength, flank load capacity, and transmission efficiency relative to the involute reference gears considered, while subsequent work has investigated the associated sensitivity to centre-distance variations [10], [11], [12]. A complementary synthesis route prescribes the contact path directly. General curved-contact-path formulations allow the pressure-angle and curvature histories to be shaped throughout engagement [13]; the circular-contact-path gearing examined here belongs to this class. It should not be confused with Wildhaber-Novikov gearing, which uses circular-arc profiles in the normal section of helical teeth and relies on axial overlap and the progression of the contact zone to maintain continuous engagement, unlike the transverse-plane spur-gear construction considered here [14], [15], [16].

Although numerous studies have reported advantages for individual non-involute profiles relative to an involute reference, the available results do not establish whether these advantages persist under a common transmission specification and modelling framework. Published studies frequently differ in tooth numbers, module, centre distance, face width, material, load, speed, lubrication model, and geometric or operational admissibility criteria. In other 1. cases, the non-involute geometry is evaluated using family-specific load-sharing, contact, or friction formulations, making it difficult to separate the effect of tooth geometry from that of the modelling assumptions. Moreover, the involute reference is often represented by a single 2. conventional design rather than by an optimized involute control. Consequently, an apparent benefit of a non-involute profile cannot always be distinguished from an improvement that could also be obtained through conventional pressure-angle adjustment. Reviews of non-3. involute gearing have therefore emphasized both its potential advantages and the unresolved challenges

associated with manufacturability, alignment sensitivity, durability, and experimental validation [1].

A further limitation is that tribological performance and transmission-error-based dynamic performance are frequently assessed separately. At a given mesh phase, the instantaneous tooth-flank frictional power loss is obtained by integrating the product of local tangential traction and relative sliding velocity over all active contact regions; the mean mesh loss follows from averaging over one mesh period. It depends on tooth-pair load sharing, the local reduced radius of curvature, sliding and entrainment velocities, lubricant rheology and thermal state, and the film thickness relative to the composite surface roughness. Loaded STE, by contrast, is the quasi-static deviation of the actual relative angular position of the gears from ideal conjugate motion under load. It is governed by local contact compliance, tooth and gear-body deformation, load sharing, and the phase-dependent lever arms of the normal contact forces required to satisfy torque equilibrium. Dynamic transmission error (DTE) is the time-dependent relative displacement error obtained when the resulting periodic, nonlinear, and potentially unilateral mesh relation is embedded in the drivetrain dynamic model. It may be strongly amplified near a resonance excited by a gear-mesh harmonic, even when the quasi-static transmission-error amplitude is comparatively small [17], [18], [19]. Consequently, a geometry optimized solely for low frictional loss cannot be assumed to exhibit favourable STE or DTE, while a stiffness-oriented design may incur a tribological penalty.

Accordingly, the present study provides an objective-driven, model-consistent comparison of fundamentally different profile-synthesis approaches rather than proposing another isolated tooth form. The central research question is: *how do the preferred tooth-profile family and its design parameters change when gear-mesh frictional power loss and loaded STE are minimized separately and jointly under a common transmission specification, operating conditions, and feasibility constraints, and how do the resulting quasi-static optima perform dynamically?*

Five profile families are evaluated in detail: involute, Cubic-CP, S-gear, involute-cycloid composite, and circular-contact-path gearing. The involute family is represented by both a standard 20° baseline, which defines the common nominal dimensions, and a pressure-angle-optimized involute. The latter serves as an internal control for determining whether improvements attributed to a non-involute family could instead be achieved by adjusting the pressure angle within the conventional involute framework.

The principal methodological contributions are:

- 1. a common contact-path and contact-event representation that maps rack-generated and contact-path-defined geometries onto consistent descriptors of conjugacy, local curvature, sliding and entrainment kinematics, and tooth-pair engagement intervals;
- 2. an event-resolved, unilateral multi-pair load-sharing formulation in which the same converged active-contact set, normal-load history, and phase-dependent force lever arms are used in the final compliance/STE and thermal partial-elastohydrodynamic-lubrication (EHL) evaluations;
- 3. a periodic nonlinear mesh-torque surface, expressed as a function of mesh phase and relative torsional displacement and reconstructed from the quasi-static solutions, that

provides a dimensionally consistent interface with the DTE model; and

4. three separately interpretable constrained optimization formulations—loss-only, STE-only, and joint loss-STE—implemented through a two-level numerical strategy: repeated lower-fidelity searches explore the design domain, whereas adaptive high-fidelity loaded-contact and EHL evaluations determine the final ranking under common geometric, meshing, strength, lubrication, and numerical-validity constraints.

The study is intentionally restricted to a single lubricated steel spur-gear benchmark, and the resulting ranking should therefore be interpreted as benchmark-specific rather than universal. The reported mesh efficiency accounts only for tooth-flank frictional losses and excludes bearing, seal, windage, and lubricant-churning losses. Strength is assessed using candidate-level analytical screens for flank contact

and tooth-root bending, followed by post-selection continuum finite-element analysis (FEA) under common loading and boundary conditions. DTE is evaluated only within the specified torsional drivetrain model and should not be interpreted as a direct prediction of gearbox-housing vibration or radiated noise. The complete information flow and the distinction between search-level and final evaluations are summarized in Fig. 1.

2 Unified contact, tribological and transmission-error formulation

All geometries are evaluated with the same gearset, material, surface and operating specification summarized in Table 1. Only the family-specific flank variables are changed during optimization.

Table 1: Gearset, material, lubricant and operating parameters held fixed in the cross-family comparison.

Quantity	Fixed value	Quantity	Fixed value
<i>Gear geometry and operating point</i>			
Tooth numbers, z_1/z_2	19/49	Module, m	3 mm
Pitch radii, r_{o1}/r_{o2}	28.5/73.5 mm	Centre distance, a_w	102 mm
Face width, b	55 mm	Profile shifts, x_1/x_2	0/0
Addendum coefficients	1/1	Dedendum coefficients	1.25/1.25
Tooth-thickness coefficients, $C_{s,1}/C_{s,2}$	0.495/0.495	Cutter-fillet coefficients, $c_{c,1}/c_{c,2}$	0.38/0.38
Standard-involute reference angle, α_0	20°	Pinion torque, T_1	60 N m
Pinion speed, n_1	3000 min ⁻¹	Input power, $T_1 \omega_1$	18.85 kW
<i>Material, surface and lubricant</i>			
Elastic modulus, $E_1 = E_2$	200 GPa	Poisson ratio, $\nu_1 = \nu_2$	0.30
Steel density, ρ_s	7850 kg m ⁻³	Surface roughness, $Rq_1 = Rq_2$	0.30 μ m
Lubricant / inlet temperature, T_{in}	ISO VG 68 / 60°C	Inlet dynamic viscosity, η_0	0.0303699 Pa s
Pressure-viscosity coefficient, α_{pv}	1.66852×10^{-8} Pa ⁻¹	Traction parameters, $\mu_b/\Lambda/\mu_{max}$	0.070/0.047/0.110
<i>DTE reference card</i>			
Rotational inertias, J_i	solid pitch-cylinder disks	Damping ratio, ζ_d	0.005
Steady-state statistics	final 20 mesh cycles		

Table 2: Notation used throughout the common formulation. Subscripts 1 and 2 denote the pinion and driven gear, respectively.

Symbol	Definition	Symbol	Definition
$i = 1, 2$	Pinion and driven-gear indices	z_i, m	Tooth number and module
r_{oi}, a_w, b	Pitch radius, centre distance and face width	$C_{s,i}, c_{c,i}$	Tooth-thickness and cutter-fillet coefficients
x_i, α_0, α_p	Profile shift, reference angle and pitch pressure angle	T_i, n_1, ω_i	Member torque, pinion speed and angular speed
i_{12}, g	Driven/pinion and pinion/driven transmission ratios	ξ	Normalized involute coordinate in the Sánchez model
$q, \gamma(q)$	Monotone contact-path parameter and fixed-frame contact locus	θ_i, ϑ_2	Signed member rotation and positive output-rotation magnitude
$\mathbf{n}, \mathbf{t}$	Unit common normal and tangent	κ_i, ρ_i, R	Signed curvature, local radius and equivalent transverse radius
h_i	Signed normal-force moment arm about member i	ε_α, ψ	Transverse contact ratio and mesh phase, $0 \leq \psi < 2\pi$
$\mathcal{A}_c, \mathcal{A}$	Candidate and accepted active contact-pair sets	u_i	Signed tangential surface speed
u_s, u_e, SRR	Sliding speed, entrainment speed and slide-to-roll ratio		
F_j, τ_j	Normal force and driven-side contact torque of pair j	δ_n, δ_θ	Local normal approach and driven-side angular approach
E_i, ν_i, E_r	Elastic constants and reduced modulus	w', a_H, p_0	Line load, Hertz half-width and maximum Hertz pressure
U, G_{EHL}, W_L	EHL speed, material and line-load groups	h_c, h_{min}	Central and minimum lubricant-film thickness
Rq_i, σ	Member rms roughness and composite rms roughness	λ_c, λ_{min}	Central- and minimum-film ratios

Symbol	Definition	Symbol	Definition
f_a, f_f	Asperity and fluid load fractions		
$\eta_0, \alpha_{pv}, \Lambda$	Inlet viscosity, pressure-viscosity coefficient and limiting-shear coefficient	μ_b, μ_f, μ	Boundary, fluid and mixed traction coefficients
P_{loss}, η_m	Mean mesh loss and mesh efficiency	e_θ, STE_{pp}	Loaded angular error and peak-to-peak STE
x, ϕ_m	Relative driven-side coordinate and normalized mesh phase	T_{el}^+, T_m	Positive elastic torque surface and instantaneous mesh torque
$J_i, K_{\theta,m}, C_i$	Rotational inertia, mean mesh stiffness and damping coefficient	ζ_a, b_θ	Damping ratio and one-sided angular backlash
T_{20}	Final 20-cycle steady-state window		
$DTE_{AC,rms}$	Peak-to-peak and zero-mean AC-rms DTE		
x_f, Ω_f	Family design vector and feasible design domain	J_L, J_E, J_D	Normalized loss, STE and post-selection DTE measures
$\ell_{ij}, s_{f,i}, \zeta_r$	Root-section force offset, thickness and local coordinate	N_{ij}, V_{ij}, M_{ij}	Root-section axial, shear and bending resultants
A_i, I_i, Z_i	Root-section area, second moment and section modulus	$\Sigma_{ij}^{scr}, \Sigma_{max}^{scr}$	Pairwise and maximum nominal root-section stress indices
$p_{H,lim}, \Sigma_{lim}^{scr}$	Common pressure and root-section screening caps		

2.1 Contact-path parameterization as the common geometric state

The compared geometries are not defined in a common native coordinate system. An involute, S-gear or involute-cycloid composite is generated by a rack, whereas the Cubic-CP and circular-arc constructions are defined by their fixed-frame contact loci. A unified physical comparison therefore requires a representation that is independent of the original geometric synthesis. The path of contact is adopted for this purpose because it identifies the instantaneous contact position and provides the parameter through which curvature, surface velocity, normal load and elastic deformation are evaluated.

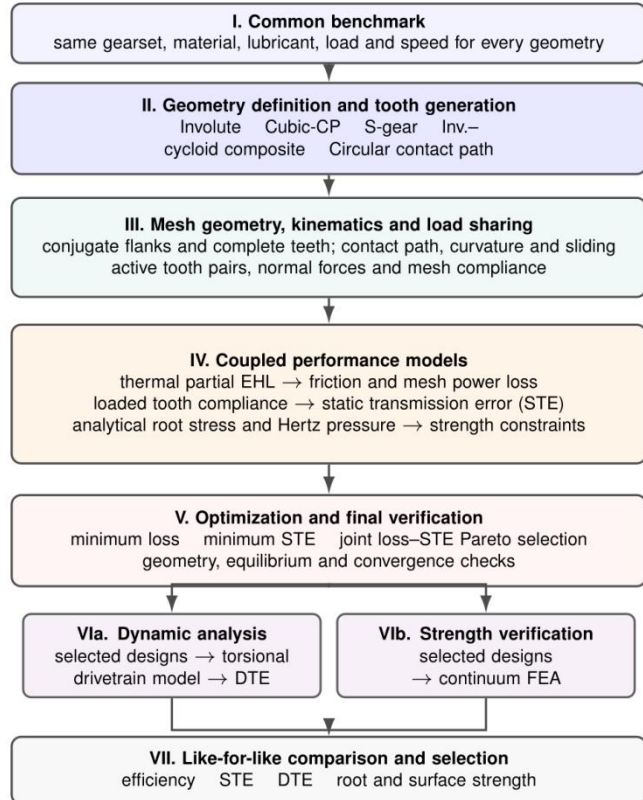


Figure 1: Methodological workflow linking geometry generation with contact mechanics, EHL, STE, DTE and FEA-based strength assessment.

Let $\mathcal{F}_0$ be the fixed transverse plane, with the pitch point C as origin and gear centres O_1 and O_2 . The theoretical contact path is the regular curve

$$\gamma(q) = [x(q) \ y(q)]^T, \quad q \in \mathcal{I}, \quad (1)$$

where q is any monotone family parameter. The working interval $[q_A, q_E] \subset \mathcal{I}$ is obtained after geometrically identifying the first and last point of tooth contact. The two native representations used in this study are transformed to Eq. (1) as follows.

Directly prescribed contact path.

For a family defined by $\gamma(q)$, the fixed-frame locus is available without an intermediate generating surface, by implementing the analytical theory of gearing. The angular laws of the two gears are obtained from conjugate action, and the path is subsequently trimmed by the complete tooth outlines. This is the representation used for the circular-arc-contact-path construction of Sec. 3.5.

Rack-generator representation.

Consider a rack flank $x_r = x_r(y_r)$ expressed in a frame translating along the pitch line. With the coordinate convention adopted here, the envelope condition gives the fixed-frame contact abscissa and pinion roll as

$$x_{CP}(y_r) = \frac{y_r}{x_r(y_r)}, y_{CP}(y_r) = y_r, \quad (2)$$

$$K(y_r) = x_r(y_r) + x_{CP}(y_r), \theta_1(y_r) = \frac{K(y_r)}{r_{o1}},$$

where $r_{o1} = mz_1/2$ is the pinion pitch radius. Reversing the rack abscissa changes the signs of x_r , x_{CP} and K but leaves the manufactured geometry and scalar contact metrics invariant. Equation (2) therefore expresses the adopted coordinate convention rather than an additional physical assumption. The external-gear rotation follows from $\theta_2 = -(z_1/z_2)\theta_1$ up to the selected assembly phase. The same mapping is applied to the straight involute rack and the curved S-gear rack.

Once a working contact path and the rotations $\theta_i(q)$ are known, the corresponding body-fixed flank points are recovered by the inverse planar rigid-body transformation

$$\bar{\mathbf{r}}_i(q) = \mathbf{T}_i^{-1}(\theta_i(q))\bar{\mathbf{y}}(q), \quad \mathbf{T}_i(\theta_i) = \begin{bmatrix} \mathbf{R}(\theta_i) & \mathbf{O}_i \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad (3)$$

where $\bar{\mathbf{r}} = [\mathbf{r}^T, 1]^T$ and $\mathbf{R}(\theta)$ is the planar rotation matrix. This $SE(2)$ map generates both conjugate working flanks from the same contact locus and angular laws.

Because C is the pitch-point origin, the common normal is radial with respect to that origin,

$$\mathbf{n}(q) = \pm \frac{\mathbf{y}(q)}{\|\mathbf{y}(q)\|}, \quad \mathbf{t}(q) = [-n_y \quad n_x]^T, \quad (4)$$

with the sign fixed by the outward normals of the two flanks. At $\mathbf{y} = \mathbf{0}$, Eq. (4) is evaluated by the one-sided regular limit of the contact path. At each contact position, the member rotations, common normal, local curvatures, moment arms and surface velocities define the state supplied to the load-sharing, tribological and compliance models.

2.2 Local differential geometry and rolling-sliding kinematics

For a body-fixed flank $\mathbf{r}_i(q) = [X_i(q), Y_i(q)]^T$, the signed curvature is evaluated in parameter-invariant form,

$$\kappa_i(q) = \frac{X_i' Y_i'' - Y_i' X_i''}{(X_i'^2 + Y_i'^2)^{3/2}}, \quad \rho_i = \kappa_i^{-1}. \quad (5)$$

Curvature is evaluated analytically for profiles defined in closed form; scale-aware centred differences are used only for discretized geometric input. With both curvatures oriented by the common contact normal, the transverse equivalent radius is

$$\frac{1}{R(q)} = \kappa_{1c}(q) + \kappa_{2c}(q) > 0. \quad (6)$$

The sign convention in Eq. (6) ensures that admissible external contact has positive relative curvature; branches that do not satisfy this condition are excluded.

The normal-force moment arm about gear centre i is retained at each event,

$$h_i(q) = (\mathbf{e}_z \times \mathbf{r}_i(q)) \cdot \mathbf{n}_i(q) = X_i n_{iy} - Y_i n_{ix}. \quad (7)$$

For an involute flank, every contact normal is tangent to the same base circle, so $h_i = r_{bi}$ is constant. A general non-involute flank has no analogous single base circle; its actual phase-dependent $h_i(q)$ must therefore be used in both torque equilibrium and the conversion from normal approach to angular transmission error.

The rigid surface velocity of member i at contact is

$$\mathbf{v}_i = \omega_i \mathbf{e}_z \times (\mathbf{y} - \mathbf{O}_i), \quad \mathbf{u}_i = \mathbf{v}_i \cdot \mathbf{t}. \quad (8)$$

The signed sliding velocity, positive entrainment velocity and slide-to-roll ratio (SRR) are

$$u_s = u_1 - u_2, \quad u_e = \frac{|u_1| + |u_2|}{2}, \quad (9)$$

$$\text{SRR} = \frac{u_s}{\max(u_e, u_{e,\min})}.$$

The small denominator floor is used only to avoid an undefined diagnostic; the physical u_s is retained in the frictional-power calculation. Accordingly, the pitch point remains a zero-sliding event.

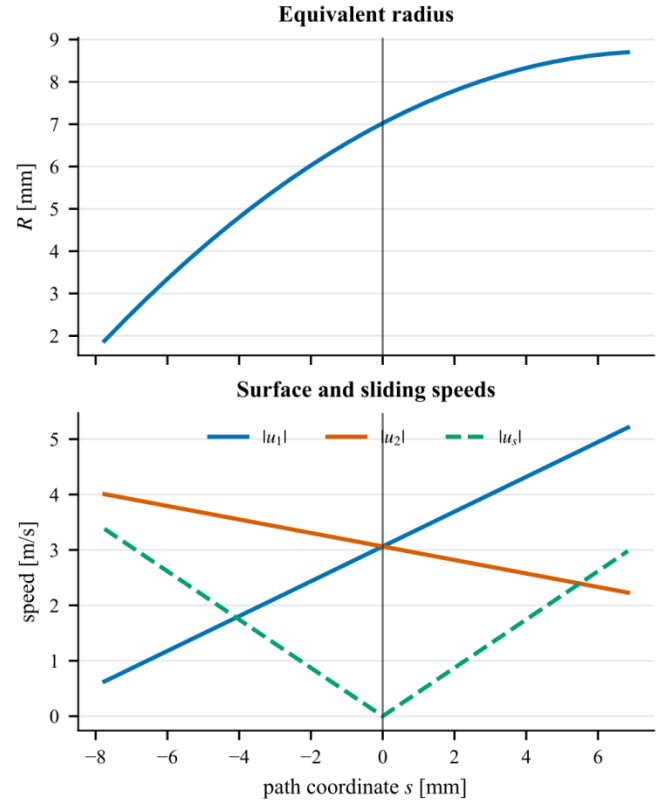


Figure 2: Equivalent radius and rolling-sliding kinematics of the involute reference.

2.3 Engagement extent, periodic occupancy and unilateral force solution

For a curved contact path, its Euclidean arc length is not an angular contact ratio. The transverse contact ratio is instead obtained from the pinion rotation traversed by one tooth pair,

$$\varepsilon_\alpha = \frac{|\theta_1(q_E) - \theta_1(q_A)|}{2\pi/z_1}. \quad (10)$$

At mesh phase $\psi = \text{mod}(z_1 \theta_1, 2\pi)$, all tooth-shifted copies of $[q_A, q_E]$ containing ψ form the candidate set $\mathcal{A}_c(\psi)$. Pair-entry and pair-exit phases are inserted explicitly in the numerical phase grid, so a short double-contact interval cannot be missed by uniform sampling.

For the standard involute, the analytical meshing-stiffness and load-sharing formulation of Sánchez *et al.* provides an independent comparison [20], including its subsequent extensions for profile modifications and non-nominal loads [21], [22]. The formulation separates tooth-pair and local Hertzian compliances,

$$\frac{1}{K_{TP}} = \frac{1}{k_x} + \frac{1}{k_s} + \frac{1}{k_n},$$

$$\hat{K}_{TP}(\xi) = \frac{K_{TP}(\xi)}{K_{TP,\max}} \simeq \cos[b_0(\xi - \xi_m)],$$

$$k_H = \frac{\pi E b}{4(1-\nu^2)}, \quad K_M(\xi) = [K_{TP}^{-1}(\xi) + k_H^{-1}]^{-1},$$

$$R_{M,j}(\xi) = \frac{K_M(\xi+j)}{\sum_{k \in \mathcal{A}_c} K_M(\xi+k)}, \quad F_j = F R_{M,j}, \quad (11)$$

where $b_0 = \{1/2[1.11 + \varepsilon_\alpha/2]^2 - 1.17\}^{-1/2}$, $\xi_m = \xi_{\text{inn}} + \varepsilon_\alpha/2$, and $K_M = 0$ outside the active interval. The integer shift represents successive tooth pairs in the normalized involute coordinate. Because this closed load-sharing law was derived for involute spur gears, it is not applied to the non-involute families. Their normal forces are obtained instead from the geometry-resolved unilateral solution used for loaded STE.

At each phase, candidate $j \in \mathcal{A}_c(\psi)$ must satisfy positional closure, parallel tangents with opposing normals, active-flank bounds and local non-penetration. Let $g_j^0 \geq 0$ be its unloaded gap expressed in the positive driven-member angular coordinate, and introduce the non-negative contact torque

$$\tau_j = F_j |h_{2j}|. \quad (12)$$

The nonlinear elastic angular approach given by the adopted tooth, foundation and Hertz compliance formulation is

$$\begin{aligned} \delta_{\theta,j}^{\text{el}}(\tau_j) &= |\delta\theta_{2j}(\tau_j)| = \frac{\delta_{nj}(F_j)}{|h_{2j}|}, \\ \delta_{\theta,j}^{\text{el}}(0) &= 0, \frac{d\delta_{\theta,j}^{\text{el}}}{d\tau_j} > 0. \end{aligned} \quad (13)$$

For the scalar driven-side angular approach δ_θ , the bounded active-set problem solved at each phase is

$$\begin{aligned} \tau_j &\geq 0, w_j = g_j^0 + \delta_{\theta,j}^{\text{el}}(\tau_j) - \delta_\theta \geq 0, \\ \tau_j w_j &= 0, \sum_{j \in \mathcal{A}_c} \tau_j = T_2. \end{aligned} \quad (14)$$

The specified nominal pinion torque is expressed in the driven-side load coordinate as $T_2 = (z_2/z_1)T_1$. Equation (14) then determines the common angular approach and the non-negative contact torques while satisfying the prescribed total torque. The phase-resolved normal force follows as

$$F_j(\psi) = \frac{\tau_j(\psi)}{|h_{2j}(\psi)|}. \quad (15)$$

Thus the same unilateral equilibrium solution determines both load sharing and the normal forces supplied to the STE and EHL models.

For the standard involute, the Sánchez and unilateral solutions agree closely (Fig. 3): the normalized RMS force difference is 2.94%, and their path-integrated forces differ by 0.07%. The unilateral formulation is used subsequently because it applies consistently to every profile family.

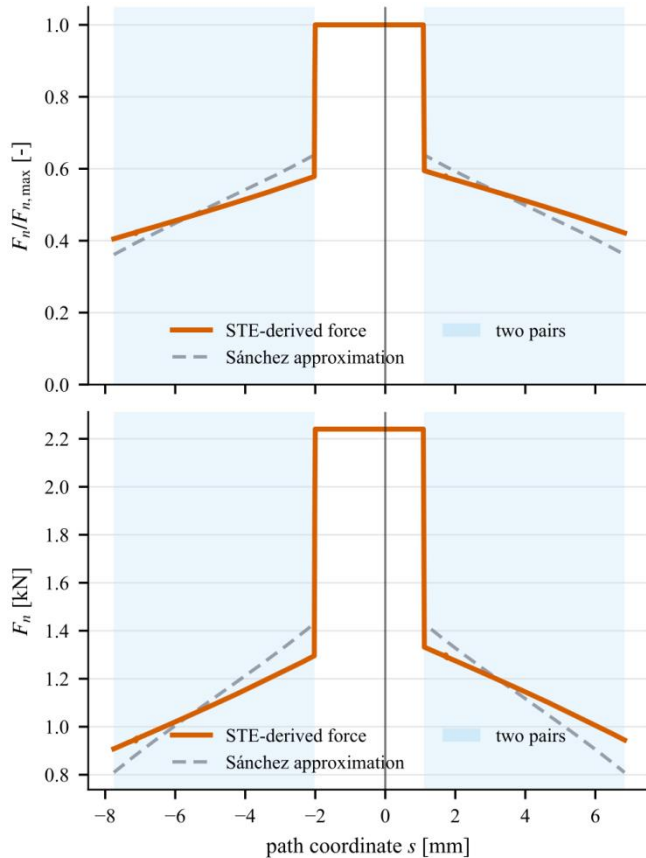


Figure 3: Involute load sharing normalized by each model's maximum (top) and absolute normal force (bottom).

Torque equilibrium is imposed independently on both members,

$$\begin{aligned} R_{T_2}(\psi) &= \sum_j F_j |h_{2j}| - T_2, \\ R_{T_1}(\psi) &= \sum_j F_j |h_{1j}| - T_1. \end{aligned} \quad (16)$$

For a conjugate pair of constant ratio, virtual work gives $|h_{2j}|/|h_{1j}| = z_2/z_1$; hence both equations close for the same F_j . The corresponding contact point, normal, moment arms, curvature and force are then supplied to the tribological calculation. Entry and exit values follow from the one-sided limit of the contact branch.

2.4 Line-contact partial-EHL traction with thermal correction

At each contact event, the tribological state of an active tooth pair is evaluated from the phase-resolved normal force in Eq. (15), together with the local curvature and surface kinematics. The coupling is sequential: the loaded-contact/STE problem determines load sharing, after which the EHL calculation determines film thickness, traction and temperature. These quantities are not fed back into the quasi-static contact equilibrium. Contact equilibrium, both torque balances in Eq. (16), phase-resolution convergence and thermal convergence are verified before integration over a mesh cycle. For elastic data (E_i, ν_i) , the reduced modulus is

$$E_r = \frac{2}{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}. \quad (17)$$

With line load $w' = F/b$ and equivalent radius R , the Hertz half-width, maximum pressure, mean pressure and transverse distribution are

$$\begin{aligned} a_H &= \sqrt{\frac{8w'R}{\pi E_r}}, p_0 = \frac{2w'}{\pi a_H}, \bar{p} = \frac{\pi p_0}{4}, \\ p(X) &= p_0 \sqrt{1-X^2}, -1 \leq X \leq 1. \end{aligned} \quad (18)$$

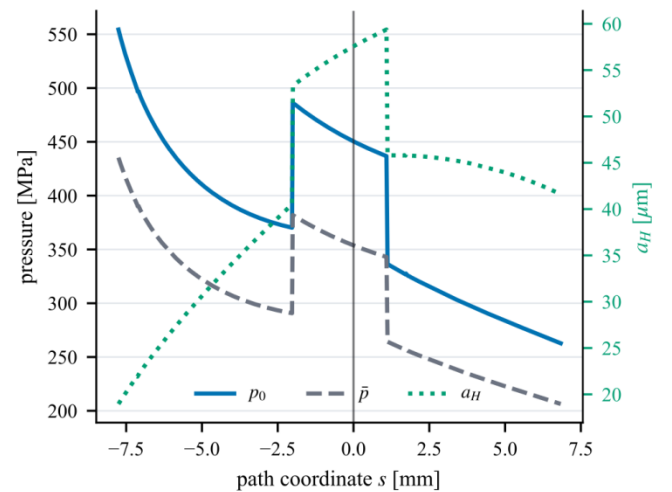


Figure 4: Hertzian pressure and contact half-width for the involute reference.

Each active tooth pair is treated locally as a fully flooded, quasi-steady transverse line contact. The central and minimum film thicknesses in the isothermal limit follow from

$$U = \frac{\eta_0 u_e}{E_r R}, \quad G_{\text{EHL}} = \alpha_{\text{pv}} E_r, \quad W_L = \frac{w'}{E_r R} = \frac{F}{b E_r R}, \quad (19)$$

and the classical smooth-surface line-contact correlations

$$\begin{aligned} h_{c,\text{iso}} &= 3.06R U^{0.69} G_{\text{EHL}}^{0.56} W_L^{-0.10}, \\ h_{\text{min},\text{iso}} &= 2.65R U^{0.70} G_{\text{EHL}}^{0.54} W_L^{-0.13}. \end{aligned} \quad (20)$$

The correlations describe fully flooded, steady, isothermal line contact and are evaluated locally using the event-

resolved force, equivalent curvature and entrainment speed [23], [24], [25]. Inlet shear heating is represented by the following empirical thermal film correction [26], [27]:

$$\begin{aligned}
 L_T &= \frac{\eta_0 \beta_T u_e^2}{k_{oil}}, \\
 \Phi_T^{\text{raw}} &= \frac{1 - 13.2(p_0/E_T)L_T^{0.42}}{1 + 0.213[1 + 2.23\text{SRR}]^{0.83}L_T^{0.64}}, \\
 \Phi_T &= \begin{cases} 0.05, & \Phi_T^{\text{raw}} < 0.05, \\ \Phi_T^{\text{raw}}, & 0.05 \leq \Phi_T^{\text{raw}} \leq 1, \\ 1, & \Phi_T^{\text{raw}} > 1, \end{cases} \quad (21) \\
 \tilde{h}_c &= \Phi_T h_{c,\text{iso}}, \quad \tilde{h}_{\min} = \Phi_T h_{\min,\text{iso}}, \\
 h_c &= \max\{\tilde{h}_c, h_c^{\text{floor}}\}, \\
 h_{\min} &= \max\{\tilde{h}_{\min}, h_{\min}^{\text{floor}}\}, \\
 (h_c^{\text{floor}}, h_{\min}^{\text{floor}}) &= (5, 2) \text{ nm}.
 \end{aligned}$$

This thermally corrected partial-EHL closure is not a fully coupled solution of the Reynolds and energy equations. The two thickness floors regularize the constitutive expressions and prevent a singular shear-rate evaluation. Any state activating either bound is excluded, and both bounds remain inactive in all reported responses.

The composite rms roughness and the associated central- and minimum-film ratios are

$$\begin{aligned}
 \sigma &= (Rq_1^2 + Rq_2^2)^{1/2}, \\
 \lambda_c &= \frac{h_c}{\sigma}, \quad \lambda_{\min} = \frac{h_{\min}}{\sigma}. \quad (22)
 \end{aligned}$$

An engineering fluid-asperity load-sharing interpolation, applied identically to every geometry, uses the central-film ratio from Eq. (22),

$$f_a = 1 - \text{erf}(\lambda_c), \quad f_f = \text{erf}(\lambda_c), \quad \lambda_c \geq 0. \quad (23)$$

The central-film ratio is the argument of this interpolation, whereas $\lambda_{\min}$ is retained as a minimum-separation diagnostic. The continuous form avoids an artificial traction discontinuity at a selected film-ratio threshold.

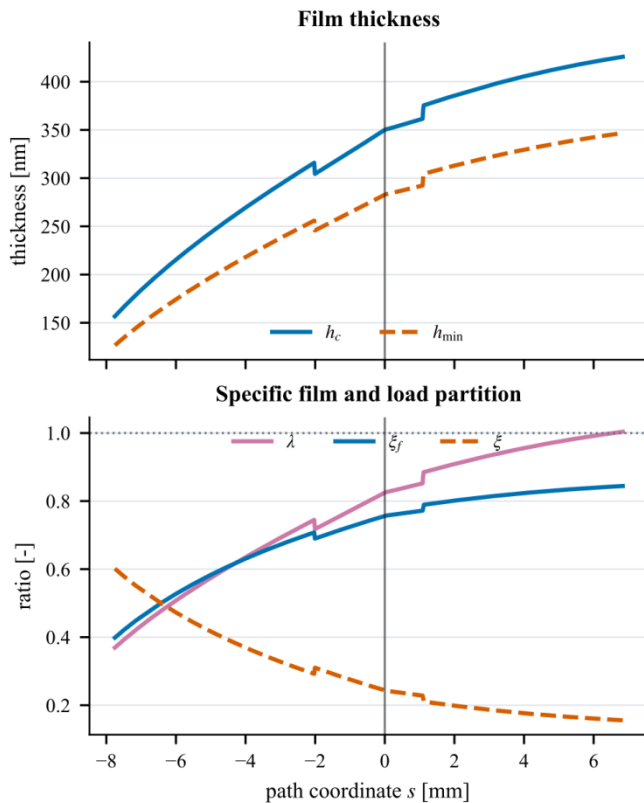


Figure 5: Film thickness and partial-EHL load partition for the involute reference.

The pressure–temperature viscosity response follows the exponential form of Roelands [28],

$$\eta_R(p, T) = \eta_{\text{ref}} \exp \left\{ \ln \left[\frac{\eta_0(T)}{\eta_{\text{ref}}} \right] \left(1 + \frac{p}{p_{\text{ref}}} \right)^Z \right\}. \quad (24)$$

The universal reference values are $\eta_{\text{ref}} = 6.31 \times 10^{-5}$ Pa s and $p_{\text{ref}} = 196$ MPa. Unless an independently calibrated exponent is supplied, Z is selected to recover the inlet pressure–viscosity slope through

$$\begin{aligned}
 Z &= \frac{\alpha_{pv} p_{\text{ref}}}{\ln[\eta_0(T_{\text{in}})/\eta_{\text{ref}}]}, \\
 \alpha_{\text{loc}}(p, T) &= \frac{Z}{p_{\text{ref}}} \ln \left[\frac{\eta_0(T)}{\eta_{\text{ref}}} \right] \left(1 + \frac{p}{p_{\text{ref}}} \right)^{Z-1}. \quad (25)
 \end{aligned}$$

With shear rate $\dot{\gamma} = |u_s|/h_c$, the Eyring response and local limiting-shear condition are

$$\begin{aligned}
 \tau_0 &= \frac{2\lambda}{\alpha_{\text{loc}}(p_0, T)}, \\
 \tau_E(X) &= \tau_0 \text{asinh} \left[\frac{\eta_R(p(X), T) \dot{\gamma}}{\tau_0} \right], \\
 \tau(X) &= \min\{\tau_E(X), \Lambda p(X)\}, \\
 \bar{\tau} &= \frac{1}{2} \int_{-1}^1 \tau(X) dX, \quad \mu_f = \bar{\tau}/\bar{p}. \quad (26)
 \end{aligned}$$

The pressure-dependent cap is applied before quadrature so that shear traction vanishes at the pressure-free edges [29]. A measured τ_0 may replace the slope-derived value, but it must be declared once in the common lubricant card rather than fitted separately to each geometry.

The film temperature is obtained by relaxed fixed-point iteration. For local frictional power $P_f = \mu F |u_s|$, the reduced flash and internal-film updates are

$$\begin{aligned}
 \Delta T_i^{\text{flash}} &= 1.06 \frac{P_f/2}{2\alpha_H b k_s} \sqrt{\frac{\chi_s \alpha_H}{\max(|u_i|, u_{\min})}}, \\
 \Delta T^{\text{int}} &= \frac{h_c P_f}{16\alpha_H b k_{oil}}, \\
 T^* &= T_{\text{in}} + 1/2 (\Delta T_1^{\text{flash}} + \Delta T_2^{\text{flash}}) + \Delta T^{\text{int}}, \\
 \tilde{T}^* &= \min\{T_{\text{in}} + 150 \text{ K}, \max(T_{\text{in}}, T^*)\}, \\
 T^{(k+1)} &= 1/2 T^{(k)} + 1/2 \tilde{T}^*, \quad \eta_0(T) = \eta_0(T_{\text{in}}) e^{-\beta_T (T - T_{\text{in}})}. \quad (27)
 \end{aligned}$$

The solution is accepted only after the temperature residual has converged and none of the prescribed temperature, viscosity, friction or film-thickness bounds has been reached.

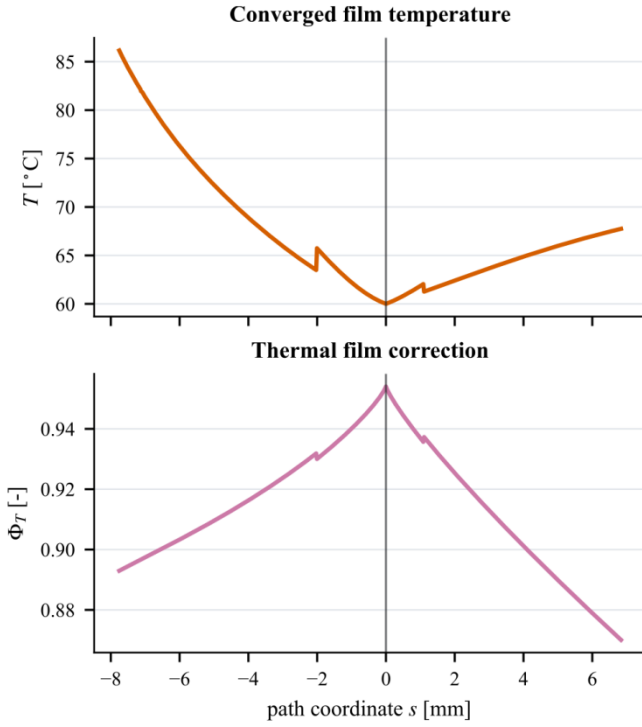


Figure 6: Converged film temperature and thermal film correction.

2.5 Mesh-cycle loss and efficiency

The mixed traction coefficient and the instantaneous loss of active pair j are

$$\mu_j = \min \left\{ \mu_{\max}, f_{a,j} \mu_b + f_{f,j} \mu_{f,j} \right\}, \quad P_j = F_j |u_{s,j}| \mu_j. \quad (28)$$

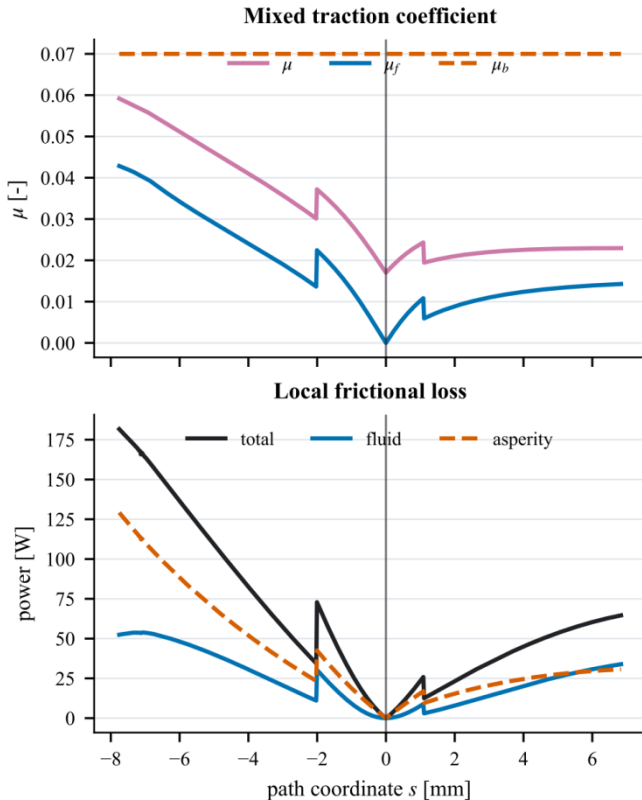


Figure 7: Mixed traction coefficient and local frictional loss.

The event-aware mean over one mesh period is

$$P_{\text{loss}} = \frac{1}{2\pi} \int_0^{2\pi} \sum_{j \in \mathcal{A}(\psi)} P_j(\psi) d\psi, \quad \eta_m = 1 - \frac{P_{\text{loss}}}{T_1 \omega_1}. \quad (29)$$

Fluid and asperity contributions, and the approach and recess portions of the integral, are evaluated separately. The efficiency η_m is that of the analysed tooth mesh; bearing,

seal, churning and windage losses lie outside the defined system boundary.

2.6 Tooth compliance and loaded static transmission error

At each mesh phase, the loaded contact is treated quasi-statically. Following the classical strain-energy decomposition for spur-gear compliance [30], the normal approach combines tooth bending, transverse shear, direct compression, root/foundation deformation and local contact indentation,

$$\delta_n = \delta_b + \delta_s + \delta_a + \delta_f + \delta_H. \quad (30)$$

For a mirror-symmetric tooth with section width $w(y) = |x_R(y)| + |x_L(y)|$, contact point (x_c, y_c) , unit normal $\mathbf{n} = (n_x, n_y)$ and root level y_f , Castigliano's theorem gives the dimensionless tooth coefficients

$$B = \int_{y_f}^{y_c} \frac{12 \ell_b(y)^2}{w(y)^3} dy, \quad \ell_b(y) = x_c n_y - (y_c - y) n_x, \quad (31)$$

$$S = \int_{y_f}^{y_c} \frac{2.4(1+\nu)n_x^2}{w(y)} dy, \quad A = \int_{y_f}^{y_c} \frac{n_y^2}{w(y)} dy,$$

so that $C_b = B + S + A$ and $\delta_b + \delta_s + \delta_a = C_b F_n / (Eb)$. The adopted semi-empirical foundation correction is written directly in terms of the normal components,

$$C_f = (1 - \nu^2) \left[L \left(\frac{q_f}{t_f} \right)^2 + M n_x \frac{q_f}{t_f} + P (n_x^2 + Q_\nu n_y^2) \right], \quad (32)$$

where $q_f = n_x(y_c - y_f) - x_c n_y$, t_f is the root section width, $L = 50/(2\pi)$, $M = 2(1 - 2\nu)/(1 - \nu)$, $P = 4.82/\pi$ and $Q_\nu = [2.4(1 + \nu)]^{-1}$; hence $\delta_f = C_f F_n / (Eb)$. This vector form is algebraically equivalent to the corresponding angle-based expression but remains regular when a normal component vanishes.

For dissimilar members, the elastic constants are combined through

$$E_{\text{eq}} = \frac{2E_1 E_2}{E_1 + E_2},$$

$$\frac{1 - \nu_{\text{eq}}^2}{E_{\text{eq}}} = \frac{1}{2} \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right).$$

Define $\hat{C}_{bf,i} = (C_{b,i} + C_{f,i}) E_{\text{eq}} / E_i$; for the identical steel pair, $E_{\text{eq}} = 200$ GPa and $\nu_{\text{eq}} = 0.30$. With $R = \rho_1 \rho_2 / (\rho_1 + \rho_2)$, the line-Hertz half-width, local indentation coefficient and total normal approach are

$$a_H^2 = \frac{8 F_n R}{\pi E_{\text{eq}} b},$$

$$C_H = \frac{2(1 - \nu_{\text{eq}}^2)}{\pi} \left[\ln \left(\frac{4 d_1 d_2}{a_H^2} \right) - \frac{\nu_{\text{eq}}}{1 - \nu_{\text{eq}}} \right], \quad (33)$$

$$\delta_n = (\hat{C}_{bf,1} + C_H + \hat{C}_{bf,2}) \frac{F_n}{E_{\text{eq}} b},$$

where d_i is the distance, along the common normal, from the contact point to its intersection with the tooth centreline of member i . Equation (33) is the nonlinear load–approach law used by the unilateral load-sharing and loaded-STE calculations. The continuum FE assessment in Sec. 4.3 independently evaluates the retained geometries.

For an external pair, adopt the positive output-rotation magnitude $\vartheta_2 = -\theta_2$ and set $g = z_1/z_2$. The angular kinematic mismatch referred to the driven member is

$$e_\theta(\psi, T_2) = g\theta_1 - \vartheta_2 - e_{\text{geom}}(\psi), \quad \psi = \text{mod}(z_1\theta_1, 2\pi), \quad (34)$$

where e_{geom} collects intentional no-load flank modification and manufacturing deviations; it is zero for the nominal comparison. Loaded STE is the equilibrium value of e_θ after the active contact torques satisfy Eq. (14). For active pair j , rigid-body virtual work gives

$$\delta_{n,j} = |h_{2j}| \delta \vartheta_{2j}, \quad \boxed{\delta \vartheta_{2j}} = \frac{\delta_{n,j}}{|h_{2j}|}, \quad (35)$$

$$F_j \delta_{n,j} = \tau_j \delta \vartheta_{2j}, \quad \tau_j = F_j |h_{2j}|.$$

For an involute flank, all contact normals are tangent to the same base circle and $|h_{2j}| = r_{b2}$ is constant. A non-involute flank has no analogous single base circle, so the actual phase-dependent moment arm must be used.

The reported quasi-static metrics are

$$\begin{aligned} \bar{e}_{\text{STE}} &= \frac{1}{2\pi} \int_0^{2\pi} e_\theta(\psi) d\psi, \\ e_{\text{AC,rms}} &= \left[\frac{1}{2\pi} \int_0^{2\pi} (e_\theta - \bar{e}_{\text{STE}})^2 d\psi \right]^{1/2}, \\ \text{STE}_{pp} &= \max_{\psi} e_\theta - \min_{\psi} e_\theta, \end{aligned} \quad (36)$$

together with the mesh-order coefficients of the zero-mean waveform. Contact entry and exit are retained as explicit events when extrema and harmonics are evaluated.

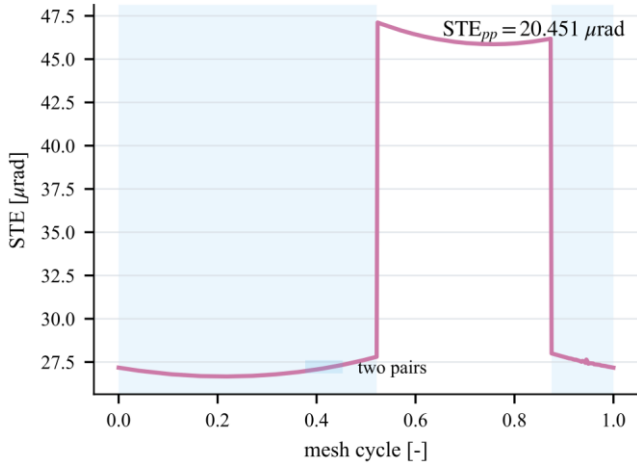


Figure 8: Loaded STE over one mesh period; shaded intervals denote two-pair contact.

2.7 Periodic mesh-torque surface and dynamic transmission error

Loaded STE provides the phase-dependent elastic excitation for the torsional model [17]. Quasi-static solutions on a phase-torque grid define the driven-side angular approach. For prescribed pinion torque T_1 , the corresponding driven-side torque is

$$T_2 = T_1 i_{12}, \quad i_{12} = \frac{z_2}{z_1}. \quad (37)$$

The normalized phase is $\phi_m = \psi/(2\pi) \in [0,1]$; the angular phase $\psi \in [0,2\pi]$ remains in the contact and STE equations. The positive loaded approach is

$$\delta_\theta(T_2, \phi_m) = |e_\theta(2\pi\phi_m, T_2) - e_\theta(2\pi\phi_m, 0)| \geq 0, \quad T_2 \geq 0. \quad (38)$$

Although the logarithmic Hertz compliance coefficient is undefined at exactly zero load, its load-approach product has the regular limit $\delta_\theta \rightarrow 0$; this limit supplies the origin. Each phasewise torque-approach curve must be strictly increasing. Its shape-preserving cubic inverse is

$$T_{\text{el}}^+(\delta_\theta, \phi_m) = [\delta_\theta(\cdot, \phi_m)]^{-1}(\delta_\theta), \quad \frac{\partial T_{\text{el}}^+}{\partial \delta_\theta} \geq 0, \quad (39)$$

over the common approach interval supported at every phase. The surface is periodic in ϕ_m and is not extrapolated. At every source state, $T_2 = \sum_{j \in \mathcal{A}} \tau_j$; hence the surface retains the resolved multi-pair load sharing and the individual moment arms.

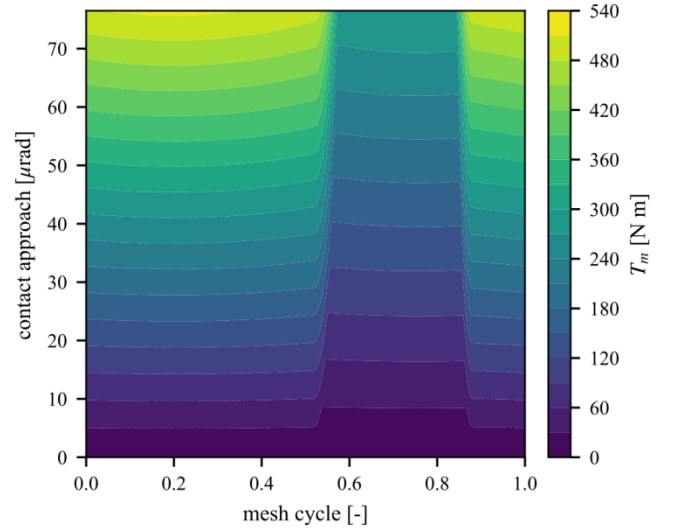


Figure 9: Periodic elastic mesh-torque surface used by the DTE model.

Figure 10 shows the corresponding steady response and its mesh-order content.

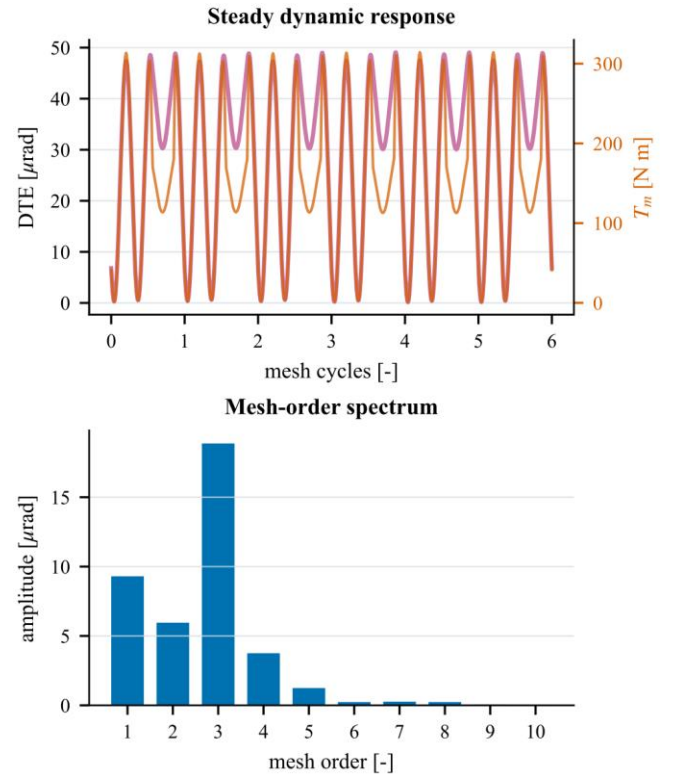


Figure 10: Steady involute DTE response and mesh-order spectrum.

The drivetrain is represented by a prescribed-pinion, single-relative- coordinate torsional model. Using the positive output rotation $\vartheta_2 = -\theta_2$, the relative coordinate and normalized mesh phase are

$$x = \frac{\theta_1}{i_{12}} - \vartheta_2, \quad \phi_m = \frac{\text{mod}(\theta_1, 2\pi/z_1)}{2\pi/z_1}. \quad (40)$$

Thus $x(t)$ is the driven-side angular DTE. For one-sided angular backlash b_θ , the contact branch, approach magnitude and mesh torque are

$$s(x) = \begin{cases} +1, & x \geq 0, \\ 0, & -b_\theta < x < 0, \\ -1, & x \leq -b_\theta, \end{cases}$$

$$\delta_\theta(x) = \begin{cases} x, & x \geq 0, \\ 0, & -b_\theta < x < 0, \\ -x - b_\theta, & x \leq -b_\theta, \end{cases} \quad (41)$$

$$T_m(\phi_m, x) = s(x)T_{el}^+[\delta_\theta(x), \phi_m].$$

The positive surface acts on the drive flank and its sign-reflected counterpart acts on the geometrically mirrored coast flank. For the adopted $b_\theta = 0$ card, the dead zone vanishes.

The recovered scalar equation of motion is

$$J_2 \ddot{x} = T_{out} - T_m(\phi_m, x) - C_2 s(x) \dot{x} + \frac{J_2}{i_{12}} \alpha_1 + s(x) \frac{C_2 - C_1 i_{12}}{i_{12}} \omega_1, \quad (42)$$

where $\omega_1 = \dot{\theta}_1$ and $\alpha_1 = \ddot{\theta}_1$ are prescribed. The input reaction required to enforce the prescribed pinion motion is

$$T_{in} = J_1 \alpha_1 - C_2 s(x) \dot{x} + T_m(\phi_m, x) + s(x) \frac{C_2 - C_1 i_{12}}{i_{12}} \omega_1. \quad (43)$$

For the common numerical card, the pitch-cylinder solid-disk inertias, mean mesh stiffness and damping coefficients are

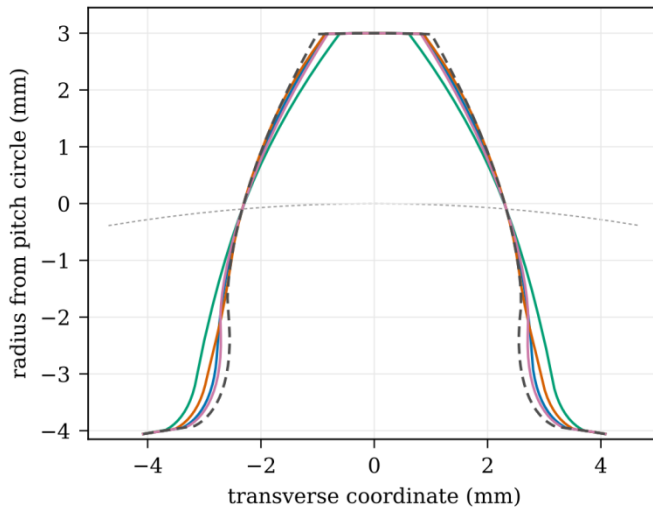
$$J_i = 1/2 \rho_s \pi b r_{oi}^4, K_{\theta, m} = \frac{T_{2, nom}}{\delta_{\theta, nom}}, \quad (44)$$

$$C_0 = 2\zeta_d \sqrt{\frac{K_{\theta, m} J_1 J_2}{J_2 r_{o1}^2 + J_1 r_{o2}^2}}, C_1 = C_0 r_{o1}, \quad C_2 = C_0 r_{o2}.$$

Here $\delta_{\theta, nom}$ is the phase-mean loaded approach at the nominal torque. These analytically defined quantities are reference parameters, not identified properties of a physical test rig.

The transient portion of each simulation is discarded. All reported DTE statistics are evaluated over the final 20 complete mesh cycles, denoted by $\mathcal{T}_{20}$. The mean over this

(a) Complete pinion tooth ($z_1 = 19$)



--- Standard involute — Cubic-CP — S-gear — Involute-cycloid composite — Circular contact path

Figure 11: Complete pinion-tooth outlines for the five investigated geometries and the assembled standard-involute 19/49 benchmark pair; the dashed tooth outline in panel (a) denotes the standard involute.

Every detailed candidate retains the tooth numbers, module, centre distance, face width and external radial envelope of the benchmark. Design variables modify the generating law within that envelope; they neither resize the transmission nor change torque, speed, material, roughness or lubricant.

stationary window is removed before the AC-rms response and mesh-order spectrum are calculated; peak-to-peak DTE is evaluated over the same window:

$$\bar{x}_{20} = |\mathcal{T}_{20}|^{-1} \int_{\mathcal{T}_{20}} x(t) dt,$$

$$DTE_{AC, rms} = \left\{ |\mathcal{T}_{20}|^{-1} \int_{\mathcal{T}_{20}} [x(t) - \bar{x}_{20}]^2 dt \right\}^{1/2}, \quad (45)$$

$$DTE_{pp} = \max_{t \in \mathcal{T}_{20}} x(t) - \min_{t \in \mathcal{T}_{20}} x(t).$$

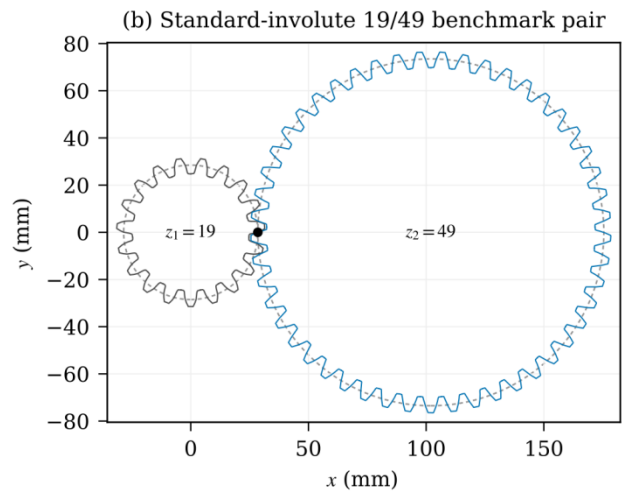
The zero-mean history also provides the first ten mesh harmonics. Peak mesh torque, dynamic load factor and branch occupancy are evaluated over the same interval. Acceptance requires a stationary final window, agreement under time-step refinement, and closure of the equation residual, energy balance and torque-surface support checks.

Each geometry has its own torque surface because compliance, contact multiplicity, event timing and moment arms all depend on the tooth form. The cross-family results in Sec. 5 therefore use separate surfaces under the same numerical drivetrain card.

3 Analytical synthesis and geometric scope of the investigated tooth systems

Five conjugate constructions are investigated: involute, Cubic Contact Path (Cubic-CP), S-gear, involute-cycloid composite and circular-arc-contact-path gearing. Figure 11 compares their complete pinion-tooth outlines on the common 19/49 geometric card and shows the assembled standard-involute benchmark pair. All geometries presented in this section and all reported performance metrics use this same gearset.

The standard 20° involute is included as a faint grey dashed reference in each detailed construction plot.



3.1 Involute reference and pressure-angle control

The involute is generated by a straight-sided rack. With rack ordinate v measured from the pitch line, the adopted sign convention is

$$x_r(v) = v \tan \alpha_0. \quad (46)$$

Equation (2) therefore yields the straight line of action

$$y_{CP} = x_{CP} \tan \alpha_0, \quad \mathbf{n} = (\cos \alpha_0, \sin \alpha_0)^T, \quad (47)$$

and a constant normal-force moment arm $h_i = r_{bi} = r_{oi} \cos \alpha_0$. In a body-fixed, gear-centred frame the working flank is

$$\mathbf{r}_i(\varphi) = r_{bi} \begin{bmatrix} \cos \varphi + \varphi \sin \varphi \\ \sin \varphi - \varphi \cos \varphi \end{bmatrix}, \quad \varphi \geq 0, \quad (48)$$

followed by tooth-index rotation and the rack-generated trochoidal/root-fillet connection. Contact ratio, principal radii and sliding velocity are evaluated using the common geometric formulation and agree with the corresponding involute closed-form relations. The standard 20° pair is fixed, whereas the optimized-involute control changes only α_0 . Increasing pressure angle decreases the base-circle moment arm $h_i = r_{oi} \cos \alpha_0$ and therefore increases the single-pair normal force required to transmit a given torque. It simultaneously shortens the active path and changes contact ratio, relative curvature, the radial force component and mesh compliance; during two-pair contact, the unilateral solution shares that torque between the active pairs. This one-parameter family separates gains available through conventional involute redesign from gains that require a different generating geometry.

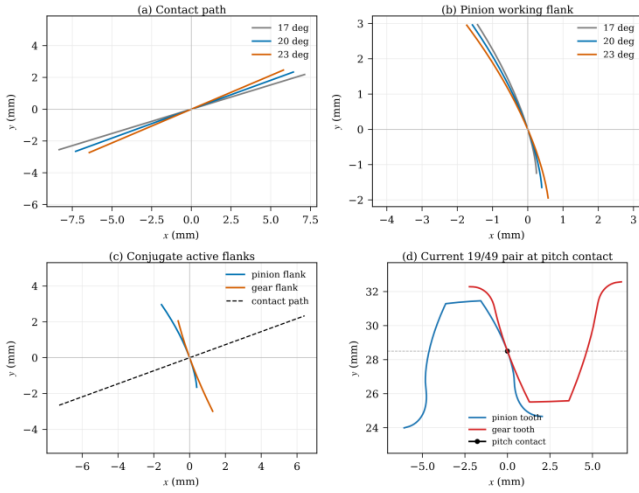


Figure 12: Standard-involute construction and pressure-angle sensitivity.

3.2 Cubic contact-path gearing

Cubic contact-path gearing replaces the straight involute line of action with a smooth, pitch-centred polynomial locus from which the conjugate tooth flanks are generated. The Cubic-CP family prescribes the fixed-frame contact locus directly [6]:

$$\mathbf{r}(x) = [a_1 x + a_2 x^3], \quad x \in [x_A, x_E]. \quad (49)$$

The common-normal direction follows from the fundamental-law ray through the pitch point rather than from the local tangent of the prescribed path. Its slope is

$$\frac{y_{CP}}{x_{CP}} = a_1 + a_2 x^2, \quad (50)$$

and analytical integration of the rotational-envelope condition gives

$$\begin{aligned} \theta_1(x) &= \frac{1}{r_{o1}} \left[(1 + a_1^2)x + \frac{4}{3}a_1 a_2 x^3 + \frac{3}{5}a_2^2 x^5 \right], \\ \theta_2(x) &= -\frac{r_{o1}}{r_{o2}} \theta_1(x), \end{aligned} \quad (51)$$

after which both body-fixed flanks follow from Eq. (3). The active interval $[x_A, x_E]$ is the connected pitch-point-containing interval that simultaneously satisfies both addendum-circle conditions and the geometric constraints of Sec. 4.2.

The dimensionless coefficient a_1 fixes the pitch-point normal limit, $\alpha_p = \tan^{-1} a_1$, whereas a_2 (mm⁻²) controls the symmetric variation of normal direction away from the pitch point. Increasing a_2 at fixed a_1 bends the path and transfers normal-direction change toward the ends of engagement. This can suppress off-pitch sliding and promote convex-concave contact, as reported for polynomial contact-path gears [6], but excessive curvature shortens the admissible interval and can reduce tip thickness or contact ratio. Loss and STE are consequently non-monotone: the same coefficient that improves local rolling also changes moment arms, curvature, pair multiplicity and the timing of load transfer.

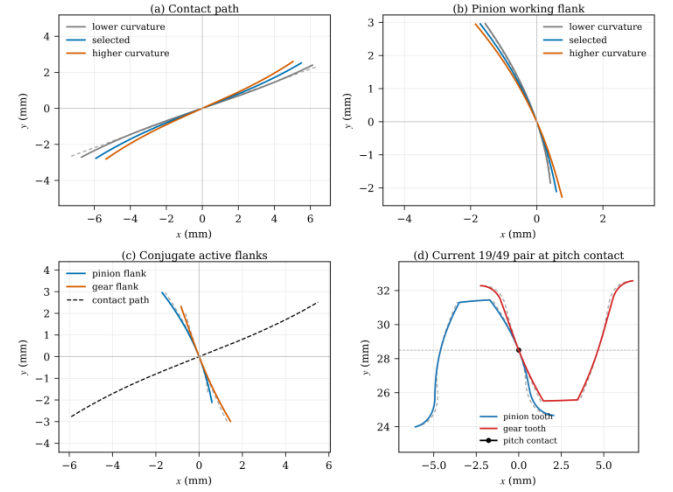


Figure 13: Cubic-CP contact path, conjugate flanks and tooth-pair sensitivity to (a_1, a_2) .

3.3 Rack-generated S-gear geometry

The S-gear family is generated from the point-symmetric curved basic rack introduced by Hlebanja et al. [7], [8]. For either half of the rack, the published dimensionless law can be written as

$$\frac{|v|}{m} = a_p \left[1 - \left(1 - \frac{|x_r|}{m} \right)^n \right], \quad 0 \leq |x_r| \leq m, \quad (52)$$

where v is the signed rack ordinate, $a_p > 0$ sets the ordinate scale and $n > 1$ controls the curvature distribution. Point-symmetric extension about the pitch point gives the inverse form used for generation,

$$x_r(v) = \text{sgn}(v)m \left[1 - \left(1 - \frac{|v|}{a_p m} \right)^{1/n} \right], \quad |v| < a_p m, \quad (53)$$

If γ_p is the rack-tangent angle at the pitch point, then

$$\tan \gamma_p = n a_p, \quad \alpha_p = \frac{\pi}{2} - \gamma_p, \quad \tan \alpha_p = \frac{1}{n a_p}. \quad (54)$$

Thus α_p is the pressure angle only at pitch; the local pressure angle varies along the curved path. The product $n a_p$ fixes the pitch tangent, while n and a_p separately control the shape away from pitch.

Substitution of Eq. (53) into the rack-envelope relation (2) generates the S-shaped contact path and both conjugate flanks. The active interval is set by the addendum intersections and the complete tooth outlines, rather than by the formal endpoint $|v| = a_p m$. Published studies show that suitable S-gear designs can combine convex-concave contact near the ends of engagement with favourable sliding and root-stress behaviour [9], [31]. These effects are design dependent: the same parameters also change contact ratio, local curvature and tooth thickness. Root strength is therefore evaluated from the generated complete outline, not inferred from the S-shaped working flank alone.

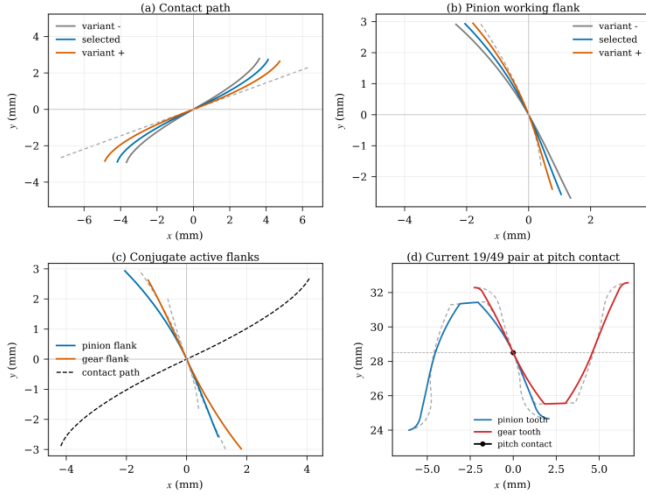


Figure 14: S-gear contact path, conjugate flanks and tooth-pair sensitivity to (n, a_p) .

3.4 Involute–cycloid composite rack

The involute–cycloid composite is generated by a single basic rack: its straight central segment produces an involute region around the pitch point, whereas its outer rolling-circle segments produce the cycloidal root and tip regions [10], [11], [32]. The rolling-circle basis of the outer regions is given by the classical epicycloid and hypocycloid of a circle of radius a rolling externally and internally on a pitch circle of radius R [3]:

$$\begin{aligned} x_e(\phi) &= (R + a)\cos\phi - a\cos\left(\frac{R+a}{a}\phi\right), \\ y_e(\phi) &= (R + a)\sin\phi - a\sin\left(\frac{R+a}{a}\phi\right), \\ x_h(\phi) &= (R - a)\cos\phi + a\cos\left(\frac{R-a}{a}\phi\right), \\ y_h(\phi) &= (R - a)\sin\phi - a\sin\left(\frac{R-a}{a}\phi\right), \end{aligned} \quad (55a,b)$$

where ϕ is the rolling angle. Equations (55) describe the classical rolling-circle curves; the composite pair itself is generated from the piecewise rack law defined below. With rack ordinate v , pitch pressure angle α_0 and rolling-circle radius a , the transition height is

$$Y_0 = a[1 - \cos(2\alpha_0)]. \quad (56)$$

The complete symmetric rack law is

$$x_r(v) = \begin{cases} v \tan \alpha_0, & |v| \leq Y_0, \\ \text{sgn}(v)\{a(\theta - \sin\theta) + 2a[\tan\alpha_0 - \alpha_0]\}, & |v| > Y_0, \end{cases} \quad (57)$$

where $|v| = a(1 - \cos\theta)$ and $\theta \geq 2\alpha_0$. Equation (57) defines the common generator of both conjugate members. At the positive transition $v = Y_0$ and, by odd symmetry, at $v = -Y_0$, the two rack branches have the same position and slope,

$$x_r(Y_0) = Y_0 \tan \alpha_0, \quad x'_r(Y_0^-) = x'_r(Y_0^+) = \tan \alpha_0, \quad (58)$$

so the transition is C^1 but not C^2 ; curvature is therefore discontinuous across the join. Applying the common rack-envelope mapping gives

$$\begin{aligned} x_{CP}(v) &= \frac{v}{x'_r(v)}, \quad y_{CP}(v) = v, \\ \theta_1(v) &= \frac{x_r(v) + x_{CP}(v)}{r_{o1}}, \quad \theta_2(v) = -\frac{r_{o1}}{r_{o2}} \theta_1(v). \end{aligned} \quad (59)$$

The envelope of this one rack therefore yields mutually conjugate flanks that are involute near pitch and cycloidal toward root and tip. The central segment provides finite pitch curvature, while the outer convex–concave contact can improve conformity and sliding behaviour. The design vector $[a_0, a/m]$ sets the transition location and the curvature of the outer branches; it consequently affects

tooth-root and contact stress, contact ratio, sliding and sensitivity to centre-distance error [10], [11], [12], [32]. The literature limit $a < z_{\min}m/4$ is applied together with the common constraints of Sec. 4.2.

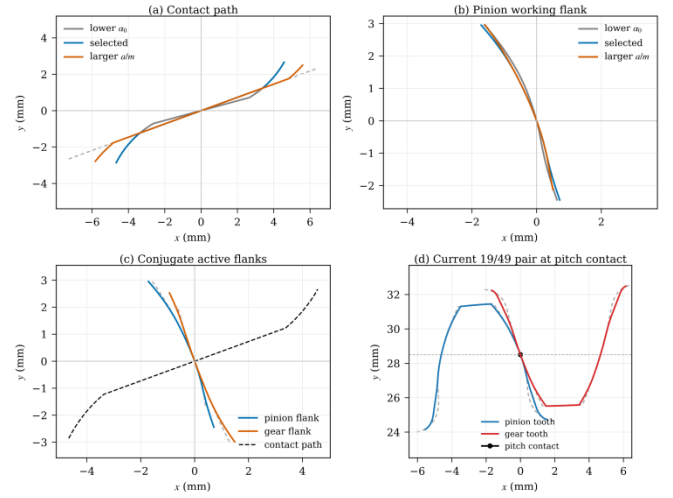


Figure 15: Involute–cycloid composite contact path, conjugate flanks and tooth-pair sensitivity to $(a_0, a/m)$.

3.5 Geometry generated from a circular-arc contact path

Here the prescribed circular curve is the fixed-frame contact path, not a circular tooth flank. Let α_t be the limiting pitch-point normal direction and R_c the path radius. With

$$\mathbf{K} = R_c [-\sin\alpha_t \quad \cos\alpha_t]^T, \quad (60)$$

the path is

$$\boldsymbol{\gamma}(u) = \mathbf{K} + R_c \begin{bmatrix} \sin(\alpha_t + u) \\ -\cos(\alpha_t + u) \end{bmatrix}, \quad u \in [u_A, u_E]. \quad (61)$$

The path passes through the pitch point and has constant curvature,

$$\boldsymbol{\gamma}(0) = \mathbf{0}, \quad |\kappa_{CP}| = R_c^{-1}. \quad (62)$$

The dimensionless curvature variable is

$$\kappa_c = \frac{m}{R_c}, \quad \kappa_c \rightarrow 0 \Rightarrow \text{involute line of action at } \alpha_t. \quad (63)$$

The pitch angle α_t fixes the limiting normal direction, whereas κ_c distributes a constant path curvature over approach and recess. Small κ_c remains close to the involute; increasing it changes sliding, relative curvature, moment arms and active-path endpoints simultaneously, so a constant path curvature alone does not ensure lower STE or power loss. This separation between a prescribed path and its conjugate flanks is central to curved-contact-path synthesis [13]. The body-fixed flanks are generated by the common $SE(2)$ mapping. The construction is kinematically distinct from Wildhaber–Novikov gearing, where circular arcs are imposed directly on the tooth flanks.

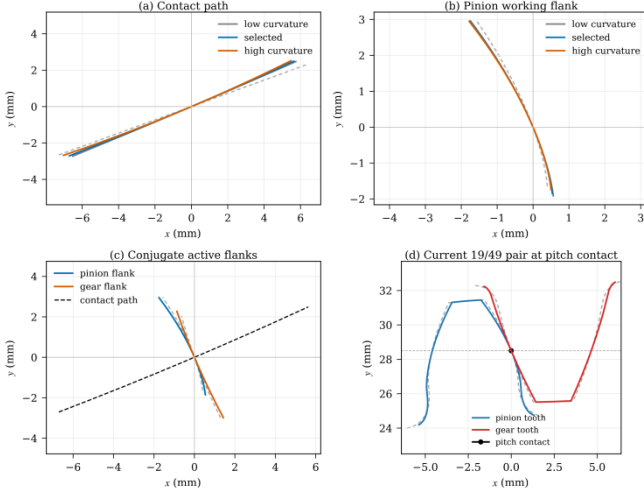


Figure 16: Circular-arc contact-path construction and curvature sensitivity at fixed $\alpha_t = 23^\circ$.

4 Constrained objective-dependent optimization framework

4.1 Design problem and objective separation

For family f , let $\mathbf{x}_f \in [\mathbf{x}_f^L, \mathbf{x}_f^U]$ denote the geometry vector and let Ω_f be the subset satisfying all hard constraints in Sec. 4.2. The two primary responses are the cycle-mean mesh loss and the event-resolved peak-to-peak loaded STE. They are reported relative to the standard 20° involute evaluated with the same physical and numerical benchmark,

$$J_L(\mathbf{x}_f) = \frac{P_{\text{loss}}(\mathbf{x}_f)}{P_{\text{inv}}^{\text{loss}}}, \quad J_E(\mathbf{x}_f) = \frac{\text{STE}_{pp}(\mathbf{x}_f)}{\text{STE}_{pp}^{\text{inv}}}. \quad (64)$$

Both objectives are normalized by the corresponding standard-involute response under the same benchmark. The fixed positive reference aids comparison without changing dominance.

Three objective formulations are defined for every performance-admissible non-involute family and for the one-dimensional optimized-involute control:

$$\begin{aligned} \text{Minimum loss: } \mathbf{x}_L^* &= \underset{\mathbf{x} \in \Omega_f}{\text{argmin}} J_L(\mathbf{x}), \\ \text{Minimum STE: } \mathbf{x}_E^* &= \underset{\mathbf{x} \in \Omega_f}{\text{argmin}} J_E(\mathbf{x}), \end{aligned} \quad (65a-c)$$

$$\text{Pareto set: } \mathcal{P}_f = \text{ND}_{\mathbf{x} \in \Omega_f} \{J_L(\mathbf{x}), J_E(\mathbf{x})\},$$

where ND is the nondominated feasible subset. Every selected geometry is evaluated for all reported responses. The joint design is selected at the knee of the nondominated loss–STE front. A weighted sum is avoided because it can hide non-convex trade-off regions and introduces an arbitrary scale.

STE harmonics remain diagnostics rather than a hidden third objective:

$$A_H(\mathbf{x}) = [\sum_{h=1}^H w_h |\hat{e}_h(\mathbf{x})|^2]^{1/2}, \quad (66)$$

where $\hat{e}_h$ comes from the zero-mean waveform. The post-selection DTE criterion is the peak-to-peak response at the reference comparison point, $n_{\text{ref}} = 3000$ rpm and $T_{1,\text{ref}} = 60$ N m,

$$J_D(\mathbf{x}) = \frac{\text{DTE}_{pp}(\mathbf{x}; n_{\text{ref}}, T_{1,\text{ref}})}{\text{DTE}_{pp}^{\text{inv}}(n_{\text{ref}}, T_{1,\text{ref}})}, \quad (67)$$

After the loss, STE and Pareto searches, DTE_{pp} is evaluated at the reference point for every feasible design retained by the multi-fidelity campaign. “Lowest retained DTE” denotes the member of this candidate set with the smallest DTE_{pp} ; it is not restricted to the three objective-specific endpoints and is not an additional evolutionary objective.

AC RMS, mesh harmonics, resonance location, mesh-force amplification, dynamic load factor and the zero-contact fraction are reported as companion diagnostics.

4.2 Geometric, meshing, strength and model-validity constraints

Conjugacy alone does not ensure a manufacturable, continuously engaged and load-carrying tooth. For normalized constraints $g_k(\mathbf{x}) \leq 0$, the aggregate violation is

$$V(\mathbf{x}) = \sum_{k=1}^{N_g} \max \left[0, \frac{g_k(\mathbf{x})}{s_k} \right], \quad (68)$$

where s_k is fixed and feasibility requires $V = 0$. Deb’s rule ranks feasible solutions first and compares objectives only within that set [33].

The retained engineering conditions are:

1. *Conjugate-flank uniqueness.* The working contact must progress monotonically from root to tip. In the mating-member frame, the radial coordinate of successive contact points must therefore remain strictly monotonic with mesh phase; a reversal makes the conjugate outline multivalued and can introduce separate branches, corners or loss of C^1 continuity [34].
2. *Tooth thickness.* The root transition must be manufacturable and $t_{\text{tip,min}} \geq 0.2m = 0.60$ mm.
3. *Undercut and interference.* The generated flank must retain the intended root section without undercutting. During a complete mesh cycle, the mating tooth outlines must not intersect outside the intended contact [35], [36].
4. *Continuous engagement.* The same $\varepsilon_\alpha \geq 1.20$ floor applies to every performance family, with connected one- and two-pair intervals at the calculated entry and exit.
5. *Contact and lubrication.* Force is compressive, reduced radius positive, $p_{0,\text{max}} \leq p_{H,\text{lim}}$, and the film, viscosity, traction and temperature remain inside the partial-EHL correlation domains.
6. *Strength.* Local Hertz pressure and an actual-outline nominal root-section index are constrained during the search; continuum FEA is applied to the retained designs.

ISO 6336-3 and ANSI/AGMA 2101 provide tooth-root bending ratings for involute spur and helical gears within prescribed geometry domains; their tooth-form and stress-correction factors cannot be transferred directly to the arbitrary flanks and fillets considered here [37], [38]. Studies of non-involute gears therefore employ geometry-specific nominal-stress models or FE-based root-stress evaluation [31], [39], [40]. The present optimization uses a rapid nominal-section index, whereas the retained designs are assessed with the continuum model of Sec. 4.3.

At mesh phase ψ , active pair j loads its associated tooth on member i by the event-resolved normal force $F_j \mathbf{n}_{ij}$. The load-sharing solution determines F_j separately for every simultaneously active tooth pair. Consistent with the frictionless FE strength comparison, tangential flank traction is omitted from this structural calculation. Let $\mathbf{r}_{c,ij}$ be the contact point, $\mathbf{r}_{f,i}$ the centroid of the root section extracted from the complete tooth outline, and $(\mathbf{e}_{x,i}, \mathbf{e}_{y,i})$ a local basis directed respectively across the section and from root to tip. With $n_{x,ij} = \mathbf{n}_{ij} \cdot \mathbf{e}_{x,i}$ and $n_{y,ij} = \mathbf{n}_{ij} \cdot \mathbf{e}_{y,i}$, the signed section resultants and line-of-action offset are

$$\begin{aligned} V_{ij} &= F_j n_{x,ij}, N_{ij} = F_j n_{y,ij}, M_{ij} = F_j \ell_{ij}, \\ \ell_{ij} &= [(\mathbf{r}_{c,ij} - \mathbf{r}_{f,i}) \times \mathbf{n}_{ij}] \cdot \mathbf{e}_z. \end{aligned} \quad (69)$$

Thus $|\ell_{ij}|$ is the perpendicular distance from the root-section centroid to the normal-force line and follows directly from the local contact geometry.

For the spur-gear screen, the load is taken as uniform across the face width b . If $s_{f,i}$ is the actual-outline thickness at the root cut, the local $b \times s_{f,i}$ rectangular-section properties are

$$A_i = b s_{f,i}, \quad I_i = \frac{b s_{f,i}^3}{12}, \quad Z_i = \frac{I_i}{s_{f,i}/2} = \frac{b s_{f,i}^2}{6}. \quad (70)$$

For a local coordinate $\zeta_r \in [-s_{f,i}/2, s_{f,i}/2]$ across the section, the nominal axial-bending and transverse-shear stresses are

$$\sigma_{n,ij}(\zeta_r) = \frac{N_{ij}}{A_i} - \frac{M_{ij}\zeta_r}{I_i}, \quad \tau_{ij}(\zeta_r) = \frac{3V_{ij}}{2A_i} \left[1 - \left(\frac{2\zeta_r}{s_{f,i}} \right)^2 \right]. \quad (71)$$

The maximum magnitudes within this idealized section are

$$\max_{\zeta_r} |\sigma_{n,ij}| = \frac{|N_{ij}|}{A_i} + \frac{|M_{ij}|}{Z_i}, \quad \max_{\zeta_r} |\tau_{ij}| = \frac{3|V_{ij}|}{2A_i}.$$

The search uses the corresponding equivalent-stress envelope

$$\begin{aligned} \Sigma_{ij}^{\text{scr}} &= \sqrt{\left(\frac{|N_{ij}|}{A_i} + \frac{|M_{ij}|}{Z_i} \right)^2 + 3 \left(\frac{3|V_{ij}|}{2A_i} \right)^2}, \\ \Sigma_{\max}^{\text{scr}} &= \max_{\psi, ij} \Sigma_{ij}^{\text{scr}}. \end{aligned} \quad (72)$$

The normal-stress maximum occurs at an outer fibre, whereas the shear maximum occurs at the section centre. Equation (72) therefore bounds $\sqrt{\sigma_n^2 + 3\tau^2}$ only within this nominal rectangular-section model; it is neither a local fillet stress nor an ISO/AGMA rating or fatigue-life prediction. It retains the effects of root thickness, force direction, moment arm and multi-pair load sharing. The subsequent FE analysis resolves the fillet concentration and the non-prismatic tooth and gear-body stress field.

For every family, the optimization caps are $p_{H,\text{lim}} = 1500$ MPa and $\Sigma_{\text{lim}}^{\text{scr}} = 430$ MPa. They are common

screening thresholds, not allowable material stresses or rated load capacities. The principal scalar constraints entering Eq. (68) are

$$\begin{aligned} g_\varepsilon &= 1.20 - \varepsilon_\alpha, \quad g_t = \frac{0.2m}{t_{\text{tip,min}}} - 1, \\ g_H &= \frac{p_{0,\text{max}}}{p_{H,\text{lim}}} - 1, \quad g_{F,\text{scr}} = \frac{\Sigma_{\text{max}}^{\text{scr}}}{\Sigma_{\text{lim}}^{\text{scr}}} - 1. \end{aligned} \quad (73)$$

Interference, monotonicity and model-domain checks are Boolean; grid, event and time-step convergence are verified independently for every retained design.

4.3 Continuum-FE strength assessment

Post-selection strength is assessed directly on the complete analytical outlines. An in-house adapter transfers the gear geometry, material data, boundary conditions and torque history to Gmsh and Kratos Multiphysics [41], [42]. Gmsh provides the body-fitted discretization, with local refinement of the working flanks and root fillets; Kratos Structural Mechanics and Contact Structural Mechanics provide the nonlinear finite-element solution. The two deformable gears are coupled by a frictionless augmented-Lagrange mortar interface in an updated-Lagrangian, finite-sliding formulation. The contact search is updated during every load increment, so the active facets, normal gap, contact pressure and contact area follow the deformed flanks rather than a prescribed point of action.

The plane-strain model uses $E = 200$ GPa, $\nu = 0.30$ and $b = 55$ mm, with the bores constrained and $T_1 = 60$ N m applied incrementally. Thirteen mesh positions span the active path of each retained pair, thereby resolving the movement of contact and the governing root and surface states without assuming their locations in advance. Figure 17 illustrates the mesh and fields for the involute reference; the same geometry-to-solver and post-processing chain is used for the non-involute profiles.

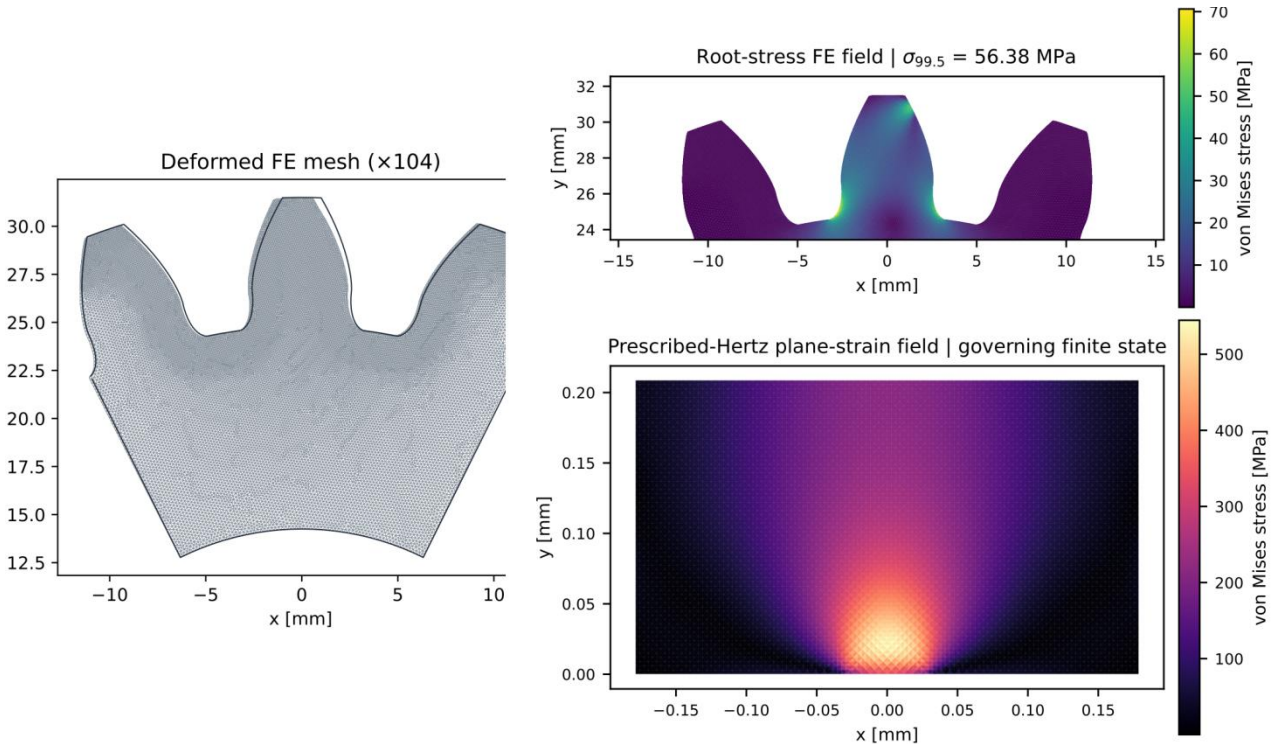


Figure 17: Involute FE reference on the common 19/49 card: locally refined mesh, governing root-stress field and contact-zone stress field. Mesh deformation is magnified for visibility.

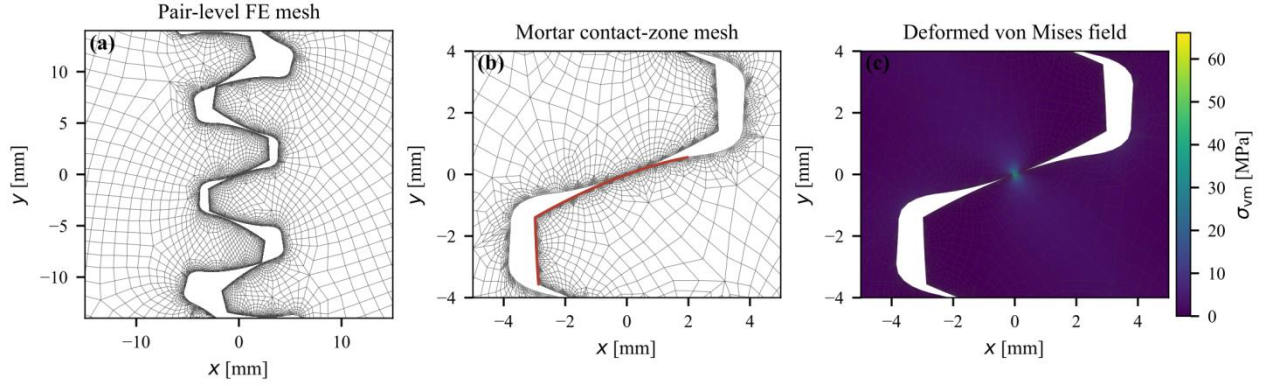


Figure 18: Nonlinear finite-sliding contact verification: pair-level FE mesh, local mortar discretization and deformed von Mises field.

The governing state is recomputed with nominal root sizes 0.32 , 0.24 , 0.16 and 0.10 mm. Mesh quality, nonlinear convergence, contact residuals, force–reaction and torque equilibrium, and the root-stress field are checked at every level [43]. The reported root measure is the area-weighted 99.5th percentile of von Mises stress in a fixed root region, which avoids basing the comparison on a single element or boundary point. At the common torque, the signed percentage difference from the involute is

$$\sigma_{F,f}^{\text{FE}} = P_{99.5}^A(\sigma_{vm}), \quad \Delta\sigma_{F,f} = 100 \left(\frac{\sigma_{F,f}^{\text{FE}}}{\sigma_{F,f}^{\text{inv}}} - 1 \right) \%. \quad (74)$$

The surface solution is checked against the exact line-Hertz pressure computed from the local curvature, moment arm and solved normal load. The analytical pressure and an independently refined contact patch provide complementary checks on the nonlinear contact field [44]. The same-load signed pressure difference is

$$\Delta p_{0,f} = 100 \left(\frac{p_{0,f}}{p_{0,\text{inv}}} - 1 \right) \%. \quad (75)$$

The implementation was first exercised on the official Kratos Hertz and large-sliding contact examples [45]. The 19/49 involute then provides the gear-specific regression anchor. The exact- curvature calculation gives $p_{0,\text{max}} = 870.739$ MPa, compared with 870.7 MPa reported here (0.0045%). At a conventional outer single-pair state, the ISO-style analytical root stress is approximately 37.0 MPa and an independent FE model gives 37.4 MPa. Repeating the conservative full-torque envelope independently gives approximately 56.20 MPa, within 0.32% of the present 56.379 MPa result. These checks supplement the mesh study, for which the governing position is unchanged and the fine-to-reference root indicator changes by 2.6% [37], [46]. Table 3 summarizes these comparisons. The reported percentages compare stress and pressure under the specified torque.

Table 3: Independent checks of the 19/49 involute strength reference.

Matched quantity	Present [MPa]	Independent [MPa]	Difference [%]
Exact-curvature	870.7	870.739	0.0045
$p_{0,\text{max}}$			
ISO-style root state	~37.0	~37.4 FE	~1.1
Conservative FE root envelope	56.379	~56.20 FE	0.32

For the composite rack, pitch contact is finite and each active C^1 /non- C^2 join is evaluated from both adjacent branches.

4.4 Design variables and search domains

Table 4 expresses each construction in its native analytical coordinates, since rack-generated and prescribed-contact-path geometries are not parametrically interchangeable. The active ranges combine the defining source formulations with the common admissibility conditions of Sec. 4.2.

Table 4: Design coordinates and investigated search domains. The geometric formulations follow the corresponding primary sources [6], [7], [8], [10], [11], [13]; the numerical bounds define the present search domains and are supplemented by the feasibility conditions of Sec. 4.2.

Geometry	Design parameter	Lower bound	Upper bound
Standard involute	reference pressure	20° (fixed)	
	angle, α_0		
Optimized involute	transverse pressure	14.5°	36°
	angle, α_0		
Cubic-CP	pitch-normal slope,	tan12°	tan28°
	α_1		
Cubic-CP	cubic coefficient, a_2 [mm ⁻²]	0	1.2
S-gear	rack exponent, n	1.25	3.25
S-gear	normalized rack	1.2	1.6
	height, a_p		
S-gear	pitch-angle	15°	25°
	constraint, α_p		
Inv.-cycloid	pressure angle, α_0	10°	20°
Inv.-cycloid	rolling-radius ratio,	1.5	4.5
	a/m		
Circular-arc CP	pitch-normal angle,	15°	30°
	α_t		
Circular-arc CP	path-curvature ratio,	0	0.1
	$\kappa_c = m/R_c$		

4.5 Constrained evolutionary search and final evaluation

Contact-branch, multiplicity and feasibility changes make the response maps non-convex and effectively non-smooth. Constrained NSGA-II therefore uses tournament selection by constraint, nondomination and crowding rank, together with simulated binary crossover and bounded polynomial mutation [33].

A low-fidelity NSGA-II search first identifies promising regions of the admissible design space. Space-filling and search-selected points are then evaluated with the event-resolved contact model, using the resulting unilateral force histories in the thermal-EHL calculation. These evaluations guide successive adaptive-refinement batches [47]. The selected designs are finally recomputed at the dense resolutions of Table 5.

Table 5: Common multi-fidelity search and evaluation levels for the integrated non-involute families.

Level	Search	Evaluation resolution and purpose
Low fidelity	3 seeds; 24×15 generations	401 EHL points, proxy STE, 801 flank points, analytical load; domain exploration.
Adaptive HF	1 + 4 batches; $24 + 4 \times 6$ evaluations	2001 EHL points, 65 STE phases, 4801 flank points, event-resolved contact load; adaptive refinement.
Final HF	5 independent evaluations	16001 EHL points, 129 STE phases, 4801 flank points, event-resolved contact load; final evaluation.

Adaptive refinement continues until the relevant endpoint values change by no more than 0.5% between consecutive rounds. Each retained design is then recomputed at the final resolution using the thermally corrected partial-EHL model, 24 pressure nodes, event-resolved STE and dense outline discretization; the involute reference is treated identically. Final acceptance requires changes below 0.5% in both loss and STE_{pp} , preservation of the contact branch and satisfaction of all model-validity conditions. The Pareto knee is identified from this converged set.

5 Results

All tabulated designs use the same 19/49 operating card. The standard involute gives $P_{loss} = 105.456$ W, $\eta_m = 99.4405\%$, $STE_{pp} = 20.451 \mu$ rad and $DTE_{pp} = 49.0695 \mu$ rad. “Min. loss”, “Min. STE” and “Joint” denote the minimum-loss, minimum-STE and Pareto-knee selections, respectively. “Lowest retained DTE” is selected afterwards from all feasible designs retained by the multi-fidelity campaign and may therefore be a distinct fourth design. The phase-resolved histories presented later are independent response evaluations rather than substitutes for the Pareto-knee selections.

5.1 Objective-specific selections

5.1.1 Pressure-angle-optimized involute

Increasing the pressure angle shortens the active path and reduces sliding, but the smaller base-circle moment arm requires a larger single-pair normal force to transmit the same torque. For the common tooth and cutter specification, the admissible angle-only range ends at $\alpha_0 \approx 23.16^\circ$; this boundary also minimizes mesh loss. Higher pressure angles have been examined with non-standard optimized root geometries, but those configurations lie outside the fixed cutter and root specification adopted here [48]. The corresponding contact-ratio, STE and DTE results are reported in Table 6.

Table 6: Manufacturing-feasible pressure-angle control on the common card.

Selection	Design variables	$\varepsilon\alpha$	P [W]	η [%]	STE [μ rad]	DTE [μ rad]
Reference	$\alpha_0 = 20^\circ$	1.647	105.5	99.441	20.45	49.07
Min. loss	$\alpha_0 = 23.156^\circ$	1.526	72.3	99.616	19.53	36.69

5.1.2 Cubic contact path

The two polynomial coefficients redistribute curvature, sliding and load away from pitch without changing the common gearset card. The loss optimum nearly halves dissipation. The STE and DTE minima move toward a more involute-like path and sacrifice part of the tribological gain. The knee-point design is the most even three-response compromise: it lowers loss by 37.3%, STE_{pp} by 17.8% and DTE by 27.9%.

Table 7: Objective-specific Cubic-CP designs.

Selection	Design variables	$\varepsilon\alpha$	P [W]	η [%]	STE [μ rad]	DTE [μ rad]
Reference	$\alpha_0 = 20^\circ$	1.647	105.5	99.441	20.45	49.07
Min. loss	$a_1 = 0.3914$, $a_2 = 2.214 \times 10^{-3}$	1.445	56.4	99.701	17.45	37.20
Min. STE	$a_1 = 0.4090$, $a_2 = 3.417 \times 10^{-4}$	1.531	72.9	99.613	16.75	35.05
Lowest retained DTE	$a_1 = 0.4140$, $a_2 = 1.800 \times 10^{-4}$	1.557	77.8	99.587	16.83	34.25
Joint	$a_1 = 0.4069$, $a_2 = 8.091 \times 10^{-4}$	1.501	66.1	99.649	16.81	35.40

5.1.3 S-gears

S-gears provide the largest reduction in sliding loss, but the loss optimum also has the lowest contact ratio in the comparison. Its pronounced single-/double-pair transition explains why a strong steady-state efficiency gain does not automatically yield the lowest DTE. Increasing overlap recovers dynamic smoothness at the cost of loss. The joint design reduces mesh loss, STE_{pp} and DTE_{pp} by 57.1%, 19.5% and 14.8%, respectively.

Table 8: Objective-specific S-gear designs.

Selection	Design variables	$\varepsilon\alpha$	P [W]	η [%]	STE [μ rad]	DTE [μ rad]
Reference	$\alpha_0 = 20^\circ$	1.647	105.5	99.441	20.45	49.07
Min. loss	$n = 1.342$, $a = 1.599$	1.307	35.5	99.812	16.95	48.20
Min. STE	$n = 1.62$, $a = 1.55$	1.420	48.9	99.741	16.37	43.40
Lowest retained DTE	$n = 1.98$, $a = 1.46$	1.550	60.3	99.680	16.64	39.60
Joint	$n = 1.55$, $a = 1.56$	1.440	45.2	99.760	16.47	41.80

5.1.4 Involute-cycloid composite

The straight central rack segment regularizes pitch contact, whereas the outer cycloidal branches control the approach and recess portions. Designs with shorter outer branches favour loss; longer branches increase overlap and reduce dynamic response. The joint selection yields a 24.9% loss reduction, a 16.4% STE_{pp} reduction and a 17.3% DTE reduction. Its intermediate position is consistent with a geometry deliberately combining involute pitch behaviour with cycloidal off-pitch curvature.

Table 9: Objective-specific involute-cycloid composite designs.

Selection	Design variables	$\varepsilon\alpha$	P [W]	η [%]	STE [μ rad]	DTE [μ rad]
Reference	$\alpha_0 = 20^\circ$	1.647	105.5	99.441	20.45	49.07
Min. loss	$\alpha_0 = 18.2^\circ$, $a/m = 2.70$	1.500	69.8	99.630	17.53	45.10
Min. STE	$\alpha_0 = 20^\circ$, $a/m = 3.50$	1.619	94.5	99.499	17.03	41.20
Lowest retained DTE	$\alpha_0 = 19.5^\circ$, $a/m = 3.85$	1.660	98.4	99.478	16.96	38.70
Joint	$\alpha_0 = 19^\circ$, $a/m = 3.10$	1.560	79.2	99.580	17.10	40.60

5.1.5 Circular-arc contact-path comparison

Table 10 reports the objective-specific results for the circular-arc contact-path geometry of Sec. 3.5, parameterized by its pitch-normal angle and path-curvature coefficient. Its joint selection reduces mesh loss, STE_{pp} and DTE_{pp} by 19.1%, 17.4% and 16.9%, respectively.

Table 10: Circular-arc contact-path objective-specific results.

Selection	Design variables	$\varepsilon\alpha$	P [W]	η [%]	STE [μ rad]	DTE [μ rad]
Reference	$\alpha_0 = 20^\circ$	1.647	105.5	99.441	20.45	49.07
Min. loss	$\alpha = 22^\circ$, $\kappa c = 0.055$	1.470	79.6	99.578	17.60	46.00
Min. STE	$\alpha = 20.5^\circ$, $\kappa c = 0.035$	1.580	90.5	99.520	16.74	41.80
Lowest retained DTE	$\alpha = 20.5^\circ$, $\kappa c = 0.025$	1.620	95.2	99.495	16.87	38.90
Joint	$\alpha = 21^\circ$, $\kappa c = 0.042$	1.550	85.3	99.548	16.90	40.80

5.2 Direct response histories and DTE

Figures 19 and 20 show the computed phase-resolved responses at 60 N m.

Changes between one- and two-pair contact produce the visible steps in local frictional power, which vanishes at pitch. S-gears reduce this power over most of the path, while Cubic-CP reduces it on both approach and recess. The composite central segment is involute, but the cycloidal outer branches shift the contact transitions and modify load sharing; its local power trace need not therefore coincide pointwise with the standard involute.

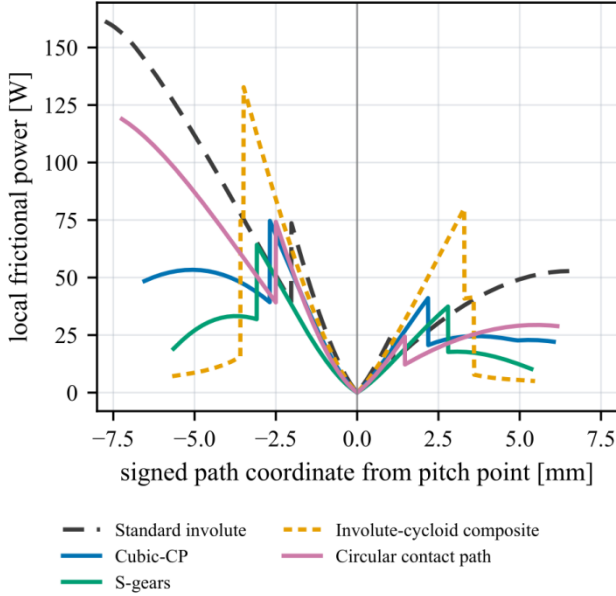


Figure 19: Phase-resolved local frictional power at 60 N m.

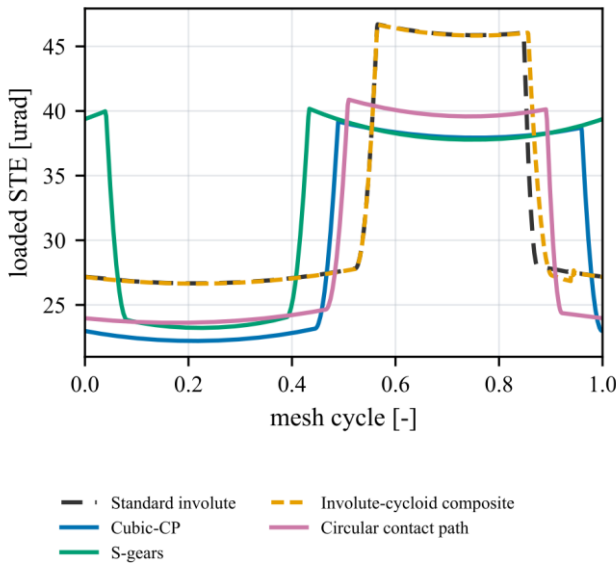


Figure 20: Event-aware loaded STE at 60 N m.

STE_{pp} describes the range of the quasi-static waveform but not its dwell, transition sharpness or harmonic distribution. DTE is obtained by applying the complete phase-dependent torque–approach relation to the drivetrain model; consequently, a lower STE_{pp} does not necessarily imply a lower dynamic response.

5.2.1 Cross-family loss–dynamics synthesis

Figure 21 provides the principal cross-family synthesis of the **joint** selections on the common operating card. Table 11

reports the underlying values, and Figure 23 compares their multiphysics fingerprints.

Table 11: Common-card performance of the Pareto-knee (joint) selections.

Geometry	$\varepsilon\alpha$	$P_{\text{loss}} [\text{W}]$	$STE_{pp} [\mu\text{rad}]$	$DTE_{pp} [\mu\text{rad}]$
Standard involute	1.64745	105.456	20.451	49.0695
Cubic-CP	1.501	66.1	16.81	35.40
S-gears	1.440	45.2	16.47	41.80
Inv.-cycloid	1.560	79.2	17.10	40.60
Circular contact path	1.550	85.3	16.90	40.80

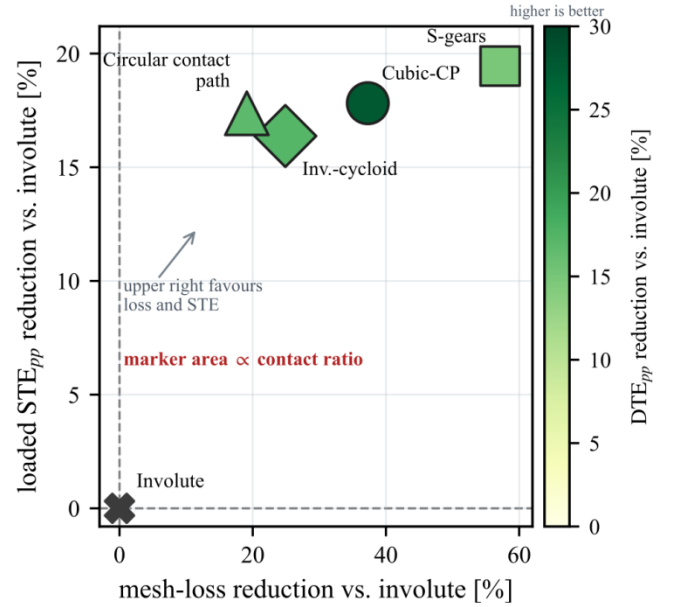


Figure 21: Loss–dynamics map.

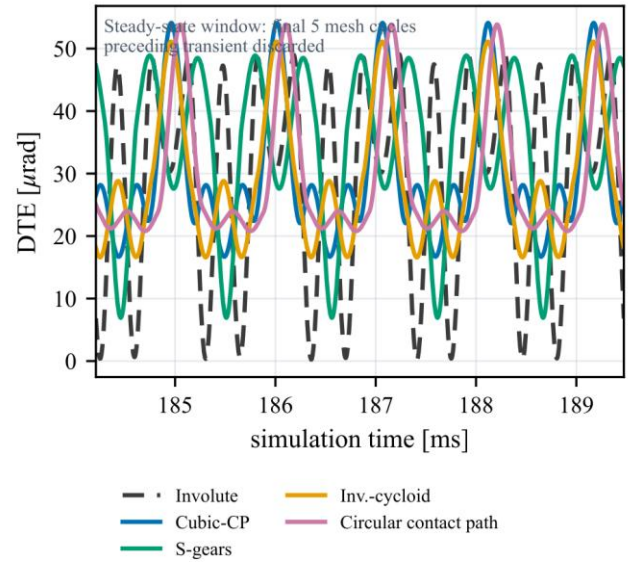


Figure 22: Stationary DTE histories over five mesh cycles sampled from the final 20-cycle analysis window; the circular trace denotes the circular-arc contact-path geometry.

For the joint selections, Cubic-CP reduces mesh loss, STE_{pp} and DTE_{pp} by 37.3%, 17.8% and 27.9%, respectively. The corresponding reductions are 57.1%/19.5%/ 14.8% for S-gears, 24.9%/16.4%/ 17.3% for the involute–cycloid composite and 19.1%/17.4%/ 16.9% for the circular-arc contact path.

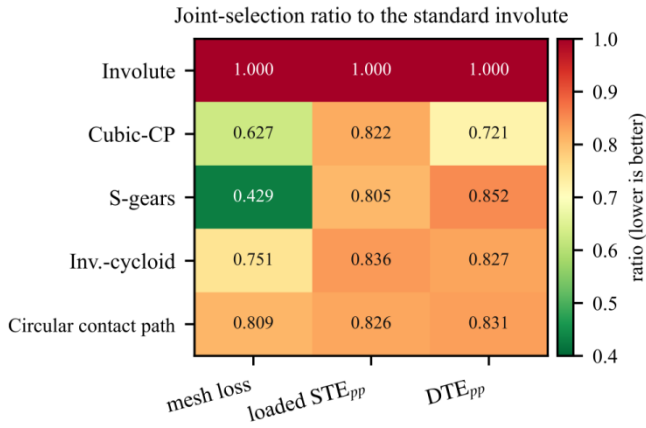


Figure 23: Performance ratios of the joint selections relative to the standard involute; lower values indicate improvement.

The joint-selection fingerprint shows that every selected non-involute design improves all three performance measures relative to the standard involute. S-gears provide the largest loss and STE reductions, whereas Cubic-CP provides the largest DTE reduction. The composite and circular-arc contact path occupy intermediate regions, with similar STE and DTE improvements but smaller loss gains.

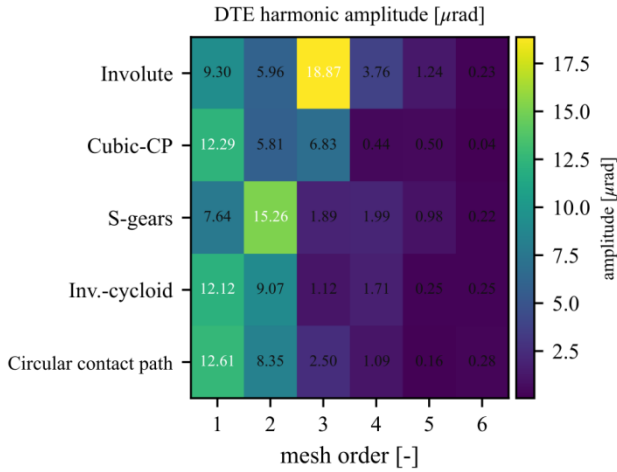


Figure 24: DTE mesh-order fingerprint; the circular row denotes the circular-arc contact-path geometry.

Figure 24 shows why peak-to-peak reduction is not a sufficient dynamic objective. The standard involute is dominated by the third mesh order ($18.87 \mu\text{rad}$), whereas S-gears are dominated by the second ($15.26 \mu\text{rad}$). Cubic-CP retains a comparatively strong third order ($6.83 \mu\text{rad}$). The composite and circular-arc contact-path spectra are concentrated mainly in the first two orders and have weaker higher-order content. A drivetrain resonance near a specific mesh order can therefore reverse a ranking inferred from DTE_{pp} alone.

5.3 Post-selection strength and continuum-FE comparison

Every optimization candidate is evaluated with the analytical Hertz/root equations of Sec. 4.2; direct FEA is computationally reserved for retained designs. The involute reference and its independent checks are presented in Sec. 4.3; the following one-column figures retain the same root-field and prescribed-Hertz layout for the four non-involute constructions.

5.3.1 Cubic-CP

Cubic-CP gives $\sigma_{F,99.5}^{\text{FE}} = 48.974 \text{ MPa}$ (13.1% below the involute) and $p_0 = 450.6 \text{ MPa}$ (48.3% below the involute).

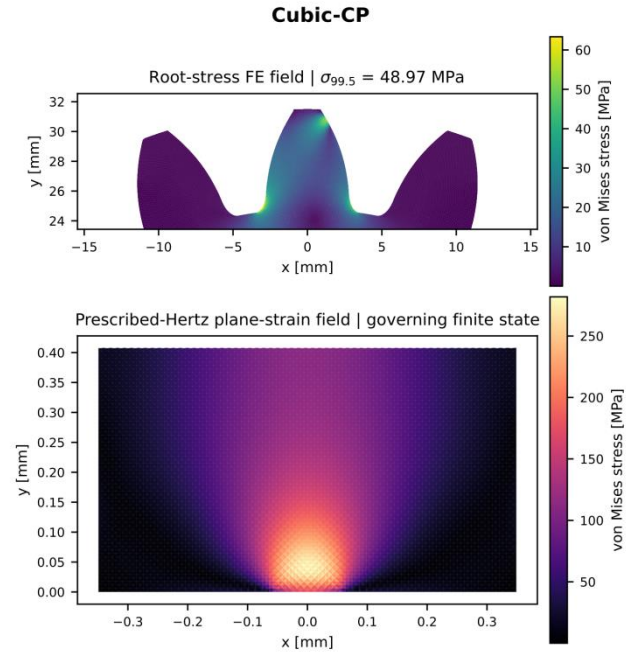


Figure 25: Cubic-CP root-stress and prescribed-Hertz fields.

5.3.2 S-gear

S-gear is the strongest root case: $\sigma_{F,99.5}^{\text{FE}} = 45.738 \text{ MPa}$, 18.9% below the involute. Its $p_0 = 412.7 \text{ MPa}$ is 52.6% lower. The gear governs; the pinion governs the other four families.

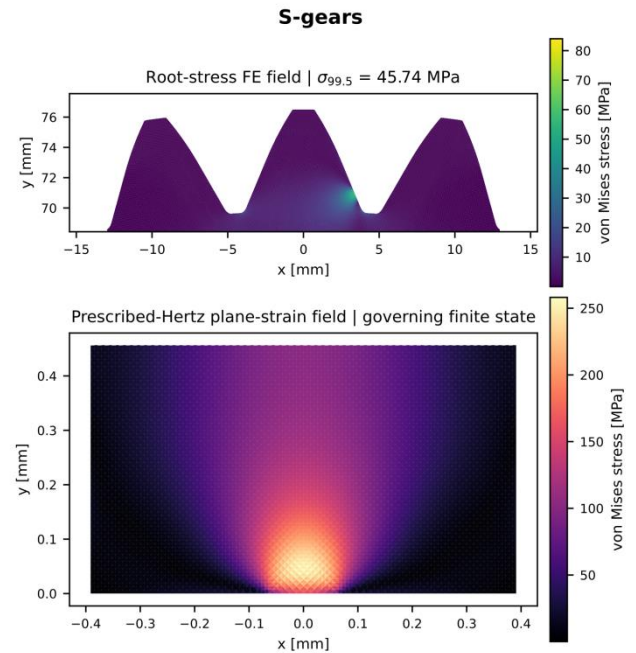


Figure 26: S-gear root-stress and prescribed-Hertz fields. The root-field view uses the same module-relative tooth-zone limits as the other families.

5.3.3 Involute-cycloid composite

The composite gives $\sigma_{F,99.5}^{\text{FE}} = 46.058 \text{ MPa}$ (18.3% below the involute) and $p_0 = 530.1 \text{ MPa}$ (39.1% below the involute). Its governing pressure is the explicit cycloidal-side approach value at the C^1 /non- C^2 transition; pitch pressure is finite and lower.

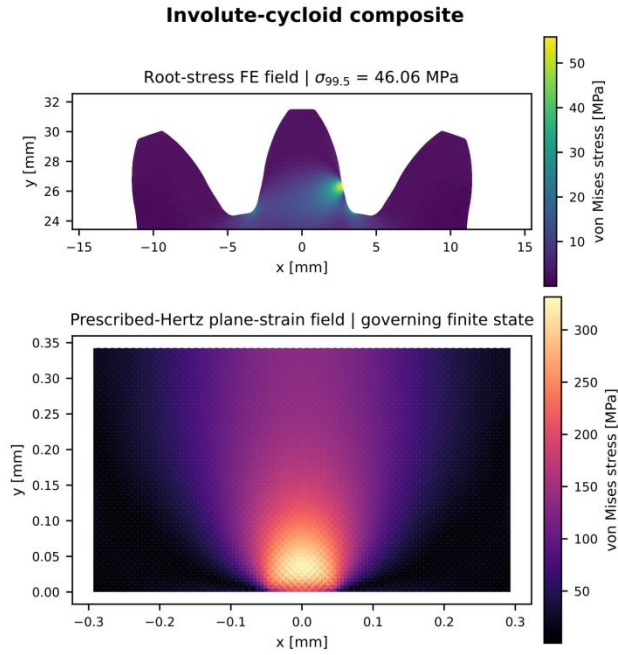


Figure 27: Involute–cycloid composite root-stress and prescribed-Hertz fields; both transition pressures are evaluated from their exact adjacent branches.

5.3.4 Circular contact path

The structural card gives $\sigma_{F,99.5}^{FE} = 52.078$ MPa (7.6% below the involute) and $p_0 = 693.5$ MPa (20.4% below the involute). This fixed-frame path is not a Wildhaber–Novikov or double-circular-arc gear.

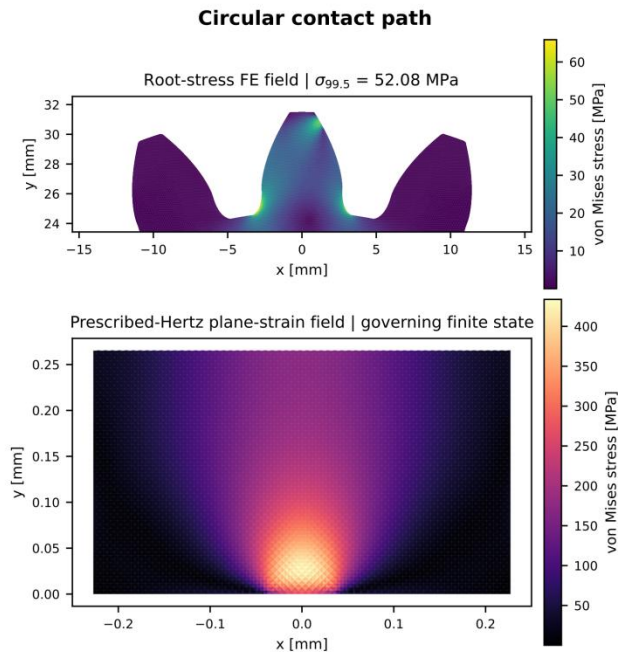


Figure 28: Circular-path root-stress and prescribed-Hertz fields.

5.3.5 Exact-curvature surface-pressure envelope

Across the full path, Cubic-CP, S-gear, composite and circular-path geometries reduce the pressure maximum by 48.3%, 52.6%, 39.1% and 20.4%. The composite curve is finite at pitch; both piecewise transitions retain their exact adjacent-branch limits.

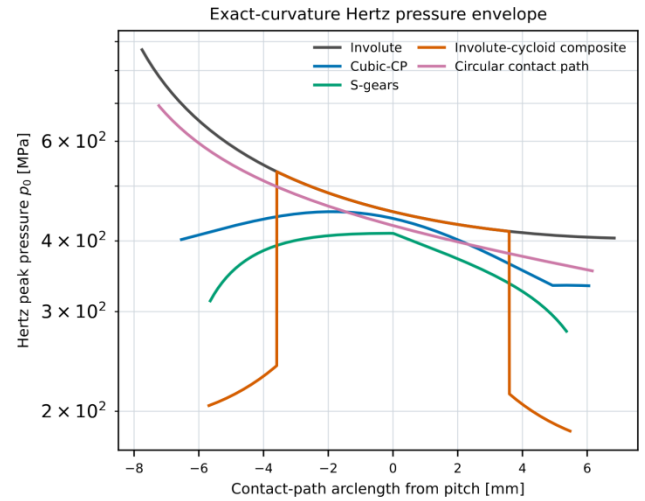


Figure 29: Exact-curvature single-pair Hertz pressure envelope for all five common-card geometries. Composite transition contacts retain explicit adjacent-branch values.

Table 12: Common-card linear-elastic strength comparison at the same applied torque. The listed member governs the root response; signed changes are relative to the standard involute.

Geometry	Member	$\sigma_{v,99.5}$ [MPa]	$\Delta\sigma_F$	$p_{0,max}$ [MPa]	Δp_0
Involute	pinion	56.4	0.0%	870.7	0.0%
Cubic-CP	pinion	49.0	−13.1%	450.6	−48.3%
S-gear	gear	45.7	−18.9%	412.7	−52.6%
Composite	pinion	46.1	−18.3%	530.1	−39.1%
Circular path	pinion	52.1	−7.6%	693.5	−20.4%

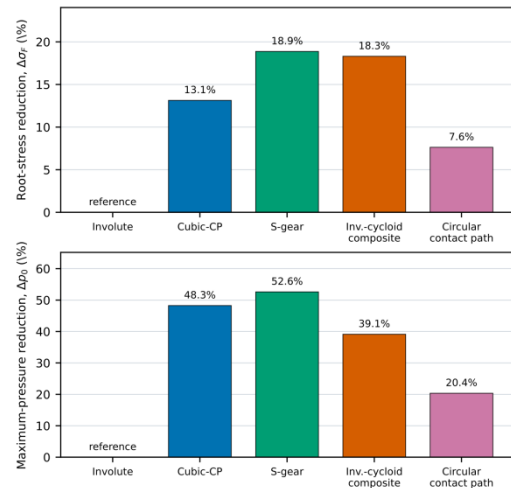


Figure 30: Same-load reductions in governing root stress and maximum Hertz pressure relative to the standard involute.

Published S-gear studies report root-stress reductions of comparable magnitude, supporting the direction of the present FE results [8], [9]. Previous composite-gear studies likewise report improved bending and surface-strength indicators, although differences in geometry, loading and material preclude direct numerical comparison [10], [32].

6 Discussion

6.1 There is no geometry-independent optimum

The central result is an objective-dependent comparison rather than a universal tooth form. Mesh loss, STE, DTE and strength are distinct functionals of the same evolving contact state. Loss integrates traction power along the path; STE follows instantaneous compliance and load sharing; DTE transmits the complete phase-dependent excitation

through the drivetrain; and root and surface stress depend on the position, direction and distribution of the contact load. A profile change therefore modifies several coupled mechanisms rather than one scalar property.

The common benchmark is essential to this interpretation. Tooth numbers, module, face width, material, lubricant, roughness, torque and speed are fixed, and each family is evaluated using the same contact, tribological and compliance formulations. Its own geometry-specific phase-load relation is then used in the DTE model. The resulting differences therefore quantify the effect of profile geometry within the adopted models, rather than a simultaneous change in transmission or operating conditions.

6.2 Efficiency-dynamics mechanisms

The S-gear reaches the largest reduction in mesh loss by combining favourable curvature with low sliding over much of the active path. Its loss-minimizing design, however, has a comparatively low transverse contact ratio. The resulting one-/two-pair transitions change both load sharing and the duration of the high-compliance portion of the mesh cycle. Contact ratio is therefore an important descriptor but not a sufficient dynamic metric: similar STE_{pp} values can correspond to different dwell times, transition sharpness and mesh-harmonic content. This explains why dynamic response must be examined after loss and STE optimization.

Cubic-CP provides two complementary geometric controls: the linear coefficient sets the pitch normal, while the cubic term redistributes off-pitch curvature and sliding. Its Pareto front therefore exhibits a gradual exchange between efficiency and load-transfer smoothness. The optimized involute quantifies how much of that improvement can be obtained through a conventional pressure-angle change while retaining established manufacturing practice.

The involute-cycloid composite combines finite-curvature involute contact near pitch with convex-concave cycloidal action toward the ends of engagement. Its response depends on both the relative extent of these regions and the curvature change at their transition, while the cycloidal portions retain greater centre-distance sensitivity than an involute flank. The fixed-frame circular path provides a separate means of controlling path curvature, but its response shows that constant path curvature alone does not determine STE or DTE: moment arms, contact events and pair multiplicity remain decisive.

6.3 Strength and tribological interpretation

The FE results show that profile design can reduce structural and surface demand simultaneously, although through different mechanisms. Root stress is governed principally by tooth-section geometry, fillet concentration and the contact-force line of action, whereas maximum Hertz pressure depends on local relative curvature and load sharing. All non-involute constructions improve both indicators at the specified torque; the S-gear gives the largest combined reduction, while Cubic-CP provides a substantial improvement without sacrificing its balanced loss-STE response.

These results describe finite-sliding nonlinear contact with a linear-elastic steel constitutive law; they are not fatigue-life predictions. Residual stress, surface finish, micropitting, scuffing, wear and plasticity are not represented. Likewise, lower peak pressure does not by

itself imply lower power loss, because loss depends on the complete histories of load, sliding, film thickness, viscosity and traction. The FE and EHL assessments are therefore complementary rather than interchangeable.

6.4 Design implications and limitations

For the investigated duty point, the S-gear is favoured when efficiency and strength carry the greatest weight, provided that its stronger dynamic excitation is acceptable for the surrounding drivetrain. Cubic-CP provides the most balanced loss-STE compromise. The pressure-angle-controlled involute retains the clearest advantage when tooling, interchangeability and centre-distance tolerance dominate, whereas the composite and fixed-frame circular path provide additional control of conformity and load transfer.

These conclusions apply to the specified steel spur-gear pair and operating point. The FE model resolves two-dimensional finite-sliding contact and contact nonlinearity, while the material response remains linear elastic; the DTE model is torsional. Housing and shaft flexibility, bearing dynamics, flank modifications, manufacturing scatter, misalignment, wear and temperature-dependent bulk deformation are outside the present scope. These effects can shift the numerical Pareto fronts and the preferred design, so the reported ranking should be interpreted within the stated benchmark.

6.5 Future work

Further work should experimentally identify and validate the drivetrain parameters used by the torsional DTE model, extend the geometry-specific phase-load surfaces across broader speed and torque ranges, and compare the predicted mesh-order response with measured vibration and dynamic mesh-force data. Robust optimization should also include centre-distance, profile and lead errors rather than nominal geometry alone. Profile deviations can alter both tooth-root stress and STE, particularly for non-standard root geometries [49]. Double-flank roll testing coupled with free-form meshing simulation provides a practical route for identifying geometric deviations in manufactured gears [50].

An important extension is to re-synthesize and compare the investigated families as helical gear pairs. The additional face contact ratio and progressive engagement can alter load sharing, STE and DTE more strongly than a transverse-profile change alone, while the resulting three-dimensional contact, lead sensitivity and axial force require full-face contact and structural models. Such a comparison would be directly relevant to high-load industrial transmissions and cannot be inferred from the present spur-gear ranking.

Polymer gears provide another relevant application route for non-involute profiles because precision moulding and additive manufacturing can reproduce a prescribed free-form flank without conventional rack- or hob-generating kinematics. Free-form polymer flanks have also been optimized, additively manufactured and tested for improved wear resistance [51]. The manufacture and testing of S-shaped POM gears already demonstrate the transfer of such profiles to polymer hardware [9]. This geometric freedom does not remove process constraints: mould shrinkage, tool compensation, anisotropy, surface finish and dimensional stability remain part of the design problem [52]. Application of the framework to polymer gears should therefore

incorporate nonlinear, temperature- and rate-dependent material behaviour, viscoelastic dissipation, evolving contact and thermal expansion, with nonlinear FE verification[53]. Manufacturability, metrology, wear and durability should then become explicit objectives in the selection of designs for experimental validation.

7 Conclusions

A common event-resolved framework has been used to compare the standard involute with four non-involute spur-gear constructions under the same gearset, material and operating conditions. The principal conclusions are:

1. No geometry is uniformly superior. Power loss, STE, DTE and strength are distinct objectives; in particular, similar STE ranges can produce different DTE because load-transfer timing and mesh-harmonic content also govern the dynamic response.
2. The minimum-loss S-gear gives the largest individual reduction in frictional power loss (66.38%). At the joint selection, the same family reduces mesh loss, STE_{pp} and DTE_{pp} by 57.1%, 19.5% and 14.8%, respectively. The distinction between these two selections demonstrates the trade-off between maximum efficiency and dynamic smoothness.
3. According to the adopted Pareto-knee selection criterion, Cubic-CP provided the most balanced loss-STE compromise. The selected design reduced gear-mesh frictional power loss by 37.3% and STE_{pp} by 17.8% relative to the standard involute baseline. In the subsequent dynamic evaluation, the same design also reduced DTE_{pp} by 27.9%, with all three performance measures remaining below their corresponding baseline values.
4. The pressure-angle-optimized involute served as a conventional design-space control, allowing improvements specific to non-involute synthesis to be distinguished from those attainable through pressure-angle adjustment within the involute family. The selected Pareto-knee design of the involute-cycloid composite reduced gear-mesh frictional power loss, STE_{pp} and DTE_{pp} by 24.9%, 16.4% and 17.3%, respectively, relative to the standard involute baseline. The circular-arc contact-path joint design gives corresponding reductions of 19.1%, 17.4% and 16.9%.
5. FEA predicts lower governing root stress and maximum Hertz pressure for all four non-involute constructions. The respective root/surface reductions are 13.1%/48.3% for Cubic-CP, 18.9%/52.6% for S-gears, 18.3%/39.1% for the composite and 7.6%/20.4% for the circular path. The S-gear gives the largest combined reduction and agrees in direction and magnitude with the primary literature.

The preferred geometry is therefore application dependent: S-gears favour efficiency and strength, Cubic-CP provides the most balanced reduction of loss and excitation, the composite and circular path offer targeted control of conformity and load transfer, and the involute retains the strongest case when manufacturing simplicity and standardization dominate.

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The Evolving Role of Planetary Transmissions in Collaborative and Humanoid Robotics

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Abstract. Mechanical transmissions have evolved from primarily power-conversion devices into increasingly integrated elements of robotic actuator systems. Servo-controlled machinery added requirements for accurate motion transmission, while collaborative and humanoid robotics introduced the need for bidirectional power flow, mechanical transparency and physical interaction. This paper examines how these cumulative requirements are reshaping the role of planetary transmissions in robotics. Particular attention is given to the trade-offs between transmission ratio, torque density, efficiency, backdrivability, dynamic behaviour and manufacturability. The emergence of quasi-direct-drive actuators demonstrates the potential of conventional low-ratio planetary gearsets combined with high-torque-density motors, while recent developments in compound planetary architectures provide an alternative route towards high-ratio, compact and backdrivable transmissions. Rather than identifying a universally optimal solution, the paper argues for application-specific optimization of transmission architecture and ratio within the complete actuator system. Finally, recent developments in integrated torque sensing and condition monitoring are discussed as evidence of a further evolution towards observable transmissions, in which mechanical architecture and sensing are increasingly considered together. These developments suggest an expanding role for planetary transmissions as scalable, application-optimized mechatronic systems for next-generation collaborative and humanoid robots.

1 Introduction

The requirements imposed on mechanical transmissions have evolved together with the machines in which they operate. New applications have progressively added functions and constraints without replacing their fundamental role as power-transmission devices. In robotics, this evolution can be viewed as a cumulative transition from power conversion, to accurate controlled motion, to bidirectional physical interaction and, increasingly, observability.

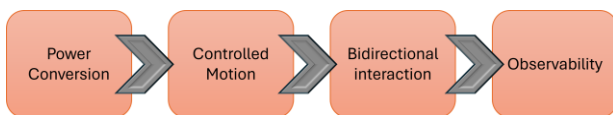


Fig. 1. Cumulative evolution of transmission requirements considered in this paper

1.1 Transmissions as power-conversion devices

The history of geared transmissions extends over more than two millennia, and their earliest sophisticated applications were not necessarily associated with power transmission. The Chinese south-pointing chariot, historically documented in the third century AD, used wheel motion to maintain a figure pointing in a predefined direction, effectively performing mechanical dead reckoning [1,2]. An even earlier surviving example is the Antikythera Mechanism, constructed around the end of the second century BC [3]. Its complex bronze gearing reproduced astronomical cycles, including solar and lunar motion and eclipse prediction [3,4]. Reconstructions have identified epicyclic mechanisms

within the device, showing that principles underlying modern planetary transmissions were already being exploited more than two thousand years ago, primarily to encode and reproduce motion.

This kinematic tradition continued through the Middle Ages and Renaissance. Giovanni de Dondi's fourteenth-century Astrarium, for example, used numerous gear trains, including planetary arrangements and non-circular gears, to reproduce celestial motions [5]. Such mechanisms demonstrate that sophisticated gearing was used to generate prescribed motion relationships long before industrial power transmission became its dominant function.

The Industrial Revolution introduced a fundamentally different need: adapting the power of prime movers to general-purpose machinery. Steam engines, initially used mainly for reciprocating pumping, had to provide continuous rotary motion to drive mills and industrial equipment. Because the crank-and-flywheel solution had been patented by James Pickard, William Murdoch developed the sun-and-planet mechanism patented by James Watt in 1781 [6]. This epicyclic arrangement converted beam motion into continuous rotation and illustrates the transition of gearing towards an interface between prime mover and working machine.

With industrialisation, this power-conversion role became dominant. Planetary transmissions became particularly attractive because their coaxial architecture, load sharing and multiple simultaneous gear contacts allow high torque density and high mechanical efficiency within compact packages. Their use subsequently expanded across automotive, aerospace and

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industrial applications, where efficient and reliable adaptation of speed and torque remains their fundamental function.

1.2 Servo systems and the transition to controlled motion transmission

The emergence of servo-controlled machinery did not replace this fundamental power-conversion function, but added a new requirement: the transmission was now also expected to preserve the accuracy with which motion was commanded and controlled. In a servo system, the transmission became part of a closed-loop control chain in which motion commanded at the motor had to be reproduced accurately at the driven load, while still adapting speed and torque and transmitting power efficiently and reliably. Consequently, the quality with which motion was transmitted became as important as the ability to transmit power.

This transition was particularly relevant for machine tools and, subsequently, industrial robots, where geared servo drives enabled relatively small, high-speed electric motors to generate the high torques required at low output speeds. The transmission ratio also became important from a dynamic-design perspective through the classical principle of inertia matching. The load inertia reflected to the motor shaft decreases with the square of the reduction ratio, allowing the transmission to bring the effective load inertia closer to the rotor inertia and thereby improve acceleration capability and servo response. Transmission selection consequently became a coupled motor–load problem rather than merely a means of obtaining the required output torque. The importance of load-to-motor inertia ratio remains well established in servo-system design and in the optimal selection of motor–reducer combinations.

At the same time, placing a gearbox between the controlled motor and the load introduced mechanical effects that directly influenced positioning performance. Backlash, torsional compliance, friction and transmission error became relevant to positioning and trajectory accuracy [7,8]. Backlash introduces a nonlinear relationship between input and output position during motion reversals, while limited torsional stiffness introduces load-dependent deviations and resonances that constrain achievable control bandwidth.

These requirements strongly influenced the development and adoption of specialized precision reducers for robotics. Transmission performance could no longer be characterized only by classical power-transmission metrics such as load capacity, efficiency and durability, but increasingly required parameters describing the relationship between input and output motion, including lost motion, transmission error, torsional stiffness and hysteresis. The gearbox had thus evolved from primarily a power-conversion machine into an integral element of a controlled motion system.

1.3 Collaborative robotics and bidirectional power transmission

The emergence of collaborative and human-centred robotics introduced another fundamental extension to

transmission requirements. Conventional servo systems were primarily conceived to accurately impose a commanded motion on a load, with power predominantly flowing from the actuator towards the environment. Collaborative robots, exoskeletons, prostheses and, more recently, humanoid robots must instead physically interact with humans and unstructured environments. External forces and motions therefore need to be accommodated by the actuator, making the transmission of mechanical power and motion inherently bidirectional.

This requirement places increasing importance on the mechanical transparency of the transmission. External loads acting at the robot joint must be able to propagate through the gearbox towards the motor with sufficiently low mechanical impedance, making backdrivability a fundamental actuator characteristic. Friction, reflected inertia and transmission ratio consequently influence not only efficiency and dynamic performance, but also the response of the robot to contact, impacts and externally imposed motion. A transmission optimized for accurate forward motion can therefore exhibit very different behaviour when driven from its output.

At the same time, physical interaction creates a need to determine the forces exchanged between the robot and its environment. Torque can be estimated indirectly from motor current in sufficiently transparent actuators, or measured through dedicated sensing elements at or within the joint. Transmission characteristics such as friction, hysteresis and internal load distribution consequently influence not only mechanical performance but also the accuracy with which external interaction forces can be observed. Modern robotic transmissions must therefore combine the accumulated requirements of efficient power conversion and accurate motion transmission with bidirectional mechanical transparency and, increasingly, the ability to support reliable force and torque estimation.

2 Transmission requirements for collaborative and humanoid robotics

Table 1. Cumulative evolution and relative importance of selected transmission requirements.

Requirement	Evolution in robotics
Torque density / ratio	Core throughout; increasingly optimized at actuator and joint level.
Efficiency / backdrivability	Forward efficiency remains essential; bidirectional efficiency and low mechanical impedance become critical.
Accuracy / stiffness	Dominant in servo systems; retained but balanced against compliance and interaction performance.
NVH	From application-dependent refinement to an increasingly important requirement in human environments.
Manufacturability / scalability	From cost and quality considerations to repeatable, scalable production for humanoid deployment.
Observability	From motor/joint position sensing towards transmitted torque and gearbox-condition information.

The historical evolution described above has produced a set of closely interconnected—and frequently

conflicting—requirements. Modern collaborative and humanoid actuators must simultaneously provide efficient power conversion, accurate and predictable motion transmission, sufficiently transparent bidirectional interaction and a realistic pathway towards industrial manufacturing. Table 1 summarizes the characteristics considered particularly relevant to this design space.

2.1 Torque density and transmission ratio

High torque density is one of the fundamental requirements of robotic actuation, particularly in mobile systems where actuator mass contributes directly to the mass that other joints must support and accelerate. This effect becomes especially important in humanoid robots, where actuators are distributed along serial kinematic chains and mass located in distal joints increases the torque and energy requirements of upstream joints. Minimizing actuator mass is therefore not only a local design objective, but can result in significant reductions in overall robot mass and energy consumption.

Electric motors generally achieve their highest power densities at speeds considerably higher than those required at robotic joints. Mechanical transmissions bridge this mismatch by reducing speed and multiplying torque, allowing smaller and lighter motors to satisfy the joint torque requirements. Increasing the transmission ratio can consequently reduce the motor torque—and therefore its required size and mass—making transmission ratio an important lever for increasing actuator torque density.

The benefits of increasing transmission ratio are, however, accompanied by important penalties. Most notably, motor inertia reflected at the gearbox output increases approximately with the square of the transmission ratio, progressively reducing the mechanical transparency of the actuator. Higher ratios also increase motor operating speed and can amplify the influence of gearbox friction and internal power losses. The transmission ratio therefore represents a fundamental compromise between motor mass and the dynamic and energetic behaviour of the complete actuator.

2.2 Efficiency and backdrivability

Efficiency is a fundamental requirement of robotic transmissions, particularly in mobile and battery-powered systems. Transmission losses directly increase electrical energy consumption and heat generation, while the resulting thermal load can further constrain motor and actuator sizing. In humanoid robots, where a large number of actuators operate simultaneously, even moderate differences in transmission efficiency can therefore influence system-level battery capacity and operating time.

Backdrivability is closely related to efficiency but should not be considered equivalent to it. At low speeds, the torque required to backdrive a transmission can be dominated by friction, seal drag and bearing or gear preload. During rapid interactions, reflected motor inertia becomes increasingly important and may

dominate the mechanical impedance perceived at the joint. Useful bidirectional behaviour consequently requires both low dissipative losses and sufficiently low reflected inertia over the operating range relevant to the application.

This distinction is particularly important for physical interaction. A gearbox may exhibit high forward efficiency under rated operation while still requiring substantial breakaway torque when driven from its output. Conversely, a low-ratio transmission with modest losses can provide very low output impedance. Backdrivability should therefore be considered as an actuator-level characteristic resulting from transmission ratio, friction, inertia and motor properties rather than as a gearbox efficiency value alone.

2.3 Accuracy, stiffness and dynamic behaviour

Servo robotics established backlash, transmission error and torsional stiffness as key requirements for positioning. Human-interactive robotics retains these requirements but changes their relative weighting. Traditional industrial robots are frequently required to repeatedly position tools with sub-millimetric accuracy along prescribed trajectories. Human motion, by contrast, is not characterized by comparable absolute positioning accuracy; dexterous interaction relies much more strongly on continuous sensory feedback and adaptation to contact. Extremely high gearbox positioning accuracy can therefore become less dominant in collaborative and humanoid applications than predictable dynamic behaviour and mechanical transparency.

This does not imply that transmission accuracy becomes unimportant. Backlash, hysteresis and nonlinear compliance can still degrade trajectory control and destabilize interaction. However, a certain degree of compliance can be acceptable or even desirable if it improves impact tolerance and interaction behaviour and remains sufficiently predictable for the controller.

At the same time, the shift from pure position control towards physical interaction increases the importance of torque information. Human-like dexterity requires the robot not only to place an end-effector at the correct position, but also to regulate contact forces while grasping, manipulating or supporting objects. Reliable joint-torque information therefore becomes pivotal for force and impedance control. Transmission friction, hysteresis and load-dependent behaviour directly affect the relationship between motor torque and external joint torque, linking gearbox dynamics to the robot's ability to perceive and regulate physical interaction.

The design objective consequently shifts from maximizing stiffness and positioning precision alone towards achieving sufficiently low lost motion together with stable, repeatable and characterizable mechanical behaviour that supports both motion control and accurate torque observation.

2.4 Noise, Vibration, and Harshness (NVH)

Noise, vibration and harshness are becoming increasingly relevant as robots move from isolated

industrial cells into environments shared with humans. Gear-mesh excitation, time-varying mesh stiffness, tooth impacts, bearing excitation and manufacturing deviations can generate audible noise and structural vibration. In highly dynamic robots, impacts and rapid torque reversals introduce additional transient excitation. NVH consequently affects more than acoustic comfort. Structural vibration can influence perceived product quality, excite lightweight robotic structures and interfere with sensing systems used for control or condition monitoring. This creates an additional interaction between transmission design, manufacturing accuracy and actuator observability. For humanoid robots expected to operate in domestic, healthcare or service environments, acoustic behaviour may therefore become a more significant transmission-selection criterion than it has traditionally been in industrial robotics.

2.5 Manufacturability, cost and scalability

Manufacturability has traditionally influenced transmission design through its impact on production cost, quality and reliability. Different gearbox architectures can require substantially different manufacturing processes, component accuracies and assembly procedures, making manufacturing complexity an important complement to purely mechanical performance.

The emergence of humanoid robotics adds a further dimension to this requirement: scalability. While conventional industrial and collaborative robots have typically been manufactured in comparatively limited volumes, the business models currently envisioned for humanoid robots anticipate substantially larger production quantities. Transmission technologies must therefore not only achieve the required performance and cost at prototype level, but also allow repeatable manufacturing, automated assembly and quality control across potentially large production volumes.

Planetary transmissions benefit in this respect from extensive industrial heritage, mature gear-manufacturing processes and established supply chains. Their scalability advantage nevertheless depends on achieving the required robotic performance without introducing disproportionately demanding tolerances, selective assembly or complex adjustment operations. As humanoid robots progress towards industrial production, manufacturing architecture may therefore become a design variable comparable in importance to torque density, efficiency and dynamic performance.

3 The evolving role of planetary transmissions

For several decades, strain-wave transmissions—commercially dominated by Harmonic Drive—became a de-facto standard for compact rotary actuation in robotics. Their combination of very high single-stage reduction ratios, low weight and volume, high torsional stiffness and near-zero backlash matched particularly well the requirements of servo-controlled robotic joints,

where accurate positioning represented a primary design objective [9].

The requirements discussed in Section 2, however, progressively exposed some limitations of this actuation paradigm. High transmission ratios allow small motors to generate substantial joint torques, but simultaneously increase reflected motor inertia, while friction and transmission losses reduce mechanical transparency and complicate torque estimation from motor current. These limitations became particularly relevant with the emergence of highly dynamic legged robots, where actuators are repeatedly subjected to impacts and rapid bidirectional interactions with the environment. This application triggered renewed interest in an apparently opposite design philosophy: combining high-torque-density electric motors with comparatively low-ratio conventional transmissions.

3.1 The Emergence of quasi-direct-drive actuation

The development of highly dynamic legged robots challenged the conventional high-ratio actuation paradigm. Rather than using a relatively small, high-speed motor combined with a high-ratio precision transmission, researchers at MIT introduced actuators combining high-torque-density electric motors with low-ratio, typically single-stage planetary transmissions. This architecture, commonly referred to as quasi-direct drive (QDD), was a key enabling technology of the MIT Cheetah robots [10].

By limiting the transmission ratio, these actuators significantly reduced reflected motor inertia and gearbox friction at the joint, producing highly backdrivable actuators capable of responding rapidly to impacts and externally imposed motion. The transmission still provided useful torque multiplication, but a larger share of the required joint torque was generated directly by the motor. This trade-off became particularly attractive for legged locomotion because electric motors can deliver peak torques substantially above their continuous rating for short periods. Running, jumping and ground-impact events impose high joint torques, but these peaks are typically transient and separated by periods of substantially lower demand. Motor thermal capacity can therefore be exploited to generate short high-torque bursts without dimensioning the actuator for the peak torque as a continuous operating condition.

The Cheetah platforms demonstrated the potential of this architecture for dynamic physical interaction [11,12]. Low mechanical impedance also enabled output torque to be estimated from motor current with useful accuracy, supporting high-bandwidth proprioceptive force control without requiring a dedicated output torque sensor. The resulting combination of peak motor capability, low reflected inertia and mechanical transparency was especially well matched to dynamic locomotion.

This strategy nevertheless has an important limitation. As the average joint torque approaches the peak torque—for example when a robot must continuously support or transport substantial external

loads—the motor becomes thermally constrained by its continuous torque capability rather than its short-term peak capability. The motor-mass penalty associated with low transmission ratios then becomes substantially more important. QDD therefore represents an especially attractive solution for highly dynamic motion with intermittent torque peaks, but its benefits become less clear for joints subjected to high sustained loads.

The same principles subsequently became attractive in wearable robotics. Zhu et al. demonstrated a high-torque-density, low-ratio actuator for a powered knee device, illustrating how low mechanical impedance and backdrivability can support responsive human interaction [13]. Conventional planetary gearsets are particularly well suited to this approach because useful reduction ratios can be achieved within a single stage while retaining high efficiency, compactness and established manufacturing processes.

3.2 From legged robots to humanoid actuation

More recently, the quasi-direct-drive philosophy has expanded from quadruped and wearable robotics into humanoid actuation. This development is particularly significant because humanoid robots combine the dynamic interaction requirements of legged locomotion with considerably larger numbers of actuated degrees of freedom, strict packaging constraints and demanding requirements in terms of overall mass and energy consumption. Recent platforms demonstrate that high-torque-density motors combined with low- or moderate-ratio planetary transmissions can nevertheless provide remarkably dynamic humanoid actuators.

This trend is particularly visible in the rapidly developing Chinese humanoid ecosystem. Recent actuator research explicitly identifies QDD architectures combining high-torque-density permanent-magnet motors with planetary reducers below approximately 20:1 as increasingly relevant for humanoid applications [14]. In parallel, optimization studies increasingly address the planetary transmission itself as an integral part of actuator design rather than as an independently selected component [15].

As in quadrupeds, this architecture benefits from the ability of electric motors to provide large transient peak torques. Demonstrations involving running, jumping or rapid changes of direction can therefore exploit short-term motor overload while retaining low-ratio transmission transparency. Humanoids intended for useful industrial tasks, however, introduce a different operating regime. Carrying tools, boxes or other payloads increases not only peak joint torque but also the mean torque over the duty cycle, particularly in the lower limbs, waist and shoulders. Continuous motor torque and thermal dissipation can then become dominant design constraints, reducing the extent to which transient motor overload can substitute for mechanical torque multiplication.

The conventional ratio trade-off may therefore evolve differently as humanoids progress from demonstrations of agile locomotion towards productive work. Low- and moderate-ratio planetary QDD

architectures remain highly attractive where transparency and impact behaviour dominate, while joints subjected to sustained high loads may benefit from greater transmission ratios provided that efficiency and backdrivability can be preserved.

For planetary transmissions this reinforces the value of architectural diversity. The comparatively modest ratios of conventional planetary gearsets can become advantageous when paired with sufficiently torque-dense motors, while higher-ratio planetary concepts provide a complementary route for applications in which continuous motor torque and overall actuator mass become limiting. Mature manufacturing processes further strengthen both propositions as humanoid robots move from experimental platforms towards industrial production.

3.3 High-ratio planetary transmissions

The renewed success of conventional planetary transmissions in QDD actuators does not eliminate one of their fundamental limitations: the relatively moderate reduction ratios achievable within a single stage. Low transmission ratios favour efficiency, backdrivability and low reflected inertia, but require comparatively large, high-torque motors. An alternative approach is therefore to reconsider the broader design space of planetary gear trains and investigate whether substantially higher transmission ratios can be achieved while retaining the intrinsic advantages that make planetary transmissions attractive for robotic applications.

Compound planetary architectures offer such a possibility. Wolfrom-type transmissions are particularly interesting because ratios well above 100:1 can be obtained within a compact and coaxial compound stage [16]. Their traditional drawback is that the high reduction ratio is generated through small differences between internal gear ratios, producing substantial internal rolling and circulating power. Gear-mesh losses can consequently be amplified relative to the transmitted output power, resulting in poor overall efficiency in conventional implementations [17].

Recent work has demonstrated that this limitation is not intrinsic to all implementations of the architecture. Optimization of topology, macrogeometry and internal power flow has enabled Wolfrom-based transmissions with ratios above 200:1 while substantially improving efficiency and backdrivability [18,19]. These results indicate that high transmission ratio and mechanical transparency are not necessarily mutually exclusive, although reflected motor inertia remains an important limitation as the ratio increases [19].

QDD and high-ratio planetary architectures consequently represent different solutions to the same actuator-level trade-off. The former transfers a larger share of torque generation to the motor in order to minimize mechanical impedance; the latter uses greater mechanical torque multiplication to reduce required motor torque and potentially motor size. Between these extremes lies a broad planetary design space encompassing conventional single- and multi-stage systems and different compound architectures. Neither

end of this spectrum should therefore be regarded as universally optimal.

4 Towards application-optimized planetary transmissions

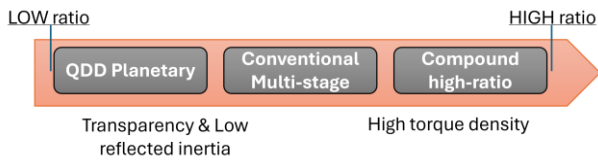


Fig. 2. Planetary actuator design space: increasing ratio shifts torque generation from the motor towards the transmission, with corresponding changes in transparency and reflected inertia.

4.1 There is no universal optimum

The developments discussed above broaden planetary transmission design from a choice between individual gearbox technologies into an actuator-level design space. The relative advantages of each architecture depend on how its characteristics interact with the motor, control system, joint duty cycle and complete robot. This suggests that future robotic transmissions should increasingly be optimized for specific actuator requirements rather than selected primarily from standardized gearbox families.

4.2 Transmission ratio as a system-level optimization variable

Transmission ratio represents one of the most influential variables in robotic actuator design because it directly couples the characteristics of the gearbox and electric motor. For a given joint torque, increasing the reduction ratio decreases the torque required from the motor, enabling the use of a smaller and lighter machine. At the same time, the motor must operate at proportionally higher speed, while its inertia reflected at the joint increases approximately with the square of the transmission ratio. Changes in ratio therefore redistribute actuator performance between electrical and mechanical components rather than simply increasing available output torque.

The contrasting approaches discussed in Section 3 illustrate this trade-off. QDD actuators deliberately employ low transmission ratios and accept a larger, high-torque motor in order to minimize reflected inertia, friction and transmission losses. High-ratio planetary architectures pursue the opposite direction, using greater mechanical torque multiplication to reduce motor torque and potentially motor mass, while accepting greater challenges in terms of reflected inertia, efficiency and mechanical transparency.

Motor speed introduces an additional system-level consideration. Higher transmission ratios increase motor and typically gear-mesh speeds for a given joint velocity. The associated electromagnetic excitation, bearing frequencies, aerodynamic effects and gear-mesh excitation generally make acoustic behaviour more challenging as rotational speeds increase. Transmission ratio therefore influences NVH not only through the gearbox itself, but also by determining the operating-

speed range of the motor and upstream transmission elements.

Consequently, the optimum transmission ratio cannot be established from gearbox performance alone. It depends on the required joint torque-speed envelope and duty cycle, but also on motor peak and continuous torque capability, rotor inertia, gearbox mass and efficiency, thermal limitations, NVH and the desired level of mechanical transparency. For mobile robots, these interactions extend further to system level, since actuator mass and efficiency influence battery requirements, structural loads and the torque demanded from other joints.

Safety provides an additional reason to consider these variables together. Humanoid robots are intended to operate without physical separation from people, making safe human-robot interaction a central prerequisite for deployment; current robotics standards similarly place inherently safe design, force limitation and risk reduction at the core of robots operating in close proximity to humans. High actuator mass and inertia increase the kinetic energy that must be managed during impacts, while poor backdrivability limits the ability of external forces to propagate through the drivetrain. Transmission architectures capable of achieving high torque density through substantial mechanical reduction without sacrificing efficiency and backdrivability can therefore contribute directly to safety by reducing actuator and robot mass while preserving mechanical transparency.

Transmission ratio should therefore be treated as a system-level optimization variable, selected together with motor and gearbox architecture for the particular joint and operating profile. The increasing diversity of planetary transmission concepts broadens this optimization space, allowing actuator designers to select where, between motor and transmission, torque generation and the associated mass, losses, inertia, noise and safety implications are most effectively accommodated.

5 Towards observable transmissions

The increasing integration of mechanical transmissions within robotic actuator systems introduces a further dimension to their design: observability. Beyond transmitting power and motion, the transmission contains information about the forces exchanged between the actuator and its environment, as well as about its own internal operating state. Torque, structural deformation, vibration and temperature can provide information ranging from external interaction forces to transmission health and degradation. The ability to access this information is becoming increasingly relevant for force control, safe physical interaction, condition monitoring and predictive maintenance.

For collaborative and humanoid robots, this evolution is closely connected to dexterity. Human dexterity does not rely on position accuracy alone: manipulation involves continuously detecting contact, estimating interaction forces and adjusting grip or joint impedance in response. A humanoid capable of handling objects,

using tools or physically cooperating with a person therefore requires reliable information about the torques transmitted through its joints. Joint-level torque observability becomes not merely a safety feature, but an enabling capability for human-like manipulation and adaptive physical behaviour.

Observability should therefore not be considered merely as the addition of sensors to an existing gearbox. The transmission architecture itself determines how loads propagate through the system, where measurable responses occur, and how reliably these responses can be related to quantities of interest. This creates an opportunity to consider mechanical design and sensing together, extending the role of the transmission from a power- and motion-transmitting component towards an increasingly integrated mechatronic system.

5.1 From controlled to observable transmissions

Position sensing provides information about joint motion but does not directly quantify physical interaction. In highly backdrivable QDD actuators, joint torque can be estimated from motor current when motor torque is well characterized and transmission friction and other disturbances remain sufficiently small and predictable. This approach is attractive because it avoids a dedicated joint torque sensor and was an important enabling characteristic of proprioceptive force control in QDD systems.

For dexterous manipulation, however, the required quantity is ultimately the torque exchanged at the robot joint or end-effector. As transmission ratio and mechanical complexity increase, the relationship between motor torque and output torque becomes progressively dependent on transmission behaviour. Friction, hysteresis, transmission error and internal load distribution can introduce load-, speed- and direction-dependent errors. Accurate force estimation can therefore become limited not by the motor-current measurement itself but by the extent to which the mechanical state of the transmission is known.

A dedicated output torque sensor provides more direct information but adds space, mechanical interfaces and potentially compliance to an already tightly packaged joint. An alternative is to use existing transmission elements as sensing structures. Hashimoto et al. measured flexspline strain in a strain-wave transmission, demonstrating that deformation of a torque-carrying transmission component could be used to infer output torque [20]. Taghirad and Bélanger subsequently showed that gear-mesh ripple and assembly misalignment had to be considered and compensated to obtain useful torque feedback [21].

The same principle can be applied to planetary transmissions. Schwenker demonstrated a compact planetary exoskeleton gearbox in which structural deformation was used for torque sensing while transmitting more than 30 Nm within a 42 mm diameter [22]. These examples illustrate an important consequence of integrated sensing: measurement performance becomes coupled to transmission

architecture, load paths, installation conditions and mechanical tolerances. The gearbox is no longer merely the object through which torque is transmitted; selected elements of its structure become part of the measurement system.

Reliable transmission-level torque information may therefore provide a direct bridge between mechanical actuator design and higher-level robotic dexterity. By enabling the controller to distinguish commanded motion from externally imposed loads and contact forces, an observable transmission can support compliant grasping, force-limited manipulation, object handling and other behaviours in which accurate interaction is more important than absolute positioning alone.

5.2 From sensorized gearboxes to observable transmission systems

Commercial developments indicate that integrated sensing is moving beyond research prototypes. Harmonic Drive has introduced strain-wave gearboxes with integrated torque sensing, while Schaeffler's RT1-T applies thin-film strain gauges directly to the flexspline. Experimental characterization has demonstrated the feasibility of using the resulting signal for closed-loop joint torque control while also highlighting transmission-specific challenges such as sensing ripple, friction and load-dependent measurement errors [23].

Observability need not, however, be restricted to transmitted torque. Gearbox operating condition itself is increasingly becoming an observable quantity. Nabtesco has pursued condition-monitoring and virtual-sensor approaches for precision reducers, while embedded wireless acceleration and temperature sensors have been experimentally demonstrated within planetary gearboxes for vibration characterization and fault diagnosis [24]. These developments extend transmission sensing from observing the interaction between robot and environment towards observing the internal state and health of the transmission itself.

Taken together, these developments suggest a broader evolution from sensorized gearboxes towards observable transmission systems. Rather than treating sensing as an additional component introduced after the mechanical transmission has been designed, transmission architecture and sensing can increasingly be considered together. Structural deformation, internal load paths, temperature and vibration contain information about both transmitted loads and gearbox condition. Designing the transmission so that these quantities can be reliably observed may therefore become an additional objective alongside efficiency, torque density, accuracy, backdrivability and manufacturability.

For humanoid robotics, the resulting value proposition extends from reliability to behaviour. Condition observability can support availability and predictive maintenance, while torque observability provides information required for compliant interaction and dexterous manipulation. The transmission consequently becomes not only part of the robot's powertrain, but increasingly part of its sensory interface with the physical world.

6 Outlook and conclusions

The requirements imposed on robotic transmissions have evolved considerably, while remaining cumulative. The fundamental requirement for efficient and reliable power conversion has been complemented first by the accuracy and dynamic behaviour required by servo-controlled systems, and subsequently by the bidirectional mechanical transparency demanded by collaborative and humanoid robots. Increasingly, transmissions are also expected to contribute to the observability of both the forces exchanged with the environment and their own internal operating condition.

These evolving requirements are reopening the design space for planetary transmissions in robotics. The emergence of QDD actuators has demonstrated that conventional low-ratio planetary gearsets, combined with increasingly torque-dense electric motors, can provide the efficiency, backdrivability and dynamic behaviour required for highly agile robotic systems. At the opposite end of the design space, recent developments in compound planetary architectures indicate that substantially higher transmission ratios can be achieved while mitigating some of the efficiency and backdrivability limitations traditionally associated with such systems. These approaches should not be regarded as competing universal solutions, but as different points within a broader actuator design space.

The rapid development of humanoid robotics makes this diversity particularly relevant. Different joints impose substantially different combinations of torque, speed, mass, inertia, interaction and packaging requirements, making a universal transmission solution increasingly unlikely. Transmission architecture and ratio should therefore be selected together with motor characteristics and the operating requirements of each joint. At the same time, manufacturability and scalability will become increasingly important as robotic production volumes grow, potentially favouring architectures that can leverage mature industrial gear-manufacturing processes.

Finally, the emergence of integrated torque sensing and gearbox condition monitoring suggests that the transmission itself is evolving into an observable mechatronic system. Future planetary transmissions may consequently be optimized not only for how efficiently and accurately they transmit power and motion, but also for how reliably their internal and external load states can be observed. The transition from power conversion, to controlled motion, to bidirectional interaction and ultimately to observability may therefore define the next stage in the evolution of planetary transmissions for robotics.

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Computer simulation of the stress-deformable standing of the bone-plate system

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Abstract. This study investigates the influence of localized porosity in an osteosynthesis plate and age-dependent bone tissue properties on the biomechanics of the bone–implant system. Finite element analysis was performed for young and elderly bone using both solid and locally porous plates. Bone tissue stresses, interfragmentary strain (IFS), strain energy density (SED), and mechanical stimulus were evaluated. The use of a porous insert reduced IFS from 11.25–12.25% to 2.78–2.85% and SED from 0.114–0.115 to 0.003–0.004 mJ/mm³, substantially altering the local mechanical environment in the fracture region. Specimen fabrication and SEM analysis confirmed the manufacturability of the developed porous structure. The obtained results demonstrate the potential of localized porosity for controlling the mechanical characteristics of the fixation system while accounting for age-dependent properties of bone tissue.

1 Introduction

Forearm bone fractures require sufficient mechanical stability of the injured segment while maintaining favorable conditions for subsequent bone tissue healing. One of the most widely used methods of internal fixation is plate osteosynthesis, in which the plate and screws form a mechanical system that carries a portion of the external load and limits the relative displacement of the bone fragments. The effectiveness of fixation is determined not only by the strength of the implant but also by the stiffness relationship between the plate and bone tissue, the load-transfer mechanism between them, and the magnitude of deformation in the fracture region [1, 2].

As early as 1892, Wolff demonstrated that bone tissue is highly sensitive to mechanical loading and proposed that mechanical forces play a key role in regulating the fracture-healing process. Throughout healing, the fracture callus remains highly responsive to the local mechanical environment.

The mechanical environment within the fracture region is one of the key factors determining the course of fracture healing. Perren demonstrated that fixation stability should be evaluated in terms of interfragmentary strain resulting from the relative displacement of bone fragments [3]. Subsequently, Claes and Heigele, based on finite element modeling and experimental data, demonstrated a relationship between local stresses and strains within the fracture region and the formation of different tissue types [4]. Therefore, the fixation system should provide not only sufficient

mechanical strength but also controlled mechanical interaction between the implant and bone tissue.

Conventional titanium plates are characterized by substantially higher stiffness than bone tissue. As a result, a considerable portion of the mechanical load may be carried by the fixation construct, thereby altering the stress distribution in the adjacent bone. Early experimental studies by Pilliar et al. demonstrated that the stiffness of metallic plates and the characteristics of load transfer at the “bone–implant” interface significantly influence bone remodeling [3]. Subsequent studies have also shown that modifying the mechanical properties of fixation plates can regulate the distribution of stresses between the plate and the bone [4, 5].

One promising approach to controlling the stiffness of fixation constructs is the incorporation of porous or functionally graded regions. Modifying the geometry and porosity of selected regions of an implant enables a local reduction in its effective stiffness without requiring a change in the material of the entire structure. In particular, finite element studies of functionally graded plates have demonstrated the potential to modify stress distribution within the fracture region compared with conventional titanium alloy and stainless-steel plates [5]. The development of additive manufacturing technologies has further expanded the possibilities for fabricating implants with complex internal architectures and controlled porosity. Recent studies on porous fixation plates have confirmed that their stiffness and the pattern of mechanical load transfer to bone tissue can be tailored through optimization of the internal structure [6, 7].

However, the mechanical behavior of the bone–plate system is determined not only by the implant design but

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also by the mechanical properties of the bone tissue itself. The elastic modulus of bone is not constant and depends on its density, microstructure, anatomical location, and tissue condition. Consequently, a reduction in bone stiffness can alter the relative load distribution among the bone fragments, fixation plate, and screws. Therefore, a fixation construct that exhibits favorable mechanical behavior for one set of bone properties may exhibit a different stress-strain state when the elastic modulus of the bone is reduced.

In this context, a comparative analysis of solid and locally porous fixation plates under different mechanical properties of bone tissue is of particular interest. The finite element method enables such an analysis to be performed under identical geometric and loading conditions while independently varying the material properties of the bone and the design parameters of the plate. This approach makes it possible to evaluate the influence of each factor on the stress distribution, displacements, and interfragmentary motion within the fixation system.

The aim of this study is to numerically investigate the effect of reduced bone elastic modulus on the mechanical behavior of the radius-fixation plate system and to comparatively evaluate a solid plate and plates incorporating localized porous regions. Finite element analysis is performed for two sets of bone mechanical properties while maintaining identical model geometry, contact conditions, and external loading. The primary comparison criteria include stresses in the bone tissue and fixation plate, displacement of the bone fragments, and parameters characterizing interfragmentary motion.

2. Materials and Methods

2.1 Mathematical Models of Bone Tissue Regeneration

Bone tissue regeneration is governed by a combination of biological and mechanical factors. Following fracture fixation, the mechanical environment within the injured region, including the magnitude of interfragmentary motion, local strains, and stresses, has a significant influence on callus formation, tissue differentiation, and the subsequent restoration of the mechanical integrity of the bone. In the present study, four interrelated theoretical approaches are used to evaluate the mechanical conditions of fracture healing: the interfragmentary strain theory, the strain energy density theory, a model describing the relationship between bone mechanical properties and bone density, and a bone remodeling model.

2.1.1 Interfragmentary Strain (IFS)

The concept of interfragmentary strain was proposed by Perren and is considered one of the principal criteria for assessing the mechanical stability of a fracture. According to this theory, successful tissue formation within the fracture region is determined by the magnitude of relative deformation between the bone fragments. The classical equation for interfragmentary

strain (IFS), proposed by Stephan M. Perren, is expressed as follows: [8-9]

$$IFS = \frac{\Delta L}{L_0} \cdot 100\% \quad (1)$$

where

IFS (Interfragmentary Strain) — interfragmentary strain, expressed as a percentage (%) or as a dimensionless quantity;

ΔL — relative displacement (change in the width) of the fracture gap under loading, *mm*;

L_0 — initial fracture gap width, *mm*.

At low levels of interfragmentary strain, favorable conditions are created for bone tissue formation and primary fracture healing. As the strain increases, fibrous or cartilaginous tissue may initially form, whereas excessive strain can inhibit osteogenesis. Thus, *IFS* is one of the principal mechanical criteria for assessing the stability of osteosynthesis.

2.1.2 Strain Energy Density (SED)

One of the most informative mechanical parameters is strain energy density (SED), which characterizes the amount of elastic energy stored per unit volume of a material under loading. For a linearly elastic material, in the three-dimensional formulation, *SED* is expressed as follows [10]:

$$SED = \frac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy} \gamma_{xy} + \tau_{yz} \gamma_{yz} + \tau_{xz} \gamma_{xz}) \quad (2)$$

where:

$\sigma_x, \sigma_y, \sigma_z$ are the normal stresses;

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ are the normal strains;

τ_x, τ_y, τ_z are the shear stresses;

$\gamma_x, \gamma_y, \gamma_z$ are the engineering shear strains (angular strains).

The SED parameter is widely used in bone remodeling models because it reflects the level of mechanical stimulation experienced by bone cells. At insufficient SED levels, stress shielding may occur, whereas excessively high values can lead to localized overloading of bone tissue.

2.1.4 Bone Tissue Remodeling Model

Bone remodeling is a continuous process involving changes in bone tissue density in response to mechanical loading. In many mathematical models, the remodeling rate is related to strain energy density [11], and the remodeling rule can be expressed in terms of the net remodeling rate as follows:

$$\begin{aligned} \frac{d\rho}{dt} &= a(\rho)\{S - (1-s)S_{ref}\}, \text{ if } S < (1-s)S_{ref}, \\ \frac{d\rho}{dt} &= 0, \text{ if } (1-s)S_{ref} \leq S \leq (1+s)S_{ref}, \\ \frac{d\rho}{dt} &= a(\rho)\{S - (1+s)S_{ref}\}, \text{ if } S > (1+s)S_{ref} \end{aligned} \quad (3)$$

where:

ρ — bone tissue density (g/cm^3 or kg/m^3);

$d\rho/dt$ — rate of change in bone tissue density over time;

t — time;

$a(\rho)$ — remodeling rate coefficient, dependent on the current bone density;

S — local mechanical stimulus, most commonly represented by the strain energy density (SED);

S_{ref} — reference value of the mechanical stimulus;

s — relative width of the lazy zone, a dimensionless parameter.

where ρ is the bone tissue density, expressed in g/cm^3 (or kg/m^3);

$\frac{d\rho}{dt}$ — the rate of change in bone tissue density over time;

t — time;

$a(\rho)$ — the remodeling rate coefficient, which depends on the current bone density;

S — the local mechanical stimulus (most commonly, strain energy density, SED);

S_{ref} — the reference value of the mechanical stimulus;

S — the relative width of the “lazy zone,” a dimensionless parameter.

If the SED exceeds the physiological level, new bone tissue formation occurs. Under insufficient mechanical stimulation, the remodeling rate becomes negative, resulting in bone resorption and a decrease in bone density.

Mechanical Criteria for Bone Tissue Regeneration

Thus, changes in mechanical load distribution resulting from the choice of implant material, geometry, or porous architecture directly influence bone remodeling processes. Therefore, the combined analysis of IFS and SED parameters together with bone remodeling models makes it possible not only to evaluate the stress–strain state of the bone–implant system but also to predict whether the mechanical environment is favorable for subsequent fracture healing.

The following ranges are then defined:

$S < 2.60 J/kg$ — reduced mechanical stimulation;

$2.60 \leq S \leq 5.40 J/kg$ — moderate mechanical stimulation;

$S > 5.40 J/kg$ — increased mechanical stimulation.

When the strain energy density (SED) exceeds the physiological threshold, new bone formation is stimulated. Conversely, when the mechanical stimulus is insufficient, the remodelling rate becomes negative, leading to bone resorption and a reduction in bone density [12].

According to Perren’s classical concept, the following approximate IFS ranges are commonly considered:

$< 2\%$ — very low interfragmentary strain, corresponding to conditions of absolute stability and primary bone healing;

$2\text{--}10\%$ — moderate interfragmentary strain, corresponding to conditions of relative stability and secondary bone healing;

$> 10\%$ — high interfragmentary strain, representing less favorable conditions for bone tissue formation.

Object of the Study

Bone plates are among the most commonly used implants for internal fracture fixation [13] and offer several advantages, including adequate mechanical

stability and resistance to tensile, compressive, shear, torsional, and bending loads when used in combination with fixation screws [14]. Currently, many bone plate designs incorporate periarticular configurations and locking holes, allowing the surgeon to select an appropriate screw type, such as locking or non-locking screws, depending on the location and characteristics of the fracture [15].

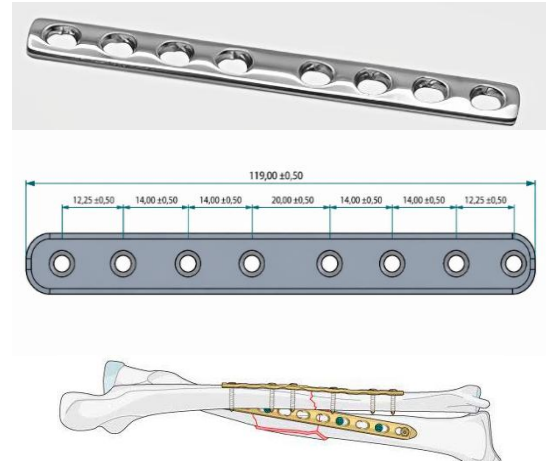


Fig. 1. DCP Plate for the Ulna and Radius.

A standard 3.5 mm Dynamic Compression Plate (DCP) was used as the control design. The DCP is a straight plate with oval holes designed for 3.5 mm cortical screws.

Design of a Locally Porous Plate

The design of the locally porous plate was preceded by numerical modeling using Digimat software to investigate the influence of the geometric parameters of the lattice structure on its mechanical properties. The simulation results showed that the effective elastic modulus is primarily determined by the unit-cell geometry and the relative porosity of the structure, whereas variations in the linear dimensions of the unit cell at constant porosity have only a minor effect on its mechanical properties. Based on the obtained relationships, a Diamond-type lattice structure with a unit-cell size of 1.5 mm, a strut thickness of 785 μm , and a relative porosity of 60% was selected for the plate design. This configuration provides reduced structural stiffness while maintaining sufficient mechanical strength and manufacturability for additive manufacturing. Fig. 2 presents the porous structure and the plate configurations incorporating the localized porous region.

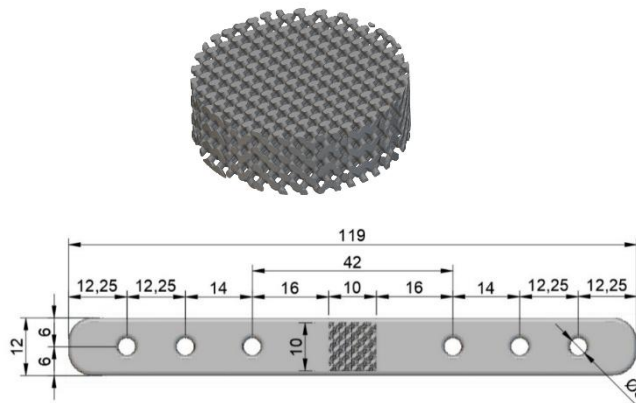


Fig. 2. Plate Design with a Localized Porous Structure.

The localized porous structure generated in nTopology was integrated with the solid plate model to form a unified geometric model without altering its external contour or contact surfaces. For subsequent finite element analysis, the geometric model of the plate was prepared for import into the MSC Patran software package.

Since nTopology generates the porous structure geometry in STL format, which represents a polygonal surface mesh, the resulting model was imported into Autodesk Fusion, where the polygonal model was converted into solid geometry. After reconstruction of the solid model, the plate was exported in Parasolid (*.x_t) format, ensuring accurate transfer of the geometry to the finite element analysis software while preserving the key design features.

FE analysis

To simplify the calculations, the cross-section of the radial diaphysis was approximated as a hollow cylinder with an outer diameter of 15.9 mm and an inner diameter of 6.37 mm. These dimensions fall within the anatomical range reported for the radial diaphysis in adults.

The three-dimensional assembly model of the commercial bone plate and bone was developed using Autodesk Inventor computer-aided design (CAD) software and subsequently imported into MSC Patran, where the finite element model was prepared (Fig. 2). To ensure accurate mechanical load transfer and to investigate stress distribution within the system, 16 contact pairs were defined. A contact search tolerance of 0.001 m was specified for the interacting surfaces. This approach enables the simulation of stable internal fixation, in which the mechanical load is transferred through the bone–screw–plate system, and allows the stress and strain distributions within the structural components to be evaluated.

Four-node tetrahedral solid elements were used to generate the finite element mesh (Fig. 3), enabling an adequate representation of the complex geometries of the bone, plate, and screws [16–17]. The finite element size was set to 0.002 m, providing the required computational accuracy while maintaining a reasonable computational cost. To assess the influence of element size on the numerical results, a mesh-independence study was performed. The analysis showed that further mesh

refinement resulted in a change of less than 5% in the maximum equivalent stress, indicating sufficient convergence of the finite element model. Fig. 3 shows the finite element model of the plate–bone assembly.

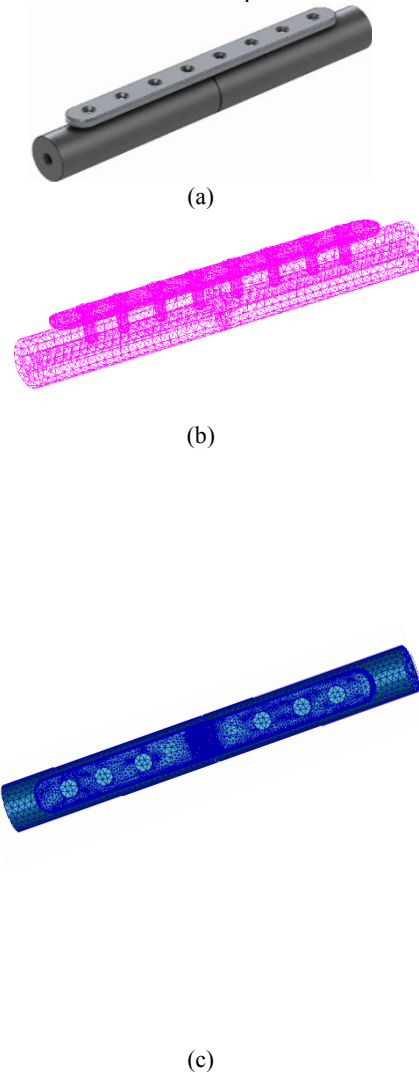


Fig. 3. Finite element meshes: (a) 3D assembly model; (b) assembly with a solid plate; (c) assembly with a plate incorporating a porous insert.

The geometric model of the bone–plate system was discretized using second-order tetrahedral solid finite elements, enabling an accurate representation of the complex three-dimensional geometry of the investigated structure. The finite element size was selected considering the geometric features of the model, the presence of the localized porous structure, and the required accuracy of the stress–strain analysis. The resulting meshes consisted of 406,893 finite elements for the plate with the porous insert and 43,624 elements for the solid plate. This discretization enabled a detailed representation of the bone, osteosynthesis plate, screws, and localized porous structure while ensuring adequate convergence of the numerical solution.

Materials

In the finite element analysis, cortical bone tissue was modeled as a homogeneous, linear elastic, isotropic material with mechanical properties corresponding to healthy cortical bone in a young patient.

This assumption is widely used in finite element studies of orthopedic implants and enables a consistent comparison of different design configurations under identical modeling conditions. In reality, the mechanical properties of cortical bone are anisotropic and depend on the orientation of its microstructure; therefore, the use of an isotropic material model may affect the absolute values of local stresses and strains. However, since identical bone tissue properties were used for all investigated configurations, this simplification does not substantially affect the comparative assessment of the mechanical behavior of the plates or the conclusions drawn [18]. This approach enables evaluation of the stress-strain state of the bone-plate system under physiological bone properties. The mechanical properties of all materials used in the finite element model are presented in Table 1.

Table 1. Material Properties.

Material	Elastic Modulus, GPa	Poisson's Ratio
Young cortical bone	24	0,30
Aged cortical bone	13	0,3
Ti-6Al-4V	110	0,34

The fracture-line geometry was defined according to the AO/OTA classification. A type 22-A3 radial shaft fracture, representing a simple transverse fracture, was selected for this study. To simulate the fracture, the bone was divided into two fragments with a 1 mm interfragmentary gap, representing conditions suitable for secondary bone healing and enabling the analysis of stress and strain distributions as well as relative displacements within the fracture region. This fracture type was selected because of its widespread use in biomechanical studies and its suitability for consistent comparison of the stress-strain state among different fixation plate designs.

Mechanical Loading Conditions

To evaluate the influence of the mechanical properties of bone tissue on the biomechanical behavior of the bone-implant system, a single standardized loading condition—axial compression—was considered in the present study. An axial compressive force of 100 N was applied to the distal region of the radius model, while the proximal region was fixed. The selected load magnitude corresponds to a standardized comparative loading condition commonly used in finite element and

experimental studies of fixation constructs for radial fractures [19]. It should be noted that this loading condition is not intended to reproduce the full range of physiological loads acting on the forearm but is used for a comparative evaluation of the mechanical response of the investigated fixation constructs under identical boundary conditions. The load was applied to the distal end of the bone, ensuring force transmission through the internal fixation system. The loading and boundary conditions are shown in Fig. 4.

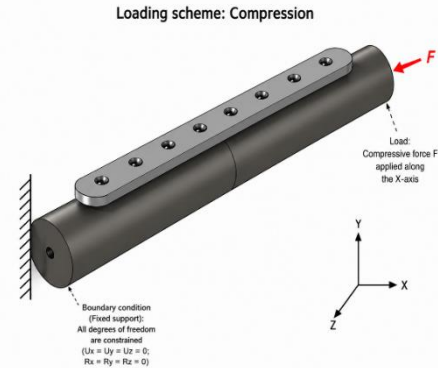
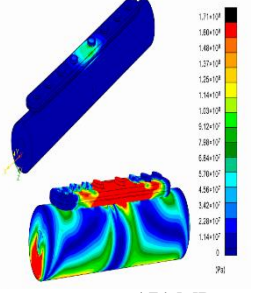
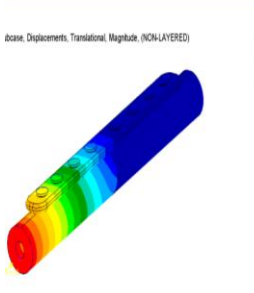
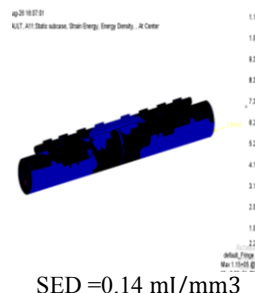
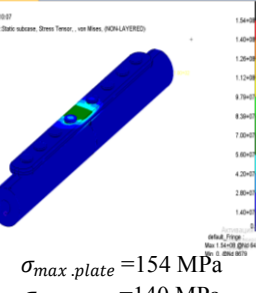
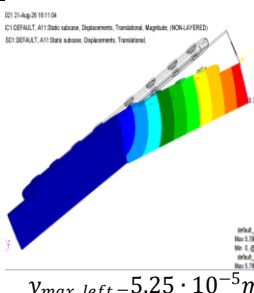
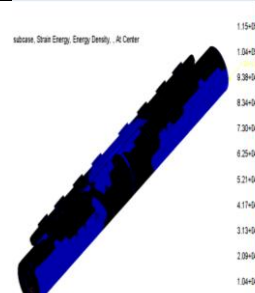
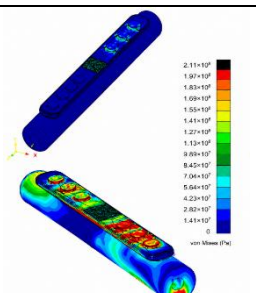
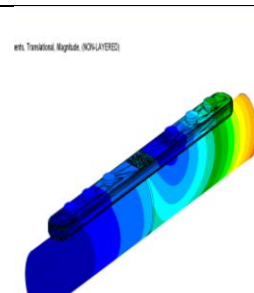
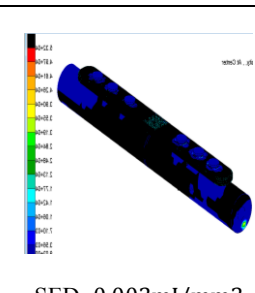
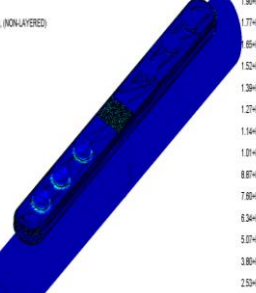
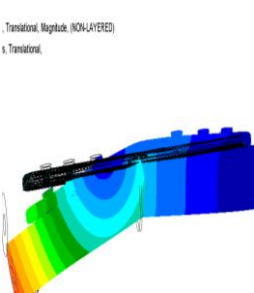
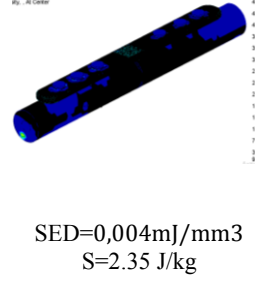


Fig. 4. Mechanical loading scheme of the bone-plate system under axial compression.

Results

Based on the finite element simulation results, a comparative assessment of the stress-strain state of the bone-implant system under axial compression was performed. Particular attention was given to evaluating the effect of a reduced elastic modulus of cortical bone on the mechanical interaction between the bone and the fixation plate. The comparison was conducted using identical model geometries, boundary conditions, and external load magnitudes, allowing changes in the mechanical properties of the bone tissue to be considered the primary variable. To evaluate the mechanical response, the von Mises equivalent stresses in the bone tissue and fixation plate, bone fragment displacements, as well as interfragmentary displacement and strain within the fracture region were analyzed. The obtained results were used to compare the behavior of the investigated fixation constructs for young and aged cortical bone. Table 2 presents the stress and displacement distributions in the bone-plate system for young and aged bone models with a solid plate and a plate incorporating a localized porous insert.

Table 2. Stress and displacement distributions in the bone-plate system for young and aged bone models.

Plate Design	Bone Age	Stress	Displacement	Strain Energy
Solid Plate	Young Bone	 $\sigma_{max,plate} = 171 \text{ MPa}$ $\sigma_{max,bone} = 114 \text{ MPa}$	 $y_{max, left} = 4.88 \cdot 10^{-4} \text{ m}$ $y_{max, rite} = 1.47 \cdot 10^{-4} \text{ m}$ IFS=11.25%	 SED=0.14 mJ/mm³ S=61.26 J/kg
	Aged Bone	 $\sigma_{max,plate} = 154 \text{ MPa}$ $\sigma_{max,bone} = 140 \text{ MPa}$	 $y_{max, left} = 5.25 \cdot 10^{-5} \text{ m}$ $y_{max, rite} = 1.58 \cdot 10^{-4} \text{ m}$ IFS=12.25%	 SED=0.115mJ/mm³ S=60.53 J/kg
Plate with a Porous Insert	Young Bone	 $\sigma_{max,plate} = 180 \text{ MPa}$ $\sigma_{max,bone} = 120 \text{ MPa}$	 $y_{max, left} = 4.43 \cdot 10^{-5} \text{ m}$ $y_{max, rite} = 5.58 \cdot 10^{-5} \text{ m}$ IFS=2.78%	 SED=0.003mJ/mm³ S=1.53 J/kg
	Aged Bone	 $\sigma_{max,plate} = 190 \text{ MPa}$ $\sigma_{max,bone} = 127 \text{ MPa}$	 $y_{max, left} = 4.96 \cdot 10^{-5} \text{ m}$ $y_{max, rite} = 4.96 \cdot 10^{-5} \text{ m}$ IFS=2.85%	 SED=0.004mJ/mm³ S=2.35 J/kg

To comprehensively assess the effects of fixation plate design and age-related changes in the mechanical properties of bone tissue on the biomechanical state of the bone–implant system, a comparative analysis of solid and locally porous plate designs was performed for young and aged bone models. The primary evaluation criteria included maximum stress in the bone tissue, interfragmentary strain (IFS), strain energy density (SED), and mechanical stimulus (S). The combined analysis of these parameters enables assessment of changes in load transfer between the implant and bone, fracture-site stability, and the resulting mechanical environment. The calculated values for the investigated configurations are presented in Table 3.

Table 3. Biomechanical Parameters of the Investigated Configurations.

Config uration n	Bone	Bone Stress	IFS	SED	Mech anical Stimu lus, S
Solid	Young	114 MPa	11.25 %	0.114 mJ/mm ³	61.26 J/kg
Solid	Aged	140 MPa	12.25 %	0.115 mJ/mm ³	60.53 J/kg
Porous	Young	120 MPa	2.78%	0.003 mJ/mm ³	1.53 J/kg
Porous	Aged	127 MPa	2.85%	0.004 mJ/mm ³	2.35 J/kg

For the solid plate, the transition from young to aged bone was accompanied by an increase in bone stress from approximately 114 to 140 MPa, corresponding to an increase of about 22.8%. At the same time, IFS increased from 11.25% to 12.25%. This indicates that, for the same fixation design, age-related changes in the mechanical properties of bone result in greater local deformation and higher stress levels. This behavior is consistent with the well-established age-related reduction in the mechanical strength and damage resistance of cortical bone [20].

A particularly noteworthy result was observed for the porous plate. In the young bone model, IFS decreased from 11.25% with the solid plate to 2.78% with the porous design, corresponding to an approximately fourfold reduction in relative interfragmentary motion. At the same time, bone stress increased slightly from 114 to 120 MPa. These findings indicate that modifying the plate design substantially redistributes the mechanical load within the bone–implant system, affecting not only fixation stiffness but also load transfer to the bone tissue. In the literature, modification of fixation stiffness is considered one of the approaches for controlling interfragmentary mechanics and reducing stress shielding [21].

Discussion

For the solid plate, the transition from young to aged bone was accompanied by an increase in bone stress from approximately 114 to 140 MPa, corresponding to an increase of about 22.8%. At the same time, IFS increased from 11.25% to 12.25%. This indicates that, for the same fixation design, age-related changes in the

mechanical properties of bone result in greater local deformation and higher stress levels. This behavior is consistent with the well-established age-related decline in the mechanical strength and damage resistance of cortical bone. A particularly noteworthy result was observed for the porous plate. In the young bone model, IFS decreased from 11.25% for the solid plate to 2.78% for the porous plate, corresponding to an approximately fourfold reduction in relative interfragmentary motion. At the same time, bone stress increased slightly from 114 to 120 MPa. These findings indicate that modifying the plate design substantially redistributes the mechanical load within the bone–implant system, affecting not only fixation stiffness but also load transfer to the bone tissue. Previous studies have similarly identified the modification of fixation stiffness as an effective approach for controlling interfragmentary mechanics and mitigating stress shielding.

For the solid plate, elevated strain energy density (SED) values were observed in the investigated bone region, resulting in calculated mechanical stimulus values of 61.26 J/kg for young bone and 60.53 J/kg for aged bone. These values substantially exceed the adopted reference range of 2.60–5.40 J/kg, indicating the formation of a localized region of intense mechanical stimulation. Such an increase reflects a high concentration of strain energy within the analyzed bone region and may indicate a potentially unfavorable mechanical environment. The introduction of a localized porous insert substantially altered the strain energy distribution, reducing the mechanical stimulus to 1.53 J/kg for young bone and 2.35 J/kg for aged bone. Thus, localized porosity reduced the pronounced concentration of mechanical stimulus observed with the solid plate and resulted in a more moderate strain-energy state within the bone tissue.

The results demonstrate that the effect of the porous plate design depends on the mechanical properties of the bone itself. For the solid plate, the difference between young and aged bone is primarily reflected in the increased stress and IFS values observed in the aged bone model. This behavior is consistent with age-related changes in cortical bone, including increased intrinsic porosity, accumulation of microdamage, and a reduction in several macroscopic mechanical strength properties. With the porous plate, the SED in aged bone was 0.004 mJ/mm³ compared with 0.003 mJ/mm³ in young bone, corresponding to an increase of approximately 33%. Thus, for the same fixation geometry, aged bone is subjected to a somewhat higher energy-based mechanical stimulus. This finding clearly demonstrates that the plate design alone is insufficient for a comprehensive assessment of osteosynthesis, as age-dependent mechanical properties of bone tissue substantially influence the resulting biomechanical environment.

The introduction of a localized porous insert substantially alters load transfer and interfragmentary mechanics compared with the solid plate, with the magnitude of this effect depending on the mechanical properties of the bone tissue. Aged bone exhibits higher local stresses and a slightly higher energy-based

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mechanical stimulus, indicating greater sensitivity to changes in the stiffness of the fixation system.

Additive Manufacturing and Morphological Analysis of Locally Porous Plates

To experimentally confirm the manufacturability of the proposed design, prototype bone plates incorporating a localized porous insert were fabricated from Ti-6Al-4V titanium alloy using additive manufacturing (3D printing). The use of additive manufacturing enabled the fabrication of the complex geometry of the localized cellular structure directly within the solid body of the plate. The fabricated specimens were used to assess the conformity of the resulting porous structure with the parameters defined during the computer-aided design stage.

Particular attention was paid to the quality of the transition zone between the solid and porous regions of the plate, as well as to the preservation of the cellular geometry after fabrication. For a more detailed assessment of the morphology of the porous region, scanning electron microscopy (SEM) was performed to determine the actual dimensions of the lattice features and evaluate the characteristics of the as-fabricated surface. The fabricated specimens and the SEM image of the localized porous structure are presented in Fig. 5.

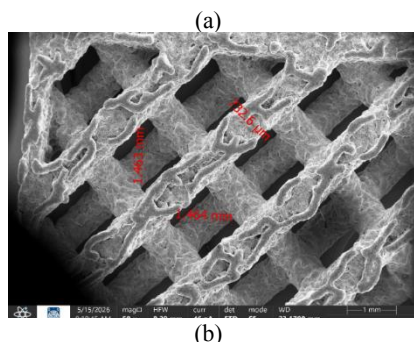


Fig. 4. (a) Titanium plate; (b) SEM image of the locally porous region.

The fabricated specimen demonstrates the technological feasibility of incorporating a localized porous insert into a solid bone plate. SEM analysis

confirmed the formation of a regular cellular structure within the porous region and the preservation of its key geometric features after fabrication. The measured cell and strut dimensions were generally consistent with the designed architecture, confirming the practical manufacturability of the proposed design and its suitability for subsequent experimental evaluation of its mechanical properties.

3. Conclusion

The present study developed and investigated a bone plate incorporating a localized porous insert designed to modify the biomechanical behavior of the bone–implant system. Numerical modeling demonstrated that the mechanical response of the system is governed by both the structural design of the plate and the age-dependent mechanical properties of bone tissue. For the solid plate, the transition from the young to the aged bone model was accompanied by an increase in the maximum bone stress from 114 to 140 MPa and in interfragmentary strain (IFS) from 11.25% to 12.25%. The introduction of a localized porous insert substantially altered the interfragmentary mechanics: IFS decreased to 2.78% for young bone and 2.85% for aged bone, while the corresponding maximum stresses were 120 and 127 MPa, respectively. Changes in the energy-related characteristics of the bone tissue were also observed. Strain energy density (SED) values were 0.003 and 0.004 mJ/mm³ for young and aged bone, respectively, indicating a substantial redistribution of mechanical energy within the investigated region compared with the solid plate configuration. These results demonstrate that localized porosity can be used as a design parameter to control fixation stiffness, load transfer, and the interfragmentary mechanical environment. However, its effect should be evaluated with consideration of the mechanical properties of bone tissue associated with different age groups. The fabrication of an experimental specimen and subsequent SEM analysis confirmed the manufacturability of the proposed design and the feasibility of reproducing the specified cellular architecture. Thus, the proposed approach provides a basis for further optimization of locally porous osteosynthesis plates considering age-related changes in bone tissue, followed by experimental and biomechanical validation.

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Pressure-free molding of powder-mesh materials

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Abstract. The process of forming powder-mesh materials (PMM) composed of a woven mesh of corrosion-resistant steel and electrolytic nickel powder without applying pressure using binders is considered. The feasibility of using starch paste and aqueous solutions of polyvinyl alcohol (PVA) and carboxymethyl cellulose (CMC) as binders is tested. The choice of PVA for use in PMM formation is substantiated. The potential applications of the resulting materials are demonstrated.

1. Introduction

The technology and equipment of modern enterprises cannot operate without liquid and gaseous atmosphere purification with filter materials (used as participants in the technological cycle in the manufacture of goods or consumer goods), which ensures the achievement of manufactured products required quality with their help or the machines and mechanisms trouble-free operation [1, 2]. The use of such materials allows for improving the quality of manufactured products, the reliability and durability of pneumatic and hydraulic systems for various purposes, and ensuring environmental protection. Filter materials (FM) are used for the purification of water, alkaline, acid and salt solutions, fuels and lubricants, milk, resins, varnish bases, molten salts and polymers, the filtration of liquefied and compressed gases during their production and practical application, and exhaust gases in various industries technological cycles [2–5].

An important place among the FMs is occupied by materials based on metal powders and meshes (PFM and SFM, respectively) due to a number of advantages. Such materials are characterized by high permeability, strength, the ability to regulate and control properties over wide ranges, high thermal and electrical conductivity, resistance to corrosion and thermal shock, the ability to be subjected to mechanical processing and welding [6, 7]. However, the existing disadvantages of PFM, caused by the design or technological features of manufacture, limit their use in practice. Thus, powder filter materials, with a comparable specific productivity in comparison with mesh filters produced by one of the world's leading FM manufacturers - the PALL company (USA), are significantly inferior to the latter in hydraulic resistance terms due to the corrugated working surface of the latter [8, 9]. In addition, the higher hydraulic

resistance of products made from PFM is also due to their greater thickness (usually 3–5 mm [10, 11]), necessary to achieve the required mechanical strength.

This paper examines the use of composite powder-mesh materials (PMM) to improve the performance of powder filter products. It's hypothesized that the use of finely dispersed powder in the structure of such materials will ensure high filtration fineness, while the mesh-containing composition will reduce the filter layer thickness to less than one millimeter. This will result in a reduction in hydraulic resistance and increased strength and ductility of the entire composite material. Furthermore, the use of mesh will enable the corrugation of the sintered porous semi-finished product during filter elements manufacture, which will also lead to a significant reduction in their hydraulic resistance. Unlike known powder-mesh materials [12, 13], this paper examines PMMs formed without the pressure application by applying finely dispersed nickel powder to a single layer of woven metal mesh made of corrosion-resistant steel using a binder.

The aim of the work is to study the process of forming powder-mesh materials without applying pressure, in particular the choice of a binder for applying nickel powder to a woven steel mesh, and to substantiate the proposed method feasibility for the filtering products manufacture.

2 Experimental results and discussion

Traditional PFM manufacturing technology typically involves the steps of preparing the charge, shaping its structure, and sintering [7, 10, 14]. The technology under consideration involves forming a porous structure by applying a binder metal powder to a woven metal mesh. Electrolytic nickel powder of the PNE-1 grade in the as-

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delivered condition [15] and woven meshes made of corrosion-resistant steel wire of the 12X18H10 grade were selected for the research. The wire diameters and cell sizes for the meshes are presented in the table below.

Selecting a binder for applying fine powder to a woven mesh is a critical step in studying the powder metallurgy process, as the nature of the binder influences the wettability of the woven mesh and powder surfaces, the adhesion strength between them, the possibility of delamination after molding, and subsequent sintering. A 40% solution of glycerol in ethyl alcohol, starch paste, paraffin, and aqueous solutions of PVA and CMC have been used in powder metallurgy practice [6, 7, 10, 16–18]. For the present study, aqueous solutions of PVA, CMC, and starch paste were used (paraffin was abandoned due to its premature cooling, and the alcohol-glycerol mixture did not initially provide high-quality, uniform adhesion).

The manufacturing process of experimental samples (ES) of PMM included five main operations: preparation of the charge (when using PVA, distilled water, a solution of polyvinyl alcohol TU BY 100295708.001-2010 and nickel powder were mixed until homogeneous in a weight ratio of 1:1:8; when using CMC, they were dissolved in distilled water with a temperature of 36 – 45 °C while maintaining a ratio of 1.0 – 12.5 (g), kept for 10 min, 20 g of nickel powder were added to the solution and the mixture was stirred until homogeneous; when using starch paste, starch and distilled water (6 – 15 °C) were mixed, maintaining a ratio of 1:4 (g), then hot water (55 – 75 °C) was added, maintaining a ratio of 1:4:1 (g), repeated mixing was performed, holding for 10 min, adding 20 g of nickel powder to the paste and stirring the mixture until smooth); applying one layer of the prepared charge to a woven mesh blank in the disk form with a diameter of 25 mm; drying the blank at room temperature; sintering the EO at a temperature of 1000 °C in an argon environment for 80 min; monitoring the properties of the EO (determining the pore sizes according to [19] and the permeability coefficient according to [20]).

Visual inspection of the obtained samples revealed that aqueous solutions of PVA and CMC can be used; starch paste causes swelling of the applied powder layer during sintering and is unsuitable for use as a binder in the manufacture of PSM. As an example, Figure 1 shows photographs of experimental samples of powder-mesh materials made from PNE-1 nickel powder and a woven mesh of 63-diameter wire with a 100-μm aperture using PVA, CMC, and starch-based binders.

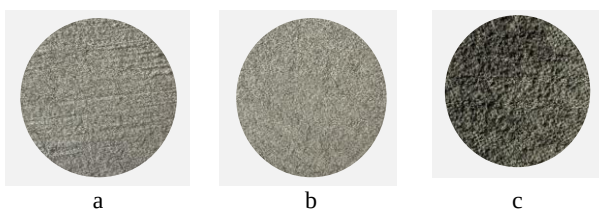


Fig. 1. EO PMM from PNE-1 nickel powder and a woven mesh of 63-diameter wire with a cell size of 100 μm per luminous aperture, manufactured using binders based on PVA (a), CMC (b) and starch (c).

Since aqueous solutions of PVA and CMC were selected as binders based on visual inspection, the Table 1 below presents the pore sizes and permeability coefficients of experimental samples prepared using them. In all cases, the metal powder same grade was used, and the woven meshes were selected arbitrarily, but with the condition that the cell size per luminous surface didn't exceed the average particle size of the powder by a factor of five.

Table 1. Properties of EO PSM.

Binder	Mesh size, mm		Permeability coefficient, $k, m^2 \times 10^{-13}$	Pore size $d, \mu m$	
	Cell size per luminous aperture	Wire diameter		Maximum d_{max}	Average d_{middle}
PVA	0,1	0,065	13,68	30,82	27,63
			12,18		
			12,02		
	0,2	0,12	10,27	20,59	16,72
			9,13		
			8,52		
	0,315	0,2	8,32	24,97	24,29
			7,37		
			7,16		
CMC	0,1	0,065	28,03	41,97	35,86
			24,82		
			24,07		
	0,2	0,12	19,05	37,93	32,66
			16,71		
			16,04		
	0,315	0,2	17,28	29,88	25,42
			13,45		
			13,21		

To select the final binder, a comparison of the homogeneity index [12] was conducted, which is the ratio of the maximum pore size to the average pore size. Calculations showed that for samples manufactured using PVA, the values of this parameter were in the range of 1.027 – 1.24, while those using CMC were in the range of 1.1–1.61. Therefore, an aqueous PVA solution was chosen as the binder for the PVM production.

It should be noted that the value of the above indicator for a two-layer material obtained by spraying carbonyl nickel powder in a layer of 200 – 400 μm onto a nickel twill mesh, sintering at a temperature of 750 – 850 °C, cold rolling at a compression ratio of up to 0.6 and re-sintering at a temperature of 1100 – 1150 °C with a holding time of 3 – 4 hours in a dry hydrogen environment is significantly higher, increasing from 1.32 to 2.0 at a compression ratio of 0.60 [13]. The same characteristics for two-layer powder-mesh materials (mesh-powder) and three-layer powder-mesh materials (mesh-powder-mesh), produced using a technology that includes preliminary sintering, rolling, and final sintering, are, according to [12], 1.16 – 1.40 and 1.40 – 1.60, respectively.

To evaluate the throughput of the developed material, a comparison was made between the permeability coefficient of powder-mesh material produced without pressure application and that of the two-layer combined material VFG-1, consisting of a twill mesh onto which a

20 – 50 μm thick powder layer was applied using low-temperature plasma [13]. It was established that the former has a throughput capacity that is more than 10 times higher (the calculation of the permeability coefficient of the latter was carried out according to the data in [13], which shows the throughput capacity of VFG-1 for kerosene; a comparison was made with the material from the table with a maximum pore size of 20.59 μm , having the same nominal purification fineness as the comparison object).

To evaluate the material's ability to be corrugated using 3D-printing, a polymer tooling was manufactured in the form of two disks with a triangular outer profile, between which a PSM blank was deformed (Figure 2).

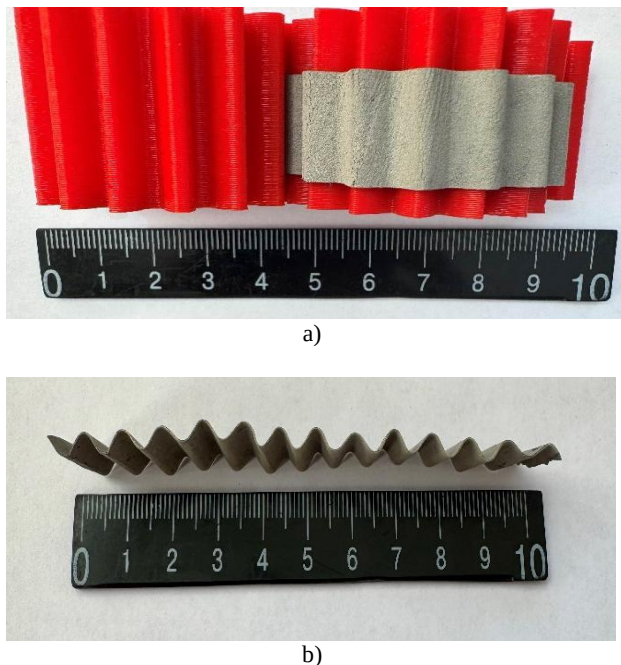


Fig. 2. Corrugation process (a) and corrugated blank (b) made of PMM. (Photographer: I. Charniak)

Visual deformed blank inspection revealed no cracks on its surface, with a bend radius of 1.5 mm at the apex.

Practical testing of PMM filter elements was conducted at a yeast production facility as part of an air purification filter. The use of a powder-mesh material, compared to traditional powder, reduced hydraulic resistance to the required level and reduced the filter's weight by more than four times, significantly reducing the labor required for installation in the air supply system. Figure 3 shows photographs of the air purification filter (3a) and the filter installed in the air supply system (3b).



Fig. 3. Air cleaning filter (a) and mounted in the air supply system (b) external view. (Photographer: I. Charniak)

Each filter element consists of four layers: a pre-filter layer made of non-woven needle-punched polymer material, a support layer made of woven metal mesh, a fine filter layer made of PMM, and a second support layer made of woven metal mesh. The remaining filter element components (flanges and the frame on which all of the above layers are located) are made of polymer material using 3-D printing.

3 Conclusion

The process of obtaining powder-mesh materials formed by the composition of a fabric net from corrosion-resistant steel and electrolytic nickel powder, without applying pressure using binders, is considered. The feasibility of using starch paste and aqueous solutions of polyvinyl alcohol (PVA) and carboxymethyl cellulose (CMC) as binders is tested. The choice of PVA for use in forming the PMMs is substantiated. It's shown that the developed PMMs possess a more uniform pore distribution and high throughput compared to known powder-mesh materials produced by rolling; they are capable of being corrugated without cracking. An example of developed material practical application in a fine air filter is given.

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Investigation of the influence of 3D printing parameters on the mechanical properties of carbon-filled composites

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Abstract. This study investigates the influence of 3D printing parameters on the mechanical properties of CR-PLA composites manufactured using fused filament fabrication (FFF). An experiment was conducted to evaluate the effects of extrusion temperature, layer thickness, and printing speed. The Young's modulus, maximum tensile stress, and elongation at break were determined through mechanical testing. The results revealed significant variations in the mechanical properties depending on the selected printing conditions. Statistical analysis of the experimental data was performed, and a regression equation describing the influence of the process parameters on the strength of the printed parts was derived. The obtained results demonstrate the significant effect of FFF printing parameters on the mechanical properties of CR-PLA composites and provide a basis for optimizing their manufacturing conditions. A comparative tribological analysis of CR-PLA and PLA specimens was also conducted. The results showed that the incorporation of carbon filler significantly affects the friction and wear behavior of the material. CR-PLA specimens exhibited a more stable variation in the coefficient of friction during testing and a lower coefficient of friction compared with unfilled PLA. Thus, the incorporation of carbon filler into the PLA matrix contributes to improved tribological performance by reducing the coefficient of friction and enhancing the stability of the friction process.

1 Introduction

Polymeric materials are widely used in high-technology industries, including the defense, aerospace, medical, mechanical engineering, and electrical engineering sectors [1,2]. The development of polymer processing technologies has led to the widespread adoption of conventional manufacturing methods such as extrusion, injection molding, and compression molding. At the same time, the introduction of additive manufacturing technologies has created new opportunities for producing polymer components with complex geometries and a high degree of design flexibility.

3D printing represents a promising area of additive manufacturing that enables the fabrication of components directly from digital models. The main advantages of this technology include the possibility of customized manufacturing, high geometric design freedom, reduced material waste, and the production of components with complex spatial structures [3,4]. Currently, a wide range of materials is used in additive manufacturing, including thermoplastics, graphite and ceramic materials, polymer- and fiber-reinforced composites, as well as metals. The expanding range of

available materials facilitates the further adoption of 3D printing technologies across various industrial sectors [5].

Depending on the physical state of the feedstock material and the principle of part formation, 3D printing technologies can be broadly classified into processes based on liquid, powder, and solid materials [6]. The most widely used additive manufacturing technologies include laminated object manufacturing (LOM), laser sintering, and filament-based material extrusion. Among these, Fused Filament Fabrication (FFF), which is based on the layer-by-layer deposition of molten polymer material, has gained widespread application due to the relative simplicity of the equipment, material availability, and its capability to manufacture both prototypes and functional components [7,8].

The most commonly used materials for FFF printing are acrylonitrile butadiene styrene (ABS) and polylactic acid (PLA) [9]. In recent years, carbon-fiber-reinforced polymer composites have also been actively developed, as the incorporation of carbon fibers can improve the strength and stiffness of printed components compared with unfilled polymers. However, the mechanical properties of components manufactured by FFF are

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determined not only by the properties of the feedstock material but also by the process parameters used during printing.

The main FFF printing parameters that influence the structure and performance characteristics of printed components include extrusion temperature, nozzle diameter, layer thickness, printing speed, infill density, material feed rate, and raster orientation [10-12]. Variations in these parameters can result in different degrees of interlayer bonding, changes in the porosity and density of the printed parts, and the development of anisotropic structures. Consequently, even when the same polymer material is used, the mechanical properties of the final components may vary significantly depending on the selected printing conditions.

Thus, identifying a rational combination of FFF process parameters is an important task, particularly for the fabrication of components from carbon-filled polymer composites, in which the material structure and the characteristics of interlayer bonding have a significant influence on mechanical strength. Establishing the relationship between printing parameters and the mechanical properties of printed components makes it possible to determine optimal manufacturing conditions and improve the efficiency of FFF technology for the production of functional parts and manufacturing tooling.

2. Materials and Methods

Carbon-filled polylactic acid, CR-PLA Carbon, manufactured by Creality (China), was used to fabricate the specimens by fused filament fabrication (FFF). This material is a PLA-based composite thermoplastic reinforced with 15% carbon fiber, which contributes to improved stiffness, strength, and dimensional stability compared with unfilled PLA. Scanning electron microscopy (SEM) was performed to characterize the morphology of the carbon filler. Figure 1 shows an SEM image of a carbon fiber within the structure of the investigated material at a magnification of 1200 $\times$. Based on the measurement, the apparent transverse dimension of the fiber was approximately 8.676 μm .

Commercial CR-PLA Carbon filament with a diameter of 1.75 mm and a spool weight of 1.0 kg was used for specimen fabrication. The material has a density of approximately 1.25 g/cm³, a tensile strength of up to 49 MPa, and a flexural strength of up to 78 MPa. The incorporation of carbon filler improves the mechanical properties of the polymer matrix and enhances the performance characteristics of components manufactured using FFF technology.

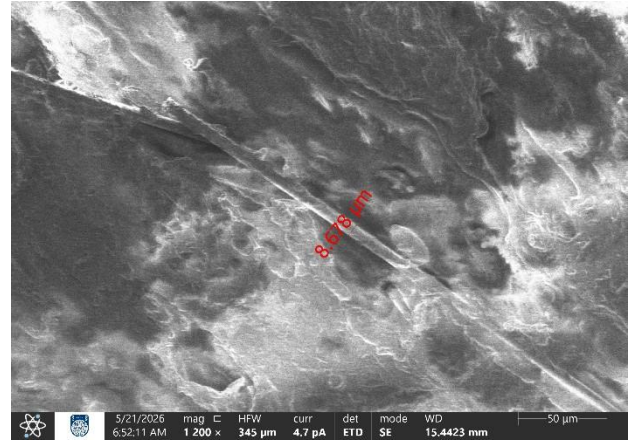


Fig. 1. SEM image of carbon fiber in the CR-PLA composite.

For the study, test specimens (Fig. 2) were fabricated with the dimensions specified in Table 1.

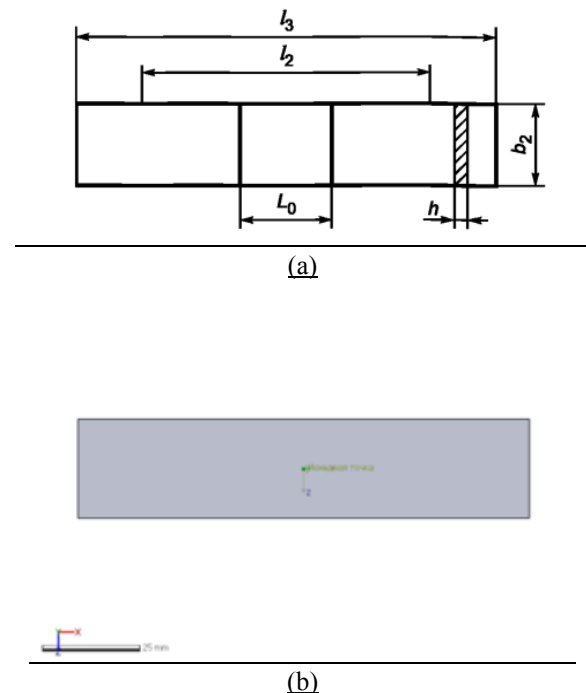


Fig. 2. (a) Test specimen; (b) schematic of the plate showing the main geometric parameters.

Table 1. Dimensions of Type 1 test specimens.

Dimension	Type 1 (mm)
l_3 Overall length	120
b_2 Width of the head	20
h Thickness	2

Carbon-fiber-reinforced polymer specimens were fabricated using a Bambu Lab X1E 3D printer with the AMS system. Fig. 3 shows the specimen fabrication process using the Bambu Lab X1E 3D printer.

The 3D printing process is carried out using specialized software that enables model preparation, generation of printing instructions, and adjustment of the

required process parameters. Bambu Studio software was used to prepare the models for printing and to configure the printing parameters.



Fig. 3. 3D printing process of the specimen using the Bambu Lab X1E with AMS.

The developed model was saved in KOMPAS software in the stereolithography (STL) file format. The STL file was then imported into Bambu Studio, where the printing orientation of the specimen and its position on the build plate were defined, as shown in Fig. 4.

Fig. 4 shows the position and orientation of the specimen on the printer build plate as configured in Bambu Studio.

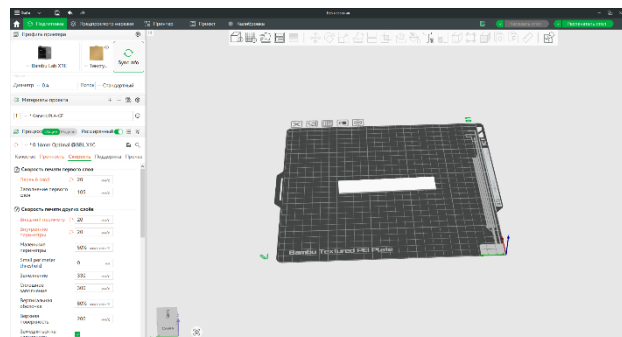


Fig. 4. Position of the specimen on the build plate of the Bambu Lab X1E 3D printer with AMS.

The aim of this study is to determine the influence of 3D printing process parameters on the mechanical properties of the fabricated components. The results obtained for the weakest and strongest printing conditions were subsequently used in an experimental study involving tooling manufactured using FFF technology.

A three-factor full factorial experimental design was developed for the study. The experimental investigation was carried out using different printed specimens by systematically varying the process parameters.

The extrusion temperature, printing speed, and layer thickness were selected as the variable factors. The experimental design matrix is presented in Tables 2 and 3.

Table 2. Experimental parameters for layer-by-layer deposition of the carbon-filled composite.

Parameters	Max	Min
Extrusion temperature °C	270	250
Layer thickness (mm)	0,28	0,16
Printing speed (mm/s)	40	20

Table 3. Parameters of the three-factor experiment for carbon-filled composite printing.

No	Extrusion temperature (°C)	Layer thickness (mm)	Printing speed (mm/s)
1	250	0,16	20
2	250	0,16	40
3	270	0,16	20
4	270	0,16	40
5	250	0,28	20
6	250	0,28	40
7	270	0,28	20
8	270	0,28	40

Eight series of three carbon-filled composite specimens were fabricated using different combinations of printing parameters, as presented in Table 8. Photographs of the fabricated specimens are shown in Fig.5.



Fig. 5. Eight series of three 3D-printed carbon-filled composite specimens.

Tensile testing

To determine the influence of additive manufacturing process parameters on the strength characteristics of carbon-filled composite specimens, an experimental design matrix was developed. Printing speed, layer thickness, extrusion temperature, and infill density were selected as the variable factors. These parameters were chosen because of their significant influence on interlayer bonding quality, material consolidation, and the formation of the internal structure of the printed components. The experiments conducted according to the developed design matrix made it possible to establish the relationship between printing conditions and specimen strength and to determine rational process parameters for manufacturing carbon-filled composite components [13,14].

The specimens intended for tensile testing were fabricated at Satbayev University in accordance with the requirements of ISO 527-2:2012, which specifies the tensile testing methods for plastics. The specimen geometry and dimensions were defined according to this standard. The shape and main dimensions of the specimens are presented in Table 1. A total of eight series were fabricated, with three specimens in each series, resulting in 24 specimens. Fig. 4 shows the fabricated specimens used for tensile testing.

Tensile tests were conducted at Satbayev University using a WDW-50E microcomputer-controlled electronic universal testing machine (Fig. 6). Prior to testing, the cross-sectional dimensions of each specimen were measured and entered into the testing machine software. The tensile tests were performed at a loading rate of 20 mm/s.



Fig. 6. WDW-50E microcomputer-controlled electronic universal testing machine.

The tensile stress–strain diagram was obtained by loading each specimen until failure. The loading rate was 2.0 ± 5.0 mm/min. The elongation at failure, Δ , was used to determine the relative elongation at break, ϵ .

The tensile strength was calculated from the maximum load, F_p .

The deformation diagram was used to determine the loads F_1 and F_2 and the corresponding elongations Δ_1 and Δ_2 at relative strains of 0.1% and 0.3%, respectively. These values were subsequently used to calculate the tensile modulus of elasticity.

When recording the deformation diagram was not possible, the modulus of elasticity was determined by cyclic loading of the specimen until stable increments were obtained within the load range from $F_1 = (0.05 - 0.1) \times F_p$ to $F_2 = 0.2 \times F_p$. At loads F_1 and F_2 , the elongation increment Δ was determined over the gauge length L_0 .

Tensile testing of the polymer materials was carried out at a temperature of 23 ± 2 °C in accordance with GOST 11262–2017.

Tensile strength along the layers of the carbon-filled composite

The stress–strain curves obtained during tensile testing are presented in Fig. 6. Photographs of the specimens before and after tensile failure are shown in Figs. 8 and 9.

The specimen dimensions were as follows: width, 20 mm; thickness, 2 mm; overall length, 120 mm; and gauge length, 50 mm. The testing speed was 2.00 mm/s. The specimens were made of carbon-filled composite.

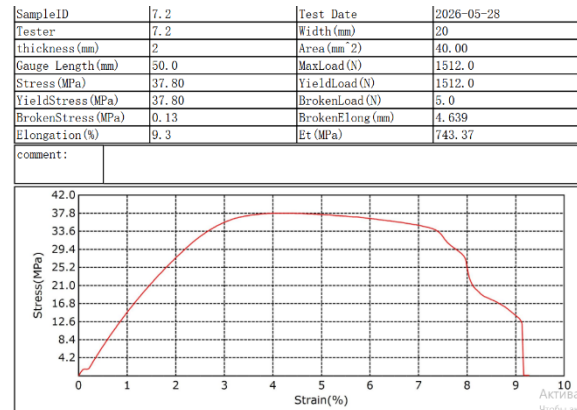
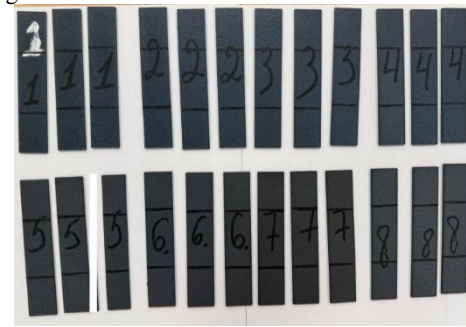


Fig. 7. Stress–strain curve of specimen No. 7.2 during tensile testing.



(a)



(b)

Fig. 8. (a) Specimens after 3D printing; (b) specimens after tensile failure along the printed layers.

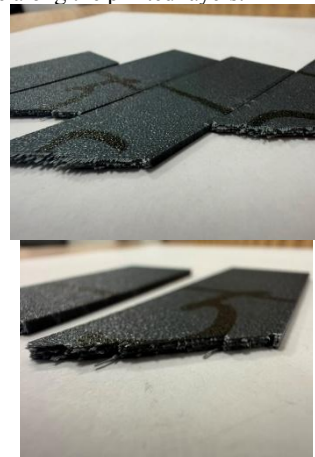


Fig. 9. Fracture surfaces of the carbon-filled composite specimens.

Table 4. Tensile strength along the layers of the carbon-filled composite.

Specimen No.	Combination of factors			Tensile strength (MPa)
	Temperature (°C)	Layer thickness (mm)	Printing speed (mm/s)	
1	250	0,16	20	34,8
2	250	0,16	40	37,5
3	270	0,16	20	40,1
4	270	0,16	40	28,0
5	250	0,28	20	23,6
6	250	0,28	40	26,2
7	270	0,28	20	37,7
8	270	0,28	40	26,5

Table 4 presents the results of uniaxial tensile tests of carbon-filled composite specimens manufactured using different printing parameters. Three replicate tests were performed for each experimental condition, allowing the reproducibility of the obtained mechanical properties and the reliability of the experimental results to be assessed.

The experimental data were subjected to mathematical and statistical analysis, and response surfaces describing the dependence of specimen strength on the process parameters, including extrusion temperature, layer thickness, and infill density, were constructed. The matrix of the three-factor experiment used for data processing is presented in Table 5.

Table 5. Experimental matrix for processing the tensile strength results of carbon-filled composite specimens.

N o	Tem perat ure (°C)	Layer thic kness (mm)	Printing speed (mm/s)	Y — Tensil e strengt h (MPa)	Y — Young's modulus (MPa)
1	250	0,16	20	34,8	1485.86
2	250	0,16	40	37,5	1581.48
3	270	0,16	20	41,62	958.71
4	270	0,16	40	39,88	1505.39
5	250	0,28	20	37,64	978.32
6	250	0,28	40	38,4	1089.38
7	270	0,28	20	37,7	735.33
8	270	0,28	40	37,37	1094.46

Statistical analysis of the experimental results was performed, and the coefficients of the linear regression model were determined. The first-order regression model including interaction effects is expressed as follows:

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3 \quad (1)$$

For the correct calculation of the regression coefficients, the process parameters were transformed into coded variables as follows:

$$x_1 = \frac{T-260}{10} \quad (2)$$

$$x_2 = \frac{h-0.22}{0.06} \quad (3)$$

$$x_3 = \frac{V-30}{10} \quad (4)$$

where:

$$T = 250 - 270 \text{ °C}$$

$$h = 0.16 - 0.28 \text{ mm}$$

$$V = 20 - 40 \text{ mm/c}$$

Tensile strength equation

After calculating the effects, the following equation was obtained:

$$Y_o = 38,11 + 1,15x_1 - 0,84x_2 + 0,42x_3 - 0,90x_1x_2 - 1,24x_1x_3 + 0,43x_2x_3 \quad (5)$$

Coefficient of determination:

$$R^2 \approx 0,98 \quad (6)$$

As a result of the statistical analysis of the experimental data using a 2^3 full factorial design, first-order regression models incorporating pairwise interactions between the factors were obtained. The coefficients of determination were $R^2=0.98$ for tensile strength and $R^2=0.97$ for Young's modulus, indicating a high goodness of fit of the developed models and their suitability for predicting the mechanical properties of the investigated material within the studied range of process parameters.

To determine the statistical significance of the investigated factors and their interactions, an analysis of variance (ANOVA) was performed on the experimental data. The effects of printing temperature, layer thickness, and printing speed, as well as their two-way and three-way interactions, were evaluated using the Fisher's F-test at a specified significance level. Based on the obtained F-statistic and p-values, the statistically significant factors affecting the tensile strength of the investigated specimens were identified.

Statistical conclusions

At a significance level of:

$$\alpha = 0,5 \quad (7)$$

The statistically significant factor is:

Printing temperature:

$$p = 0,013 < 0,05 \quad (8)$$

Interaction between printing temperature and layer thickness:

$$p = 0,011 < 0,05 \quad (9)$$

The following factors are not statistically significant:

- Layer thickness individually ($p = 0,243$)
- Printing speed ($p = 0,826$)
- Interaction $h \times V$ ($p = 0,936$)
- Three-way interaction $p = 0,627$

Interaction between printing temperature and printing speed:

$$p = 0,089 \quad (10)$$

The results of the analysis of variance showed that printing temperature had the greatest influence on tensile strength ($F = 7.89$; $p = 0.013$). In addition, the interaction between printing temperature and layer thickness was found to be statistically significant ($F = 8.37$; $p = 0.011$). The effects of printing speed and layer thickness as individual factors were not statistically significant ($p > 0.05$). These results indicate that variations in extrusion temperature play a dominant role in determining the mechanical properties of the investigated material.

To visualize and evaluate the contribution of each factor, a Pareto chart was constructed.

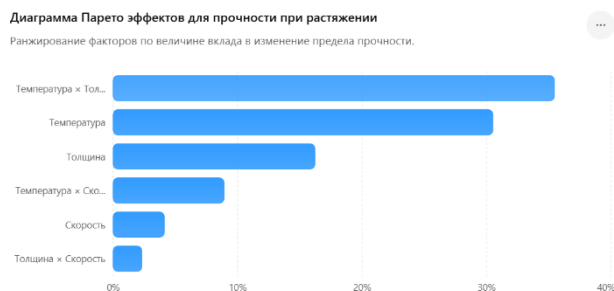


Fig. 10. Pareto chart of the standardized effects of the factors and their interactions on tensile strength.

The Pareto chart showed that the dominant factors affecting the tensile strength of the investigated material were printing temperature and its interaction with layer thickness. The combined contribution of these factors exceeded 65% of the total variation in the response. The effect of printing speed was considerably lower and did not have a significant influence on the strength characteristics of the specimens within the investigated range of process parameters.

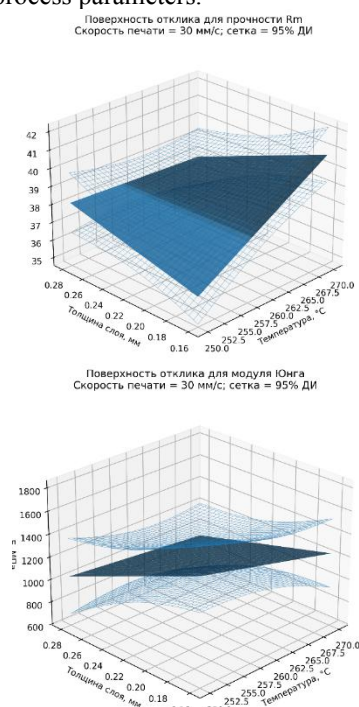


Fig. 11. Response surface for tensile strength (Rm).

The response surface illustrates the combined effect of printing temperature and layer thickness on tensile strength at a fixed printing speed of 30 mm/s. Analysis of the response surface shows that increasing the printing temperature from 250 to 270 °C results in an increase in the tensile strength of the specimens. The highest tensile strength values are observed at a temperature of approximately 270 °C and a layer thickness of 0.16 mm. Increasing the layer thickness to 0.28 mm leads to a reduction in material strength regardless of the printing temperature.

The shape of the response surface indicates an interaction between printing temperature and layer thickness, which is consistent with the results of the

analysis of variance. Increasing the printing temperature partially compensates for the negative effect of increased layer thickness on the mechanical properties of the material. The mesh surfaces bounding the main response surface represent the 95% confidence intervals for the predicted values.

Response surface for Young's modulus

The response surface for Young's modulus illustrates the effects of printing temperature and layer thickness on the stiffness of the fabricated specimens. The results show that the maximum Young's modulus is achieved at a printing temperature of 250 °C and a layer thickness of 0.16 mm.

Increasing the printing temperature to 270 °C leads to a noticeable decrease in Young's modulus, indicating a reduction in material stiffness. A similar trend is observed with increasing layer thickness. Printing temperature has the most pronounced effect on Young's modulus, as confirmed by the ANOVA results. The interaction between printing temperature and layer thickness also has a significant effect on the investigated response.

The confidence intervals indicate the range of possible variation in the predicted Young's modulus values for the specified process parameters. The widening of the confidence intervals in certain regions of the response surface is associated with greater experimental variability. Nevertheless, the intervals generally remain relatively narrow, confirming the statistical reliability of the developed model.

Analysis of the response surfaces showed that printing temperature is the dominant process parameter affecting both the strength characteristics and Young's modulus of the investigated material. Higher printing temperatures combined with a lower layer thickness are preferable for achieving maximum tensile strength, whereas maximum stiffness is obtained at a lower printing temperature and minimum layer thickness. These findings indicate the need for a compromise in selecting the process parameters depending on the required performance characteristics of the final component [15].

Tribology of polymers

To evaluate the friction and wear characteristics of the 3D-printed specimens, a cylinder-on-plate tribometer operating under reciprocating motion was used. This configuration provides high contact pressure even for small specimens and enables the simultaneous evaluation of static friction at the reversal points and dynamic friction during sliding. It also allows the material's tendency toward stick-slip behavior to be investigated through the analysis of adhesion and sliding regimes.



Fig. 12. Experimental setup for investigating the tribological characteristics of 3D-printed specimens.

For the comparative tribological study, two types of disk-shaped specimens were fabricated using additive manufacturing: unfilled PLA (white specimen) and carbon-filled CR-PLA composite (black specimen). The specimens had identical geometries and comparable printing parameters, thereby minimizing the influence of geometric factors on the test results. The materials were compared to evaluate the effect of carbon filler on the coefficient of friction and wear behavior. Fig. 13 shows the specimens used for the tribological tests.



Fig. 13. Specimens for tribological testing.

To determine the tribological properties of the 3D-printed specimens under reciprocating sliding conditions, appropriate test parameters must be selected. The tribological testing parameters used in this study are presented in Table 7.

Table 7. Parameters used in the tribological tests.

Parameter	Value
Surface roughness of the steel counterbody, Ra	0,10–0,12 μm
Load, F	150–200 N
Reciprocating motion frequency, f	4,583 Hz
Stroke length	6 mm
Relative humidity, RH	45 – 50 %
Ambient temperature, T	23 – 25 $^{\circ}\text{C}$
Test duration, t	60 min

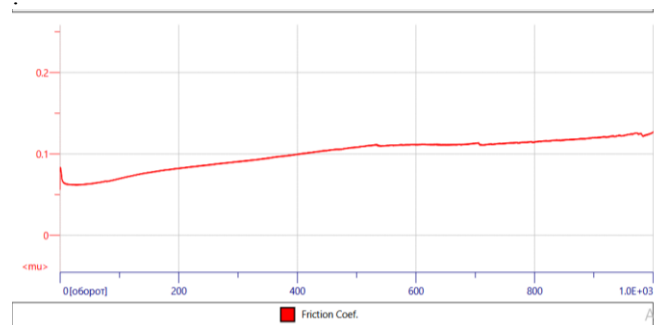
These parameters were selected based on previous studies of polymeric materials (e.g., Zsidai et al., 2002; Hanon Kovács et al., 2019; Hanon Alshammas et al., 2020; Hanon Marczis et al., 2020a). The selected surface roughness corresponds to typical values achieved by fine machining processes, such as grinding or finish cutting. A load range of 150–200 N provides a relatively high contact pressure of approximately 20–40 MPa under line-contact conditions, corresponding to industrial operating conditions. The short stroke length helps

localize the friction-induced temperature rise and its effects within the contact zone.

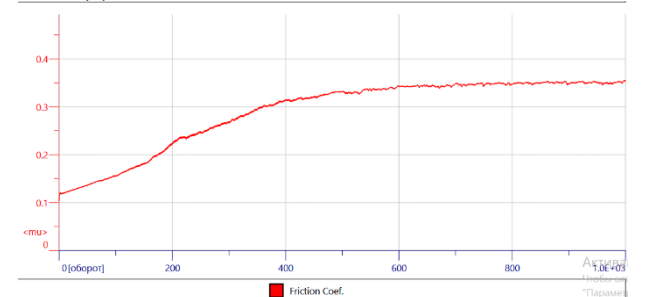
Results of tribological testing

After completion of the tribological tests, a visual examination of the tested specimens and counterbodies was performed. Distinct wear tracks and localized wear zones were observed on the surfaces of the disk specimens as a result of repeated sliding contact during testing.

The morphology and intensity of the resulting wear traces differed between the investigated materials, indicating that the presence of carbon filler affects the tribological behavior of the composite under sliding conditions. The experimental data obtained were subsequently used to determine the coefficient of friction, evaluate wear, and perform a comparative analysis of the tribological characteristics of the investigated specimens.



(a) Coefficient of friction curve for CR-PLA



(b) Coefficient of friction curve for PLA

Fig. 13. Results of tribological testing of (a) PLA and (b) CR-PLA.

During the initial stage of the test, the coefficient of friction was approximately 0.056, followed by a gradual increase and subsequent stabilization at approximately 0.11–0.13. Over the entire test period, the mean coefficient of friction was $\mu_{\text{mean}} = 0.100$, with a minimum value of $\mu_{\text{min}} = 0.056$ and a maximum value of $\mu_{\text{max}} = 0.128$; the standard deviation was 0.018. The relatively small variation in the coefficient of friction after the running-in period indicates the formation of a comparatively stable tribological contact.

The second graph shows the variation in the coefficient of friction for unfilled PLA. In contrast to the carbon-filled material, PLA exhibited a pronounced increase in the coefficient of friction over the course of the test, rising from approximately 0.11–0.12 at the beginning of the steady sliding stage to approximately 0.35 at the end of the test. The mean coefficient of friction was $\mu_{\text{mean}} = 0.291$, while the maximum value reached $\mu_{\text{max}} = 0.355$, with a standard deviation of 0.073.

Thus, PLA exhibited not only higher coefficient of friction values but also a considerably more pronounced evolution of friction behavior during the running-in process.

Comparison of the results shows that the incorporation of carbon filler led to a substantial reduction in the coefficient of friction. The mean coefficient of friction decreased from 0.291 for PLA to 0.100 for CR-PLA, corresponding to a reduction of approximately 65.6%, or nearly 2.9 times. The maximum recorded coefficient of friction also decreased from 0.355 to 0.128, representing a reduction of approximately 64%. At the same time, the standard deviation decreased from 0.073 to 0.018, indicating more stable frictional behavior of the carbon-filled material under the investigated test conditions.

The observed reduction in the coefficient of friction may be associated with changes in the interaction mechanism between the contacting surfaces resulting from the incorporation of carbon filler. The carbon phase may reduce direct adhesive interactions between the polymer matrix and the counterbody and promote the formation of a carbon-containing transfer film at the contact interface, thereby reducing resistance to sliding. In addition, the increased stiffness of the composite may reduce local surface deformation and the real contact area. In contrast, unfilled PLA may exhibit more pronounced surface deformation and a stronger adhesive contribution to friction, while the accumulation of wear debris within the contact zone may contribute to a gradual increase in the coefficient of friction during testing.

Thus, the obtained results demonstrate that, under the investigated conditions, the incorporation of carbon filler into PLA results in a substantial reduction and stabilization of the coefficient of friction.

The surface profiles of the specimens are shown in Fig. 14.

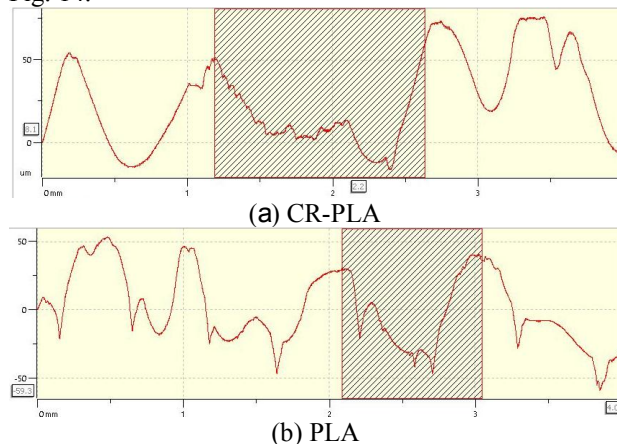


Fig. 14. Surface profiles of the specimens.

The first graph presents the surface profile of the carbon-filled CR-PLA composite specimen after the friction test. Within the selected region, a localized depression corresponding to the wear track can be observed. The profile exhibits a non-uniform morphology characterized by alternating peaks and valleys; however, the region of maximum depth is relatively localized. According to the profile analysis,

the calculated cross-sectional area of the wear track was $34,747.873 \mu\text{m}^2$. The obtained profile indicates localized material removal and redistribution within the contact zone during sliding.

The second graph presents the wear track profile of unfilled PLA. For this material, the selected region is characterized by a wider depressed area and a more pronounced surface indentation. The calculated cross-sectional area of the wear track was $66,501.022 \mu\text{m}^2$, which is substantially higher than that obtained for the CR-PLA composite. The profile also exhibits noticeable height variations at the edges of the wear track, which may be associated with plastic deformation, material removal, and local redistribution of the material during frictional contact.

Comparison of the results shows that the cross-sectional area of the wear track decreased from $66,501.022 \mu\text{m}^2$ for PLA to $34,747.873 \mu\text{m}^2$ for CR-PLA after the incorporation of carbon filler, corresponding to a reduction of approximately 47.7%. In other words, the measured wear-track area of unfilled PLA was approximately 1.9 times greater than that of CR-PLA. Combined with the previously observed decrease in the mean coefficient of friction from $\mu = 0.291$ for PLA to $\mu = 0.100$ for CR-PLA, these results indicate more favorable tribological behavior of the carbon-filled composite under the investigated conditions.

This difference may be attributed to the increased stiffness of the PLA matrix resulting from the incorporation of carbon filler, reduced deformation of the surface layer, and changes in the interaction mechanism with the counterbody. The formation of a carbon-containing transfer film may also contribute to reducing the adhesive component of friction. As a result, the CR-PLA surface undergoes less intensive frictional interaction compared with unfilled PLA.

Thus, the experimental results demonstrated that the incorporation of carbon filler into CR-PLA significantly alters its tribological characteristics by reducing and stabilizing the coefficient of friction and decreasing the cross-sectional area of the wear track. These findings confirm the potential of carbon-filled CR-PLA composites for the 3D printing of functional components intended to operate under frictional contact conditions.

Conclusion

This study investigated the influence of the main FFF process parameters, including extrusion temperature, layer thickness, and printing speed, on the mechanical properties of the carbon-filled CR-PLA Carbon composite.

The mechanical test results showed that variations in the printing parameters have a noticeable effect on the strength and elastic properties of the investigated material. Extrusion temperature and its interaction with layer thickness were found to have the most significant influence on the mechanical properties. Moreover, the printing conditions providing the highest material strength did not coincide with those resulting in the highest stiffness, indicating the need for a comprehensive selection of process parameters

depending on the required performance characteristics of the final component.

The tribological investigations provided an assessment of the friction and wear behavior of the carbon-filled composite and demonstrated the influence of carbon filler on its tribological performance. These results complement the mechanical testing data and enable a comprehensive evaluation of the material's performance characteristics.

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Research on an unconventional technological method for manufacturing gears by laser cutting – area of application, advantages, disadvantages and specific parameters

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Abstract. The report presents the results of an experimental study of the obtained parameters of gears made by laser cutting technology. For the purpose of the report, experimental samples of gears from the materials - C45 and 41Cr4 were cut on a laser cutting machine. The quality, roughness and hardness of the working profiles of the gears were analyzed. A comparative analysis was made between the results of gears from both materials and expected results with traditional gear cutting methods. Based on the results obtained, conclusions were drawn about the advantages, disadvantages of this method of manufacturing gears and possible areas of application.

1 Introduction

Gears are widely used in the drive mechanisms of all kinds of machines. Therefore, the quality of the surface of the tooth profile and the technology of its production are of great importance for the operation of gears. The main indicators that influence the operability are the material, geometric characteristics and hardness of the working profile. The basic technology for obtaining the involute tooth profile is through mechanical processing with special tools on gear-cutting machine tools. This process is slow and relatively energy-intensive and expensive.

The trends in manufacturing processes in general are to look for options to reduce the energy used for production and reduce the machine time for production. In this direction, a study has been conducted by the author, in which a more economically advantageous option is sought for the production of gears with straight teeth not in the traditional way with mechanical processing, but by laser cutting. The main limitation of this method is that only spur gears can be produced. It is not possible to produce helical gears or bevel gears, due to the technological features of laser cutting.

The purpose of the study is to manufacture and analyze test specimens of spur gears from two types of material with identical geometric characteristics. The analysis in this report presents the state of the obtained hardness and roughness of the working profile of the gears. Based on the results obtained, the positive and negative characteristics of this unconventional method for cutting spur gears and the area of application will be determined.

2 Materials, methods and experimental setup

2.1 Materials

For the manufacture of the experimental samples of cylindrical gears with straight teeth, it is assumed to use some of the most widely used materials in general mechanical engineering – steel C45 and 41Cr4(37Cr4). The chemical composition of the selected steels is shown in Table 1.

Table 1. Chemical composition of gear materials in the study.

steel	C	Si	Mn	Cr	S
C45	0,42-0,5	max.0,4	0,5-0,8	max.0,4	max.0,035
41Cr4	0,38-0,45	max.0,4	0,6-0,9	0,9-1,2	max.0,035

Steel C45 (also designated as 1.0503 or equivalent to AISI/SAE 1045) is a medium-carbon steel. C45 is a medium strength steel with good machinability and excellent tensile properties.

Steel 41Cr4 (also known as material number 1.7035 or similar to AISI 5140) is a low-alloy engineering steel defined under the EN 10083 standard.

The main difference between the two materials is the percentage of Cr, this should have a different impact on the cutting surface - roughness and hardness. The selected materials do not have special characteristics and features regarding the chemical composition and mechanical performance, [2,4,5]

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2.2 Geometric characteristics of gears

To generate the entire contour of the gear wheel, we use the 3D engineering program Autodesk Inventor. It is used to build the real involute profile, which will be cut on a laser machine.

Using the Autodesk Inventor program, 3D spur gears can be generated from the Design/Spur Gear menu. From the resulting gears, a simplified version (Export to DWG) of the involute curve of the teeth can be generated. The curve thus obtained does not visually correspond to the real curve, and it is used only for visualization and does not burden the software.

For more precise generation of the real involute curve, the command "Export tooth shape" is used. Thus, the involute contour of one interdental space is sequentially saved, and exported to a .dwg file. Then it is necessary to process it further to obtain the contours of all teeth.

It is assumed that the designed gear wheel has a relatively large module and tooth height, so that the laser beam during the cutting process does not affect the quality of the working surface. The geometric characteristics of the gear wheel are listed in Table 2.

m - Module

z - Number of teeth of the gear

x - Profile shift coefficients

d - Pitch circle diameter of the gear

d_f - Root circle diameter of the gear

d_{a1} - Addendum circle diameter of the gear

b - Facewidth

Table 2. Geometric characteristics of a spur gear

m , mm	z	x	α_o	β_o	d , mm	d_a , mm	d_f , mm	b , mm
8	16	-0,15	20	0	128	141	105,6	20

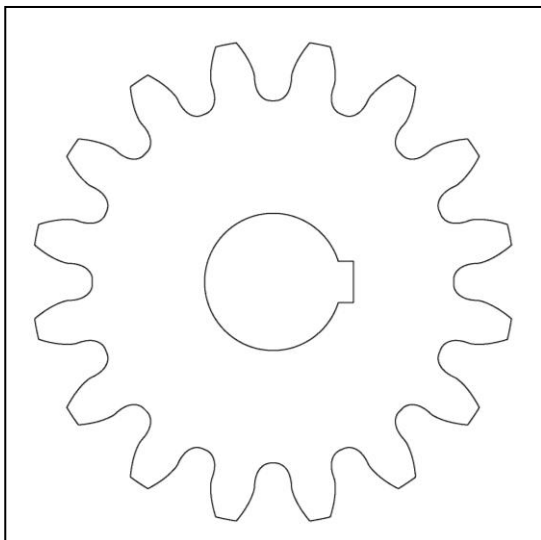


Fig. 1. Contour of a studied spur gear

This contour is used in .dxf file format, and is cut out on a laser cutting machine.[1,3]

2.3 Laser cutting machines and technology

The gears used in this study were manufactured using a high-precision laser cutting process, which is particularly suitable for creating complex profiles like involute teeth. The machine used was a BODOR A6 (2025) laser cutter, equipped with a fiber laser source optimized for cutting carbon steel plates with high accuracy.

The gear blanks were cut from C45 and 41Cr4 (6 pcs. of both materials). The material thickness was 20 mm - pre-cleaned of rust. All laser-cutting operations were performed using the following parameters, given in Table 3.

Table 3. Laser-cut parameters

	Laser cut parameters	Value
1	Thickness	20
2	Speed	0,3 m/min
3	Power	6000 W
4	Frequency	5000 Hz
5	Duty cycle	100%
6	Cut height	0,3mm
7	Nozzle	D-1,4mm
8	Cut focus	16mm
9	Gas pressure	0,6 Bar
10	Gas type	Oxygen

These parameters were chosen to achieve the best possible quality for gear contours, ensuring sharp edges and minimal thermal distortion. The cutting focus and cut height were carefully calibrated to maintain consistent kerf width and avoid excessive burr formation.

The tolerances expected from this laser cutting setup are within ± 0.1 mm for overall dimensions. While laser cutting offers a high degree of precision, small deviations may still occur due to material warping, laser beam divergence, or uneven heat distribution. These deviations are particularly relevant in gear production, where even minor geometric inaccuracies can affect meshing behavior and efficiency.[6,9]

After the laser cutting process, the gear blanks were manually de-burred using a belt sander to remove any residual slag or burrs that may have formed along the tooth edges.

3 Results and discussion

3.1 Hardness measurement

To measure the hardness of the surface obtained in the laser cutting process, the LS252C Universal Precise Leeb Hardness Tester is used (Fig. 2).



Fig. 2. LS252C Universal Precise Leeb Hardness Tester

The LS252C is an all-in-one hardness tester with a compact size for hardness measurements on various metal materials. Featuring a C-shaped impact device with low impact force, it is suitable for testing the hardness of surface hardening layers and parts with small masses or thin thicknesses. The instrument uses advanced dual coil technology for good repeatability and high measurement accuracy. The probe has a built-in orientation sensor, which

automatically compensates for measurement errors in different impact directions. The system has five hardness scales, which can be interchanged between Richter (HL), Vickers (HV), Brinell (HB), Shore (HS) and Rockwell (HRC) to avoid tedious manual calculations.

Surface hardness measurements were performed on 6 gears made of C45 material and 6 gears made of 41Cr4 material. For each test gear, surface measurements were made for all 6 teeth. The measurement results are shown in Fig. 3.

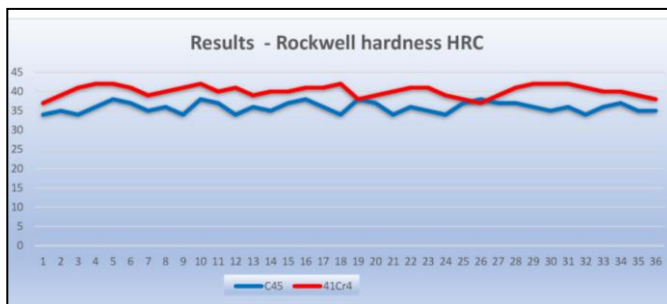


Fig. 3. Results - Rockwell hardness HRC

The obtained hardness measurement results clearly show the difference in the range of values for the two types of material. For gears made of C45 material, the hardness is in the range of 34-38HRC, while for gears made of 41Cr4, the hardness obtained from laser cutting is higher (38-42 HRC). This difference is due to the presence of the alloying element chromium (Cr), which imparts greater hardness when the material is heated by the laser beam during cutting. Before laser cutting, the hardness of the sheet material for C45 was measured to be 16-18 HRC. In the core of the gears, the hardness was not changed and was the same as the hardness of the starting material. Before laser cutting, the hardness of the sheet material for 41Cr4 was measured to be 18-20 HRC. The laser cutting process itself increases the hardness of the material in the cutting area. This improves the qualities of the working profile of the cut gear tooth without

additional heat treatment, because the laser cutting process itself is a type of heat treatment.[7,8]

3.1 Surface roughness measurement

The roughness of the resulting surfaces during laser cutting depends extremely on two factors, the quality of the sheet material being cut and the parameters of the cutting technology of the laser machine specified in point 2.3.

To obtain better results, a workpiece absolutely clean of rust or contamination is used for the study and the best experimentally established parameters of the laser machine cutting mode are selected. To measure the roughness of the obtained surfaces GSR750 - Display Surface Roughness Tester Roughness Measuring Instrument (Fig. 4)



This roughness tester measures over 12 parameters, including Ra, Rz, Rq, providing in-depth surface texture evaluation for precision manufacturing needs.

Fig. 4. GSR750 - Display Surface Roughness Tester Roughness Measuring Instrument

The results of the roughness measurement are shown in Fig. 5.

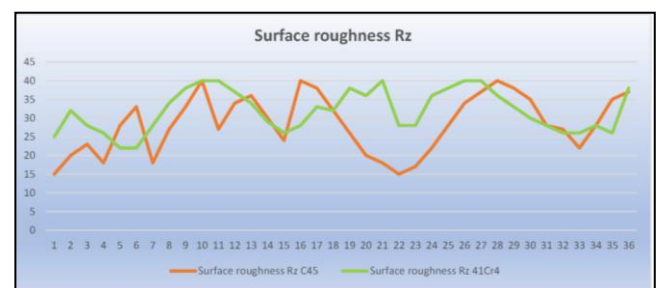


Fig. 5. Results - Surface roughness Rz

The main problem that is noticeable from the obtained results is that the roughness values vary in a relatively large range (Rz 10-40). This indicates instability and fluctuations of the laser beam during the cutting process. But overall this roughness is satisfactory as a result and would not pose a problem for the normal operation of the gears

4 Conclusions

The results obtained from the study lead to the following conclusions:

The resulting spur gears using the described technology are obtained with very good indicators of the surface hardness of the working profile (HRC 34÷42), without additional heat treatment. Of great importance for the obtained hardness is the chemical composition of the material - type of carbon steel and the presence of alloying elements. The obtained hardness results are very similar to the results of additional heat treatment with high-temperature heat treatment. Which determines that directly from the laser cutting, the working surface is strengthened without additional processing.

The resulting roughness is satisfactory (Rz15-Rz40), and this parameter is greatly influenced by the quality of the material and the cutting parameters of the laser cutting machine. With an absolutely clean cutting surface of the sheet material from which the gears are cut and correctly selected cutting parameters, minimal roughness values are obtained.

From the presented final results and conclusions it can be determined that it is economically more profitable to produce gears with straight teeth for not heavily loaded and slow-moving gears and not so precise gears. In this way, the long time for manufacturing by mechanical processing is saved with the significantly shorter machining time for laser cutting. Separately, expensive and complex tongue-cutting, metal-cutting tools are not required.

Analysis of geometric accuracy, which encompasses multiple parameters, the depth of the hard layer and the quality of the working profile are tasks of the subsequent study.

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Microstructural analysis and surface characterization of selective laser melted (SLM) metal gears

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Abstract. This study presents a comprehensive microscopic investigation of the surface morphology and microstructure of a 3D-printed metal gear fabricated using Selective Laser Melting (SLM). The analysis reveals a complex surface topography characterized by parallel scan tracks, micro-pores, and oxide inclusions. The presence of partially melted powder particles and thermally affected regions indicates localized instability during the melting and solidification process. Quantitative measurements demonstrate a relatively stable manufacturing process with dimensional deviations below 50 μm . However, the observed surface defects and morphological irregularities highlight the need for post-processing operations to improve the tribological performance, fatigue resistance, and corrosion resistance of the component. The findings of this study provide valuable insights into the microstructural characteristics of SLM-fabricated metal gears and emphasize the importance of controlling process parameters to minimize surface defects and ensure optimal functional performance.

1 Introduction

Additive manufacturing (AM) techniques, such as Selective Laser Melting (SLM), have revolutionized the production of complex metal components, including gears. SLM offers several advantages over traditional manufacturing methods, including increased design flexibility, reduced material waste, and improved mechanical properties. However, the unique characteristics of the SLM process can also introduce surface defects and microstructural irregularities that may affect the functional performance and durability of the component. From a gear fatigue performance point of view, the presence of pores in the material is particularly important, as they can act as stress concentrators and initiate cracks.

The microscopic investigations presented in this study were primarily conducted at the Bulgarian Academy of Sciences (BAS), as part of a collaborative project between BAS and Technical University Sofia. At technical University is ongoing research on developing and endurance SLM gear. This microscopic analysis work represents an important milestone in SLM gear development, enabling us to understand the tooth structure in detail successfully and possible to connect with simulations and to develop a methodology for reliable development of SLM drive trains. The gear sample utilized in this research was provided by Technical University Sofia, which had previously been employed in another scientific investigation [1]. Leveraging the synergy effect, we were able to perform an in-depth microscopic analysis of the gear, facilitating

a comparative study with existing research [2]. Notably, our analysis focused on the tooth bearing area, where pores can be present under the tooth surface, as previous studies have found them as deep as 0.1 mm to 0.6 mm beneath the surface. The question remains: are these features truly pores, or are they a result of other microstructural phenomena? This collaborative effort enabled us to gain a deeper understanding of the microstructural characteristics of the gear material and its potential implications for performance and durability.

To investigate the internal structure of the gear, we initially attempted to perform radiography or X-ray imaging. However, the machine at the Bulgarian Academy of Sciences is limited to parts with a thickness of up to 4 mm of aluminum. Since the presented gear has a thickness of 20 mm, this approach was not feasible. Instead, we decided to conduct a microscopic analysis, focusing on the top section of the gear. According to literature, this region is the contact surface between the part and the surrounding gas during printing, where thermal stresses are likely to occur.

As a next step, we plan to perform a sequential removal of the gear in 1 mm increments (from 20 mm to 19 mm, 18 mm, etc.) and repeat the analysis. Although this approach is destructive, it will provide valuable insights into the porosity underneath the tooth surface, substituting for the X-ray examination. This initial analysis will be crucial in understanding the internal structure of the gear.

This study aims to investigate the surface morphology and microstructure of a 3D-printed metal gear fabricated using SLM, with a focus on identifying

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potential defects, particularly in the subsurface region under the tooth, and optimizing process parameters for improved product quality. Furthermore, this analysis will also examine the rest of the gear's surface to provide a comprehensive overview of its microstructural characteristics, ensuring optimal functional performance, fatigue resistance, and corrosion resistance.

2 Methods

To investigate a 3D-printed gear using an Olympus microscope and Olympus Stream Basic software, it is necessary to develop a methodology following specific steps divided into preparation, technical software operation, and metallographic analysis.

2.1 Sample preparation

1) Cleaning: The surface of the specimen is cleaned with acetone to remove residual oils, dust, and contaminants from the working environment.

2) Positioning: The component is positioned on the microscope stage, ensuring that the examined surface is perpendicular to the optical axis.

2.2 Configuration of the optical system

Equipment – An Olympus BX53M metallurgical microscope integrated with Olympus Stream Basic digital image analysis software was used.

Calibration – For each magnification (10×, 20×, and 50×), the corresponding objective lens profile was selected in the software to ensure the accuracy of the integrated scale bar and all subsequent dimensional measurements.

Illumination – Reflected brightfield illumination was employed. The illumination intensity was adjusted to minimize glare from the metallic surface, thereby enabling clear observation of the 3D-printed metal gear specimen.

Objective Lens Selection – An Olympus objective lens with the appropriate magnification was used. The selection of the objective lens depended on the specific features of interest in the gear specimen to be examined. **Figure 1** shows the experimental setup with the selected objective lens.

Software Synchronization – In the Olympus Stream Basic software, the objective lens is manually selected in the Microscope Control panel. This is a critical step, as it enables the software to correctly calculate the scale bar and ensure accurate dimensional measurements.

2.3 Observation stages

Focusing – Fine focusing is performed on the upper metal layer. Due to the limited depth of field, precise adjustment is required to obtain a clear visualization of the surface morphology and laser scan track patterns.

Illumination Correction – The light intensity and exposure settings are adjusted within the software to avoid surface highlights and reflections from the metallic

specimen, which could obscure details in darker regions such as pores.



Fig. 1. Examination of a 3D-printed gear specimen using an Olympus microscope and gear sample.

2.4 Analysis of surface defects

Identification of Structures – The parallel traces resulting from laser scanning (Laser Scan Tracks) are observed [3].

Defect Detection – The specimen is examined for defects characteristic of 3D printing processes:

- Microporosity – Small dark spots associated with gas porosity..
- Lack of Fusion (LoF): Larger, irregular dark regions where adjacent layers have not fully fused.
- Investigation for microcracks in critical regions of the gear tooth.

Quantitative Measurements and Documentation

• **Metrology** – The measurement tools available in Olympus Stream Basic are used to determine:

- The distance between laser scan tracks (hatch spacing).
- The maximum size of detected pores/defects.
- Data Recording – Using the "Snap" function, digital micrographs are acquired and saved in .TIF format. A scale bar is automatically added to each image, corresponding to the calibration settings for the 20× magnification.

3 Results

The image presented in **Figure 2** shows the surface microstructure of the 3D-printed metallic gear. Clearly visible are parallel diagonal lines corresponding to the **laser scan tracks**. These features are characteristic of the **Selective Laser Melting (SLM)** process and indicate the direction of metal solidification and crystallization. The surface exhibits significant micro-roughness, which is typical for untreated (**as-printed**) components.

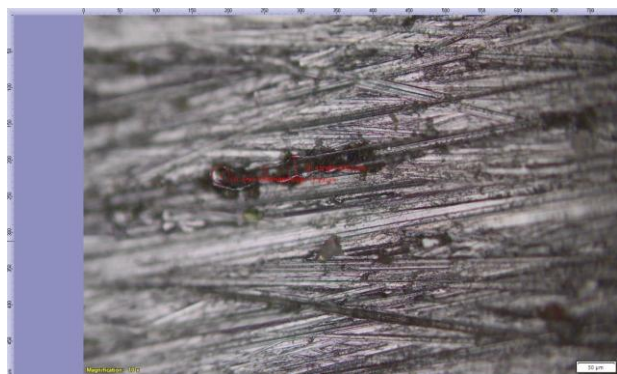


Fig. 2. Microstructural analysis and metrology of a surface defect.

At the center of the image, a specific defect was localized and measured. The defect consists of a combination of micro-pores and an unmelted particle. To the left of the defect, a highly reflective (bright) spherical feature, highlighted by a red circle, can be observed. This feature represents metal powder that was not completely melted or was deposited onto the previously solidified surface in the form of a “spatter” particle. A dark region surrounding the particle is also observed, corresponding to a lack of density or a small micro-cavity.

Using the measurement tools of the **Olympus Stream Basic** software, the following parameters of the defect shown in **Figure 2** were determined:

- The measured area of the critical unmelted powder region was $500.06 \mu\text{m}^2$. This provides a quantitative assessment of the defect scale on the surface.
- The vertical width (perpendicular to the laser scan tracks) was measured as $17.99 \mu\text{m}$.

A $50 \mu\text{m}$ scale bar is positioned in the lower right corner of the image, confirming and validating the microscopic dimensions of the observed structure.

The presence of such defects with dimensions in the range of approximately $20\text{--}30 \mu\text{m}$ indicates a relatively stable manufacturing process; however, at higher concentrations, these defects may act as stress concentrators. Since the detected defect is localized at a scale below $50 \mu\text{m}$, it is unlikely to significantly compromise the overall mechanical strength of the component. Nevertheless, it remains important for the evaluation of the surface fatigue behavior of the gear.

Figure 4 presents a detailed view of the same region of the gear surface, where two independent area measurements were performed to characterize the surface heterogeneity. The measurements reveal the

presence of features with significantly different dimensions within the micro-topography.

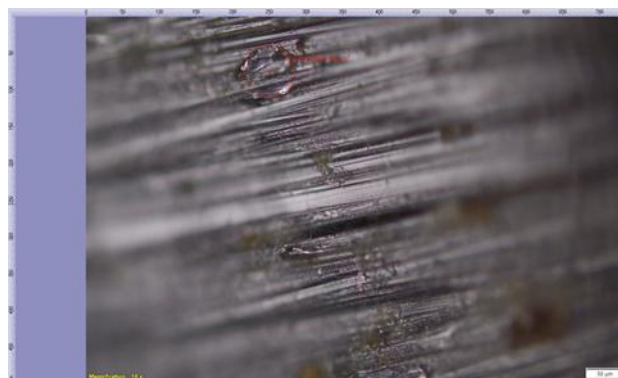


Fig. 3. Microscopic investigation of the surface morphology of a 3D-printed metallic gear.

While the first region from **Figure 3 (Zone 1)** represents a large accumulation or defect with an area of $3861.93 \mu\text{m}^2$, the second location shown in **Figure 4 (Zone 2)** identifies a considerably smaller feature with an area of only $50.55 \mu\text{m}^2$.

Zone 2 (with an area of approximately $50 \mu\text{m}^2$) corresponds in size to a single particle of the initial metal powder that remained partially sintered or adhered (“stuck”) to the surface. This is a typical phenomenon in additive manufacturing, which contributes to the final surface roughness of the component. The presence of such micro-protrusions (**Zone 2**) and larger remelted regions (**Zone 1**) provides an important indication of the thermal conditions during the printing process.

The results presented in **Figure 4** confirm that the surface of the 3D-printed gear contains numerous micro-features (also referred to as satellites) [5] with different morphologies. These surface formations represent a critical factor in determining the coefficient of friction and the necessity for subsequent finishing operations on the gear tooth surfaces++

The surface exhibits relatively high microscale roughness. Small irregularities and a granular morphology are visible, which are typical characteristics of components in the as-printed condition that have not undergone additional mechanical post-processing, such as polishing or grinding.

Figure 3 presents a micrograph of the gear surface acquired at **10× magnification** using the **Olympus Stream Basic software**. The observation reveals several key characteristics of the microstructure and surface topography. In the upper part of the image, software-based dimensional analysis was performed on a specific region highlighted by a red circle. The measured area of this feature was $3861.93 \mu\text{m}^2$.

This feature represents a surface anomaly, most likely associated with a cluster of partially unmelted powder particles or a locally

overheated region. Its dimensions are critical for evaluating the printing quality and the subsequent wear resistance of the gear. The visual analysis confirms successful layer formation while also identifying characteristic micro-roughness and specific surface

formations that may require additional mechanical finishing to improve the tribological performance of the gear.

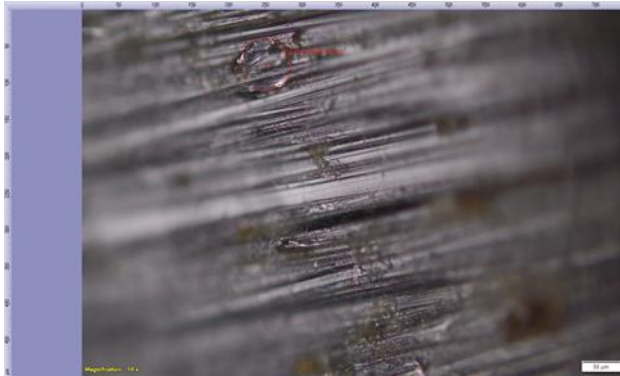


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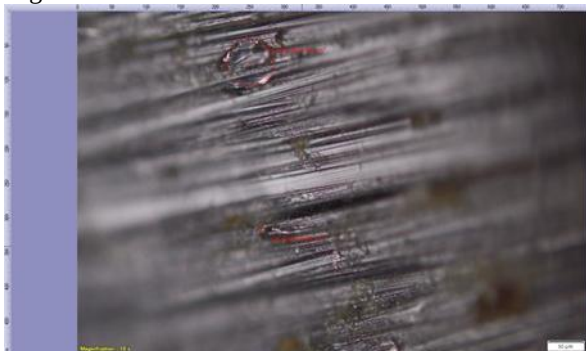


Fig. 4. Comparative microstructural analysis and quantitative characterization of surface features with different dimensions.

Figure 3 provides an overall view of the surface structure, whereas **Figure 4** is used for the quantitative assessment of variations in surface defects.

Figure 5 presents a micrograph of the peripheral region of the gear, where the boundary between the metallic body and the surrounding space (represented by the black region on the right side) is clearly visible. This image provides important information regarding edge geometry and the quality of sintering in boundary regions.

The characteristic edge irregularity associated with 3D printing processes can be observed. The edge is not perfectly straight but exhibits a serrated morphology caused by the granular nature of the initial powder material and the dynamics of the molten pool at the end of each scanning line.

Within this region, two specific dark-colored features, distinct from the main metallic matrix, were identified and dimensionally measured. These features most likely correspond to formed pores:

- **Zone 1** with a measured area of $208.09 \mu\text{m}^2$.
- **Zone 2** with a measured area of $215.12 \mu\text{m}^2$.

The nearly identical areas of the two features (with a difference of less than 4%) suggest that they represent recurring structural elements. Their dark appearance and morphology may indicate the presence of local oxide inclusions, micro-cavities, or regions with different crystallization characteristics formed during the reversal of the laser beam at the contour boundary.

It can be observed that the scan tracks near the edge are more finely defined and slightly change their orientation. This is a result of the application of specialized contour scanning strategies aimed at improving dimensional accuracy.

In metallographic analysis, dark spots on a bright, reflective metallic surface generally indicate a **void or surface depression**. The incident light from the microscope enters the cavity and is not reflected back toward the objective lens, causing the region to appear dark. When such features are perfectly spherical, they are typically associated with **gas porosity**. However, the features shown in **Figure 5** exhibit a more irregular, slightly angular morphology. This characteristic is typical of **lack-of-fusion (LoF) pores**, which are regions where the laser energy was insufficient to completely melt the metal powder or where insufficient overlap occurred between adjacent laser scan tracks.

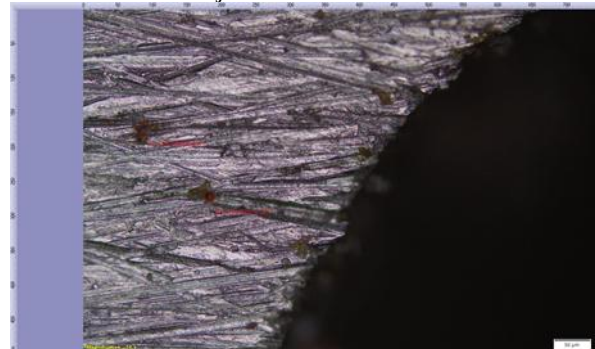


Fig. 5. Microscopic analysis of the peripheral region (contour) and assessment of surface inclusions.

Zone 1 ($208.09 \mu\text{m}^2$) and Zone 2 ($215.12 \mu\text{m}^2$) in Fig. 5 are very similar in size, suggesting that this is a systematic issue associated with this particular edge. Most likely, the laser reduces its scanning speed in this region, or the scanning strategy results in the formation of small “gaps” at the contour boundary.

Figure 6 presents a microscopic view of the surface at $10\times$ magnification, focused on a group of specific surface anomalies located along the scanning tracks. Unlike the previous figures, this image reveals features with a distinct reddish-brown coloration. This change in color compared with the grey metallic matrix is often an indication of localized metal oxidation or the presence of overheated spatter particles that have undergone oxidation upon contact with residual oxygen in the chamber [6].

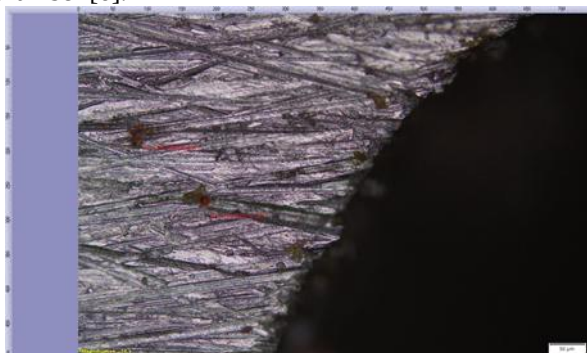


Fig. 6. Quantitative and morphological analysis of surface oxide inclusions.

Figure 6 presents a microscopic analysis of surface defects characterized by specific chromatic features (reddish-brown coloration). Based on their visual characteristics and morphology, these regions are identified as localized oxidation sites. The measured areas (Zones 1, 2, and 3) indicate the presence of corrosion products (most likely iron oxides) formed along the scanning tracks. The dark and intense coloration in the center of Zone 2 ($3843.00 \mu\text{m}^2$) suggests deeper penetration of the oxide layer or accumulation of a larger amount of oxidized material.

In metal powder-based additive manufacturing (particularly in stainless steels and tool steels), surface oxidation can occur through two main mechanisms [7]:

- In-situ (during the process) – due to the presence of residual oxygen in the build chamber, which reacts with the hot molten pool.
- Ex-situ (after the process) – as a result of atmospheric moisture penetrating into micropores or surface irregularities of the untreated surface, leading to pitting corrosion.

Software-based measurements show that the oxide spots vary significantly in size, ranging from micro-scale oxidation sites ($\sim 480 \mu\text{m}^2$) to larger oxide regions exceeding $3800 \mu\text{m}^2$. This indicates that the oxidation process is not uniform but is concentrated in areas with higher surface energy or the presence of micro-defects [8].

The presence of oxide inclusions represents a critical defect. Oxide-rich regions are brittle and can easily

detach during gear operation, acting as abrasive particles that accelerate the wear of contacting surfaces [9, 10].

Figure 7 shows a detailed view of the surface in the immediate vicinity of the gear edge. The image demonstrates the complex intersection of laser scanning tracks.

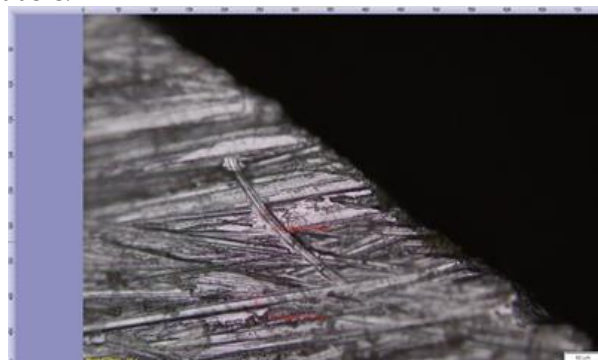


Fig. 7. Linear microscopic analysis of the width of scanning tracks in the peripheral region of the component.

Unlike the previous figures, linear measurements of specific texture features were performed in this case. The Length (1) = $13.45 \mu\text{m}$ represents the width of a specific overlap region or micro-groove within the structure. The Length (2) = $10.97 \mu\text{m}$ indicates the size of an adjacent structural feature.

These values (approximately $10\text{--}13 \mu\text{m}$) are significantly smaller than the typical laser spot diameter (usually $50\text{--}100 \mu\text{m}$), indicating that the software is capturing the fine boundaries between the solidification fronts within the melt pool (melt pool boundaries).

In the upper right region of the image, the edge of the gear tooth is visible. The characteristic serrated morphology resulting from layer-by-layer deposition can be observed. According to literature sources, this “staircase effect” is an inherent feature of additive manufacturing and is particularly pronounced on inclined surfaces, such as gear teeth [5].

The fine linear defects and the small groove dimensions measured in the figure are directly related to tribological contact behavior [10].

The linear analysis presented in Figure 7 enables quantification of the fine surface texture, confirming the high resolution of the applied printing method while also revealing the presence of micro-irregularities that could act as initiation sites for tribocorrosion.

Figure 8 presents a detailed view of the surface morphology in the tip (apex) region of the gear tooth. A distinct diagonal depression (groove) intersecting the primary surface texture can be observed. The dark coloration and lack of reflectivity in this region indicate a considerable depth of the surface relief compared with the surrounding areas. This groove was most likely formed at the interface between contour scanning and internal hatching, where the highest thermal stresses accumulate [9]. The measured groove width is $52.56 \mu\text{m}$. This dimension is comparable to the scale of the laser spot size, suggesting that the defect extends across an entire scanning track. According to literature sources, such “valleys” in the micro-relief are typical features of

additive manufacturing and represent significant geometric deviations from the theoretical tooth profile [5].

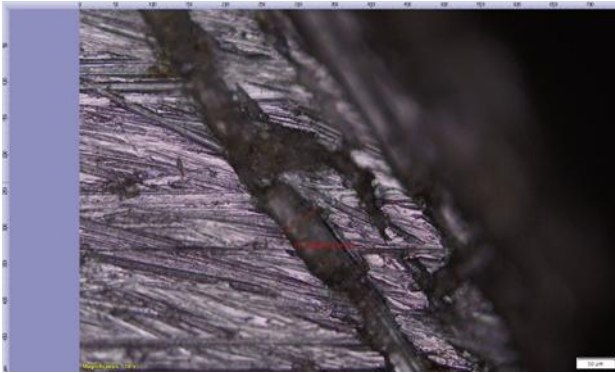


Fig. 8. Microscopic analysis of the surface topography at the gear tooth tip and dimensional measurement of a depression groove.

The location of this groove at the tooth tip is of particular concern. During gear operation, this region is subjected to intense contact stresses and cyclic loading. Such deep surface micro-grooves act as severe stress concentrators, promoting the initiation of fatigue cracks and surface pitting. Furthermore, they may accelerate tribocorrosion by facilitating the retention of moisture and oxide debris within the groove, thereby creating favorable conditions for localized corrosion-assisted wear [10].

4 Discussion and future work

The results presented in this study highlight the importance of controlling process parameters to minimize surface defects and ensure optimal functional performance. Acc. figure 5, there is a pore under the tooth at app. 0.15 mm as in [2] observed.

A significant advantage of the present microscopic analysis lies in the direct indication of the pore areas in square micrometers. For development, this precise quantitative recording is of invaluable value, as it allows for a deeper understanding of the exact dimensions of the pores and facilitates and refines the optimization process and modeling of such pores in CAD.

The presence of oxide inclusions, partially melted particles, and micro-cavities can significantly affect the tribological performance, fatigue resistance, and corrosion resistance of the component. As discussed in previous studies [3-5], these defects can act as stress concentrators, promoting the initiation of fatigue cracks and surface pitting. Furthermore, the oxidation products observed in this study can accelerate tribocorrosion by facilitating the retention of moisture and oxide debris within the groove, thereby creating favorable conditions for localized corrosion-assisted wear [10].

5 Conclusions

A comprehensive microscopic investigation of the SLM-fabricated metal gear was performed using the Olympus Stream Basic software, providing a detailed quantitative

and qualitative assessment of its surface condition. Analysis of the acquired micrographs enabled the formulation of the following principal findings:

1) Process Characteristics and Stability

The observed parallel scan tracks (Figs. 2, 3, and 7) are characteristic of the Selective Laser Melting (SLM)/Direct Metal Laser Sintering (DMLS) process and confirm the proper scanning strategy and stable guidance of the laser energy source. Detailed measurements of the micro-defect shown in Fig. 2 (area: $500.06 \mu\text{m}^2$, horizontal length: $28.85 \mu\text{m}$, and width: $17.99 \mu\text{m}$) indicate a relatively stable manufacturing process, with dimensional deviations remaining below the $50 \mu\text{m}$ threshold. These dimensions demonstrate good manufacturing precision; however, the presence of partially melted spherical powder particles indicates localized microstructural heterogeneity.

2) Analysis of Surface Defects and Morphological Irregularities

The microscopic investigation revealed a progressive increase in the severity of surface imperfections, ranging from isolated micro-defects (Fig. 2) to extensive agglomerations of partially melted powder particles and thermally affected regions with areas exceeding $3800 \mu\text{m}^2$ (Figs. 3 and 6). These observations indicate localized instability during melting and solidification, resulting in non-uniform surface morphology.

3) Chemical Degradation and Oxidation

The characteristic reddish-brown discoloration observed in the regions analyzed in Fig. 6 confirms the presence of localized oxidation products (iron oxides). These oxide-rich regions create favorable conditions for accelerated tribocorrosion. The synergistic interaction between mechanical wear and electrochemical degradation within the gear tooth contact zone represents a critical factor that can significantly reduce the service life of the component.

4) Functional Performance and Serviceability

Although the dimensional analysis of the individual scan tracks (Figs. 7 and 8) demonstrates manufacturing precision in the range of $10\text{--}50 \mu\text{m}$, the overall surface morphology is characterized by relatively high roughness and the presence of satellite particles. Without appropriate post-processing, these surface features are expected to increase the coefficient of friction and promote premature surface fatigue, ultimately leading to pitting damage under cyclic loading.

From an engineering perspective, the gear tooth edges and tip regions are subjected to the highest stress concentrations during service. Consequently, any defects located in these critical areas may serve as preferential sites for crack initiation and progressive wear.

The identified oxidation products further emphasize the necessity of maintaining a well-controlled protective atmosphere during the SLM process, followed by appropriate passivation or the application of a protective coating to minimize corrosion during service.

Overall, the obtained results clearly demonstrate that the manufactured gear requires appropriate post-processing, such as grinding, polishing, or thermochemical surface treatment/passivation. These finishing operations are essential for eliminating surface

grooves, oxide inclusions, partially melted particles, and micro-cavities, thereby improving the tribological performance, fatigue resistance, and corrosion resistance of the component.

The location of the observed groove at the gear tooth tip is particularly critical. During gear operation, this region experiences severe contact stresses and repeated cyclic loading. Such deep surface micro-grooves act as pronounced stress concentrators, promoting the initiation of fatigue cracks and surface pitting. Furthermore, they may accelerate tribocorrosion by facilitating the retention of moisture and oxide debris within the groove, thereby creating favorable conditions for localized corrosion-assisted wear [10].

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ARTIFICIAL INTELLIGENCE-ASSISTED FRAMEWORK FOR PREDICTING ELECTROCHEMICAL DETERIORATION OF REINFORCED CONCRETE STRUCTURES UNDER COMBINED CHLORIDE AND STRAY CURRENT EXPOSURE

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Abstract. Reinforcement corrosion remains a major cause of premature deterioration of reinforced concrete infrastructure, especially when chloride ingress and stray direct currents act simultaneously. This paper develops and computationally demonstrates an artificial intelligence-assisted framework for predicting a relative electrochemical deterioration score from seven measurable inputs: corrosion potential, concrete electrical resistivity, corrosion current density, chloride content, moisture, temperature and stray-current intensity. A reproducible synthetic parametric database of 1,200 scenarios was generated within physically plausible engineering ranges. Four supervised learning models—an artificial neural network, Random Forest, support vector regression and Gradient Boosting—were trained using a 75/25 stratified train-test split. Prediction accuracy was assessed by R^2 , mean absolute error and root mean square error, while permutation importance was used to interpret the dominant predictors. The ensemble models achieved the strongest performance, and the importance analysis confirmed that corrosion current density, chloride content and electrical resistivity exerted the largest influence on the predicted condition. The results are a methodological demonstration rather than validation against laboratory or field measurements. The proposed workflow is intended to complement conventional electrochemical inspection, support maintenance prioritisation and provide a transparent basis for future calibration using monitored reinforced concrete structures.

Keywords: reinforced concrete; corrosion; artificial intelligence; machine learning; chloride ingress; stray current; durability assessment

1 Introduction

Reinforced concrete structures are designed for long service lives, yet reinforcement corrosion remains one of the principal mechanisms controlling their durability and maintenance demand. Corrosion products occupy a larger volume than the original steel, generating tensile stresses in the concrete cover and progressively causing cracking, delamination, spalling, bond deterioration and loss of reinforcement cross-section [1–3]. Condition assessment is particularly difficult when several deterioration mechanisms interact and individual measurements provide partially conflicting indications.

Chloride ingress is a major cause of reinforcement depassivation in marine, de-icing-salt and industrial environments. The critical chloride level is not a unique material constant; it depends on binder composition, moisture, oxygen availability, electrochemical potential and local conditions at the steel-concrete interface [1,4]. Once local depassivation occurs, corrosion may remain highly non-uniform and can progress without clear visible symptoms during the early stage.

Stray direct currents introduce an additional deterioration mechanism in structures located near railways, metro systems, cathodic-protection installations and other direct-current infrastructure. Current leaving reinforcing steel at anodic locations can accelerate metal dissolution, while the imposed electric field can modify ionic transport through the concrete pore system [5–8]. Recent studies indicate that cracks, chloride contamination and stray-current exposure may act in a coupled manner, increasing local corrosion severity and producing asymmetric electrochemical conditions around reinforcement [6–8].

Conventional diagnostic practice commonly combines corrosion potential, concrete resistivity and corrosion current density. ASTM C876 provides a standardised basis for corrosion-potential measurements, while polarisation-resistance methods provide estimates of corrosion current density [9,10]. Concrete resistivity offers complementary information about ionic conductivity and the environment supporting corrosion kinetics. In addition, ISO 12696:2022 provides internationally recognized guidance for the cathodic

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protection of steel in concrete, emphasizing systematic monitoring and long-term durability management of reinforced concrete structures [20]. However, no single indicator provides a complete diagnosis, and externally imposed electrical fields can further complicate interpretation

Machine-learning methods have increasingly been applied to concrete durability, including prediction of chloride migration, diffusion and resistance classes [11–15]. Their main advantage is the ability to learn nonlinear interactions among multiple predictors. Their limitations include sensitivity to database quality, insufficient external validation and reduced interpretability. For engineering use, an AI model should therefore be transparent, explicitly bounded by its training domain and integrated with—not substituted for—physical inspection and professional judgement [15–17].

The research gap addressed in this paper is the limited integration of electrochemical indicators, chloride exposure and stray-current intensity within one transparent AI-assisted deterioration-assessment workflow. The objective is to develop and computationally demonstrate a reproducible framework that maps seven input variables to a relative deterioration score and engineering condition class. The contribution is methodological: a synthetic parametric dataset is used to compare representative algorithms, quantify prediction accuracy and identify influential variables. The study does not claim experimental or field validation.

2 Methodology

2.1 Framework and input variables

The workflow shown in Figure 1 consists of: (i) definition of physically plausible parameter ranges; (ii) generation of a reproducible parametric database; (iii) train-test separation; (iv) training of four regression models; (v) performance comparison; (vi) interpretation by permutation importance; and (vii) conversion of the predicted continuous score into an engineering condition class.

The selected variables were corrosion potential E_{corr} , electrical resistivity ρ , corrosion current density i_{corr} , chloride content Cl , relative moisture H , temperature T and stray-current intensity I_s . The adopted ranges in Table 1 are broad enough to represent favourable to highly aggressive conditions, but they are not proposed as universal diagnostic limits.

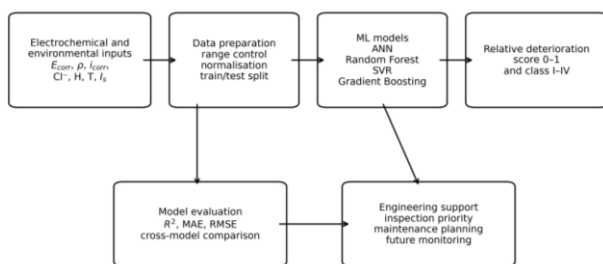


Figure 1. Artificial intelligence-assisted deterioration assessment workflow.

Table 1. Input variables and adopted parametric ranges.

Variable	Symbol	Unit	Range	Interpretation
Corrosion potential	E_{corr}	mV	−650 to −100	More negative = less favourable
Electrical resistivity	ρ	kΩcm	5 to 60	Lower = less favourable
Corrosion current density	i_{corr}	μA/cm ²	0.05 to 5.00	Higher = less favourable
Chloride content	Cl	% cement	0.10 to 1.20	Higher = less favourable
Relative moisture	H	%	50 to 100	Higher generally favours conduction
Temperature	T	°C	5 to 40	Affects kinetics and resistivity
Stray-current intensity	I_s	mA	0 to 100	Higher = greater electrical influence

2.2 Parametric target and dataset generation

A total of 1,200 independent scenarios was generated with a fixed random seed. Uniform sampling was used for all variables except corrosion current density, which was sampled log-uniformly because practical values span approximately two orders of magnitude. The synthetic target is a bounded relative deterioration score $D \in [0,1]$. It combines normalised main effects and selected interaction terms representing the expected aggravating influence of chloride with stray current, active corrosion with moisture, and negative potential with low resistivity.

The score is not a standardised corrosion index and must not be interpreted as measured steel loss, probability of failure or remaining service life. Its purpose is to create a controlled nonlinear benchmark for testing whether the AI workflow can recover physically interpretable trends.

$$D^* = 0.17E_{corr} + 0.14\rho + 0.24i_{corr} + 0.19Cl + 0.07H + 0.04T + 0.10I_s + 0.11Cl \cdot I_s + 0.07i_{corr} \cdot H + 0.05E_{corr} \cdot \rho \quad (1)$$

$$D = [1 + \exp(-5.2(D^* - 0.50))]^{-1} + \epsilon, \quad \epsilon \sim N(0, 0.022^2) \quad (2)$$

2.3 Machine-learning models and evaluation

The database was divided into 75% training data and 25% test data. Stratification by deterioration-score quintiles maintained a comparable output distribution in both subsets. Four regression algorithms were evaluated: multilayer perceptron artificial neural network (ANN), Random Forest (RF), support vector regression (SVR) with radial-basis kernel, and Gradient Boosting (GB) [16–19]. Input standardisation was applied to ANN and

SVR. Model settings were fixed before final test evaluation to preserve a transparent and reproducible comparison.

Performance was quantified by the coefficient of determination R^2 , mean absolute error (MAE) and root mean square error (RMSE). Permutation importance was calculated on the independent test subset by repeatedly shuffling one predictor and measuring the resulting loss of predictive performance.

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} \quad (3)$$

$$MAE = (1/n) \sum |y_i - \hat{y}_i| \quad (4)$$

$$RMSE = [(1/n) \sum (y_i - \hat{y}_i)^2]^{1/2} \quad (5)$$

Table 2. Principal model settings used in the reproducible demonstration.

Model	Principal settings
ANN	Two hidden layers (24, 12); ReLU; $\alpha=0.002$; max. 3000 iterations
Random Forest	500 trees; maximum depth 14; minimum leaf size 2
SVR	RBF kernel; $C=20$; $\epsilon=0.025$; standardised inputs
Gradient Boosting	350 estimators; learning rate 0.035; depth 3; Huber loss

2.4 Engineering classification

For decision-support presentation, the continuous predicted score is converted into four relative condition classes. The limits in Table 3 are study-specific and are intended only for prioritisation within the demonstrated framework.

Table 3. Relative electrochemical deterioration classes.

Class	Score interval	Condition	Indicative response
I	0.00–0.25	Low	Routine inspection; no AI-based intervention trigger
II	>0.25–0.50	Moderate	Periodic monitoring and targeted verification
III	>0.50–0.75	High	Detailed assessment and preventive maintenance
IV	>0.75–1.00	Severe	Immediate engineering investigation and action

3 Results

3.1 Predictive performance

All four algorithms reproduced the nonlinear parametric target with high accuracy on the independent test subset. SVR achieved the highest R^2 (0.950) and the lowest or near-lowest prediction errors. The ranking is reported in Table 4. Because the target was generated by a controlled analytical function, these values demonstrate computational consistency of the workflow; they do not establish real-world predictive validity.

Table 4. Test-set performance of the investigated models.

Model	R^2	MAE	RMSE
SVR	0.950	0.028	0.036
Gradient Boosting	0.940	0.032	0.040
Random Forest	0.891	0.044	0.054
ANN	0.668	0.077	0.094

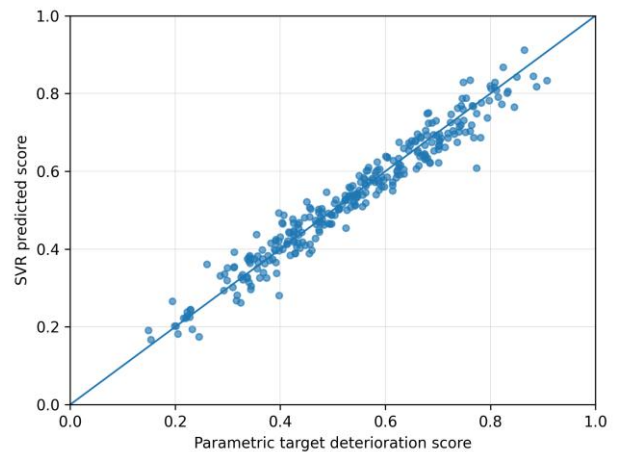


Figure 2. Predicted versus target deterioration score for the best-performing model (SVR).

3.2 Variable importance and physical interpretation

Permutation analysis identified $icorr$ ($\mu A/cm^2$), Chloride (% cement) and E_{corr} (mV) as the three most influential predictors. This result is consistent with the construction of the parametric benchmark and with engineering expectations: corrosion current density represents active kinetics, chloride content contributes to depassivation and localised attack, while low resistivity facilitates ionic current transmission. Stray current also contributed directly and through its interaction with chloride exposure.

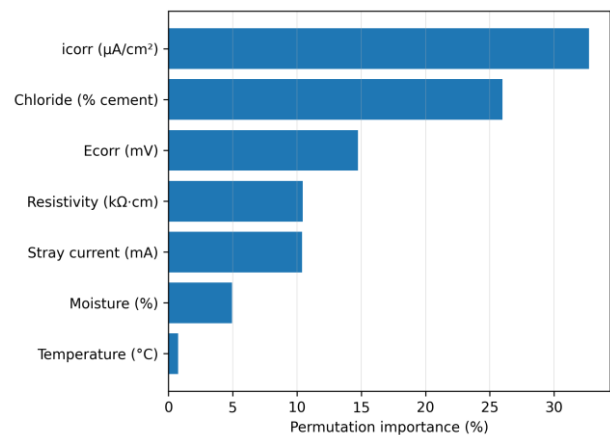


Figure 3. Test-set permutation importance for the SVR model.

3.3 Classification behaviour

After conversion of continuous predictions to the four relative classes, the best model achieved an exact class agreement of 92.0% on the test subset. Most disagreements occurred near class boundaries and generally involved one adjacent class. This supports the use of a continuous score together with the class label, rather than treating the class as an exact physical diagnosis.

4 Discussion

The principal value of the proposed framework is its integration of heterogeneous measurements into a single, interpretable workflow. Corrosion potential, resistivity and corrosion current density describe different aspects of the electrochemical state, whereas chloride, moisture, temperature and stray-current intensity describe exposure and transport conditions. Their combined use is more defensible than reliance on one threshold value, particularly in electrically disturbed environments.

The model comparison shows that ensemble tree methods are well suited to nonlinear deterioration mapping because they capture interactions without requiring strong assumptions about the mathematical form. ANN and SVR also achieved high accuracy after input standardisation. In a real application, algorithm selection should not be based solely on test error. Dataset size, missing data, extrapolation risk, uncertainty quantification and interpretability are equally important.

The feature-importance result should be interpreted carefully. Importance is conditional on the generated ranges and the adopted target formulation; a variable with a narrow range can appear less important even if it is physically critical. For field deployment, importance should therefore be recalculated after calibration and compared with corrosion science, inspection findings and sensitivity analyses.

The proposed class system supports inspection prioritisation but is not a substitute for structural safety assessment. A high predicted electrochemical deterioration score does not directly quantify reinforcement cross-section loss, bond reduction, crack width or residual load-bearing capacity. Detailed inspection, chloride profiling, crack mapping, section-loss measurements and structural verification remain necessary.

Integration with BIM, digital twins and structural-health-monitoring systems is a logical future extension. Time-stamped sensor measurements could update the input vector, while the model could provide a continuously revised condition score and flag rapid deterioration. Such integration would be especially relevant for railway and underground infrastructure exposed to changing electrical and moisture conditions.

4.1 Limitations

The most important limitation is the synthetic nature of the database. Although the parameter ranges and

qualitative interactions are physically plausible, the target score was not derived from measured corrosion damage. Consequently, high R^2 values primarily show that the algorithms learned the designed benchmark relationship.

The framework omits concrete composition, crack geometry, cover depth, oxygen availability, carbonation, reinforcement configuration, wetting-drying history and exposure duration. It also treats each scenario as independent and does not model temporal deterioration. These variables should be incorporated when experimental data become available.

External validation is essential before practical use. A future programme should combine controlled laboratory specimens exposed to chlorides and imposed direct currents with long-term monitoring of existing infrastructure. The target variable should then be linked to measured steel mass loss, pitting depth, crack development or a validated condition rating.

5 Conclusions

1. A reproducible AI-assisted framework was developed to integrate seven electrochemical and environmental variables into a relative deterioration score for reinforced concrete exposed to chloride and stray-current effects.
2. A synthetic parametric database of 1,200 scenarios enabled transparent comparison of ANN, Random Forest, SVR and Gradient Boosting models under an identical 75/25 train-test split.
3. The best-performing model was SVR, with $R^2=0.950$, $MAE=0.028$ and $RMSE=0.036$ on the independent test subset.
4. Permutation importance identified corrosion current density, chloride content and concrete resistivity as dominant predictors within the adopted parametric domain.
5. The proposed class system can support inspection and maintenance prioritisation, but it must not be interpreted as a direct structural-safety, steel-loss or remaining-service-life model.
6. The study provides a methodological foundation for future calibration using laboratory tests and long-term field monitoring, followed by integration with BIM, digital twins and structural-health-monitoring platforms.

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Comparative study of bearing losses in vehicle gearboxes using SKF digital tools

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Abstract. A comparative analysis of power loss calculations in rolling bearings of automotive gearboxes using digital tools and traditional engineering methods has been carried out. The aim is to assess the impact of the refined calculation methodology proposed by SKF on the overall efficiency of the vehicle transmission. The relevance of the study is due to the fact that classical engineering methods often do not take into account the influence of angular speed and load on the friction coefficient, which can significantly affect the evaluation of transmission energy efficiency. The SKF methodology is implemented in the company's online service. It allows consideration not only of the geometric parameters of the bearings but also of real operating conditions such as lubricant type, temperature, speed, and dynamic loads. This ensures higher accuracy compared to traditional semi-empirical methods. The research methodology was structured to consistently evaluate the impact of the refined digital model of rolling bearing losses proposed by SKF on the results of power loss calculations both at the level of an individual bearing and at the level of gearboxes of different configurations. The study was conducted in two stages. In the first stage, the behaviour of a bearing under isolated load conditions was analyzed, which made it possible to establish the sensitivity of the digital model to the way power is formed. In the first scenario, power increased due to rising angular speed at constant torque. In the second scenario, power increased at constant angular speed due to rising torque. The SKF digital model demonstrated non-invariance with respect to the way power is formed. In the second stage, the obtained results were transferred to the calculation of gearbox efficiency, which made it possible to assess the practical significance of the refined model for engineering transmission calculations.

1 Introduction

The growing demand for vehicle energy efficiency under modern conditions is one of the key factors shaping the development of the automotive industry. This necessitates the search for new transmission designs, optimization of engine performance, and more accurate calculation of power losses in vehicle mechanisms and assemblies. Among the various sources of losses, rolling bearings occupy a special place: despite their high efficiency, their large number in the vehicle design results in a significant share of total energy losses.

Modern vehicles contain dozens, and sometimes hundreds, of rolling bearings located in different units. In particular, they are widely used in vehicle gearboxes, transfer cases and differentials, starters, generators, and other auxiliary assemblies such as wheel hubs, turbochargers, air conditioning systems, and pumps. Given such widespread use, even small refinements in the calculation of power losses for each individual bearing can, in total, significantly affect the overall energy efficiency of the vehicle. For this reason, new modelling methods have been actively developed in recent years, taking into account subtle effects in rolling

contact, lubricant flow characteristics, the influence of temperature on viscosity, and the behaviour of rolling elements. These approaches provide more accurate results compared to simplified engineering methods based on semi-empirical or empirical dependencies. At the same time, other sources of power losses in vehicles are often estimated too roughly — for example, through the use of fuel consumption maps for fleet operation analysis. This raises the question of how much detailed refinements in rolling bearing calculations truly affect the final result and whether engineers should apply them in the design of future vehicles.

Previously, expensive software packages were used to model power losses, accessible only to large companies. Today, however, many online services have emerged for calculating gear transmissions, bearings, clutches, and even internal combustion engines. Some of these are free and publicly available, opening new opportunities for engineers and researchers. These services allow refinement of design calculations, modelling of individual processes, and consideration of factors that previously remained unaccounted for due to complexity or high cost of analysis.

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Thus, an important task of modern research is to analyze which refinements are provided by online services, how significant they are for practical application, and what weight they carry in the overall energy efficiency of the vehicle. This will help determine the balance between calculation accuracy and the feasibility of their use in real engineering projects.

2 Literature review and problem statement

Modern vehicles are characterized by high structural complexity and the presence of a large number of units operating in rolling motion. Such units include bearings installed in vehicle gearboxes, transfer cases, starter shafts, as well as in numerous auxiliary mechanisms. Their operation is accompanied by power losses, which in total have a significant impact on the overall energy efficiency of the vehicle.

For gear transmissions and multi-disc clutches, established methods of loss evaluation exist, based on determining torque and applying corresponding empirical dependencies. However, for rolling bearings, no standardized methodology has yet been developed [1-3]. This creates a certain gap in engineering calculations, since the absence of unified approaches complicates the prediction of energy efficiency already at the design stage of new vehicles.

In cases where detailed studies of fuel consumption and emission levels are required, engineers usually rely on so-called loss maps, which are formed on the basis of experimental measurements. Such maps make it possible to account for real operating conditions of the units; however, they can only be obtained after producing prototypes or using data from previous projects. This means that at the early stages of design, when measurements are not yet available, engineers are forced either to ignore bearing losses or to apply simplified assumptions. As a result, design calculations of bearing losses are rarely performed, which reduces the accuracy of forecasts regarding fuel economy and environmental characteristics of vehicles.

In modern studies of friction in rolling bearings, four main calculation methods are considered, each with its own features, advantages, and limitations. They are divided into two groups: global (empirical) and local (physically based) [4, 5].

The Schaeffler method is a global empirical approach based on calculating the friction torque as the sum of two components: no-load friction torque (caused by lubricant and seals) and load-dependent friction torque (caused by rolling and sliding friction). This method is simple and fast to apply, making it convenient for standard bearings. However, it has limitations, as it does not account for the specific shape of contact surfaces and may overestimate the influence of load [6, 7].

The SKF method also belongs to the global approaches but is more detailed. It considers four main sources of friction: rolling friction, sliding friction, seal friction, and friction from lubricant or air churning. This method provides a more accurate distribution of losses

and is widely used in industry due to its high consistency with experimental data [8, 9].

The Schleich method is a local approach that models friction based on physical processes in tribological contacts. It accounts for the non-Newtonian behaviour of lubricants and their temperature dependence. This method allows detailed analysis of friction in individual contacts, making it useful for non-standard or hybrid bearings. However, it is sensitive to the choice of friction coefficient and may underestimate friction under certain conditions [10, 11].

The LFP (Low Friction Powertrain) method is also local and considered the most comprehensive. It takes into account all major sources of friction, including complex lubricant behaviour, limiting shear stress, and other physical parameters. For the no-load component of friction torque, this method can incorporate results from global approaches such as Schaeffler. Due to its accuracy and versatility, LFP is the best choice for complex studies, but it requires detailed input data [12, 13].

To evaluate the accuracy of the friction calculation methods — Schaeffler, SKF, Schleich, and LFP — the experimental database FVA No. 364/IV was used. This database contains measured values of friction torque for cylindrical rolling bearings under different load conditions, rotational speeds, and lubricant viscosities [3].

The comparison was carried out by juxtaposing the calculated values obtained using each method with experimental data. The evaluation considered not only the overall accuracy of each method but also their ability to account for physical effects such as friction in rib contacts of bearings and lubricant compression in the entry zone of the contact. According to the analysis, the SKF and LFP methods demonstrated the smallest deviation from experimental data, while the Schleich method underestimated friction due to neglecting certain contact effects. The Schaeffler method provided good approximate values but with some limitations in accuracy under specific operating conditions [3].

It is widely recognized that, when modelling the driving cycle of a vehicle, the majority of transmission losses consist of constant losses (so-called “load-independent”), which arise from bearing friction, lubricant churning, ventilation resistance, and hydrodynamic effects. Load-dependent losses, associated with transmitted torque, represent a smaller share of the overall balance, especially under low and medium loads in urban cycles. Thus, it is often assumed that bearing friction losses do not depend on torque.

It should be noted, however, that this does not mean such losses are constant. It is generally assumed that their variation is small and can be neglected. This explanation, frequently found in educational literature, is incorrect [14, 15]. The issue is not a weak dependence of losses on speed — in fact, they strongly depend on speed — but rather the mutual compensation of these losses as speed increases. The main components of gearboxes are gear transmissions, rolling bearings, and sliding bearings. As shaft speed increases, losses in gear transmissions initially rise, but beyond a certain point they begin to

decrease due to the effect of fluid friction. This is even more pronounced in sliding bearings, where frictional power losses decrease significantly with increasing speed. In rolling bearings, on the contrary, frictional power losses increase with rising rotational speed [16].

Even the most advanced publicly available transmission loss calculation programs do not fully account for, or only partially consider, the dependence of power losses on torque and rotational speed, which reduces the reliability of the computations. In engineering practice, the efficiency of typical vehicle transmissions under full load is usually assumed as follows: manual gearbox 92–97%; automatic gearbox 90–95%; mechanical CVT 87–93%; hydrostatic transmission 80–86% [17]. Recent advances have introduced new tools for preliminary assessment of bearing losses. The SKF online calculator and Autodesk Inventor software allow modelling of no-load power losses.

An analysis of the design of modern multi-stage and robotic gearboxes with five or six speeds indicates the presence of significant cumulative power losses. These are mainly caused by the large number of gear meshings, the operation of synchronizers, and the complexity of control system elements [15]. To model power losses in rolling bearings and reduce the influence of external factors, the most appropriate object of analysis is the three-speed gearbox. Although three-speed transmissions have historically been associated with outdated vehicles, today this type of architecture is experiencing renewed development due to its compromise between compactness, weight, and energy efficiency. In particular, three-speed inline gearboxes are actively used in modern dedicated hybrid transmissions of the 3-speed DHT (Dedicated Hybrid Transmission) type [18]. An example of such integration is the high-tech architecture of the Chery group with the P2+P2.5 configuration, where the power flows of the internal combustion engine and two electric motors are distributed through coaxial input shafts, three clutches, and synchronizers, ensuring system operation over a wide range of modes [19, 20].

Beyond the automotive segment, three-speed gearboxes with the “three forward gears and one reverse” scheme remain standard for modern small-sized utility vehicles, such as ATVs Tao Mikilon 125, Dozer ATV 125, and Orix 150 [21].

For calculating losses in the bearing units of three-speed gearboxes, it is necessary to operate with baseline input data typical of modern hybrid vehicles and small-sized utility equipment. In three-speed dedicated hybrid transmissions (DHT of the Chery group), the typical kinematic range for three gears is: first gear from 2.5 to 3.5; second gear from 1.4 to 1.8; third gear from 0.8 to 1.0 (direct or overdrive for highway use) [20].

In both hybrid parallel mode (HEV) and pure electric mode (EV), when traction motors are operating, the angular velocity of the primary shaft (or coaxial input shafts) is maintained within 200 to 420 rad/s. In steady EV mode, the load (torque) on the input part of the transmission may range from 120 to 180 Nm. In contrast,

when switching to hybrid parallel mode (HEV), the torque on the primary shaft is within 150 to 230 Nm [22].

The SKF company has introduced an updated online service designed for calculating power losses in rolling bearings (Fig. 1). This tool allows detailed assessment of losses depending on the specific operating mode of the bearing and includes an additional function for evaluating carbon emissions, making it particularly relevant in the context of modern environmental requirements for transport systems.

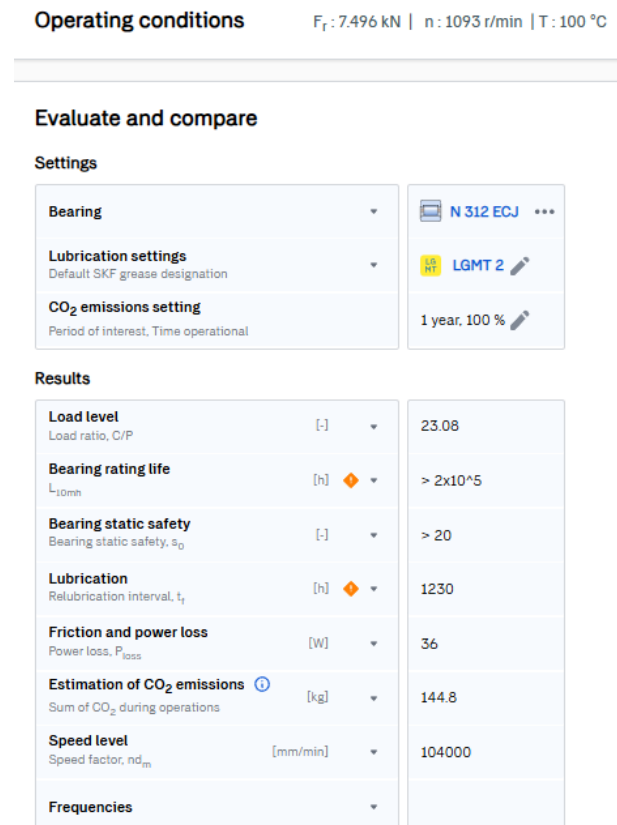


Fig. 1. Example of a technical report from SKF online-service.

3 Research method

The first stage of the study was aimed at determining whether the SKF digital model depends on the way power is formed. For this purpose, two fundamentally different scenarios were created, both providing the same transmitted power but with different combinations of torque and angular velocity.

In the first scenario, angular velocity remained constant, while power increased due to rising torque ($\omega = \text{const.}, T \uparrow$). In the second scenario, torque remained constant, while the increase in power was achieved by raising the angular velocity ($T = \text{const.}, \omega \uparrow$). The pairs of torque T and angular velocity ω values were selected so that the condition was satisfied.

$$P = T_1 \cdot \omega_1 = T_2 \cdot \omega_2, \quad (1)$$

Thus, the power in the corresponding points of both scenarios was identical. This made it possible to carry out a correct comparison, excluding the influence of the actual magnitude of power and allowing investigation of whether the loss model is invariant with respect to the way power is formed.

The calculation of bearing losses was performed using the SKF digital service, which implements a multi-component model of friction torque. In this model, the total loss torque is defined as the sum of components related to rolling of the elements, micro-sliding in contacts, hydrodynamic resistance of the lubricant, and seal losses. Power losses are then determined by multiplying the loss torque by the angular velocity. The bearing friction torque is defined as

$$T_f = T_{rr} + T_{sl} + T_{seal} + T_{drag}, \quad (2)$$

where individual friction torque contributions are accounted for separately:

T_{rr} — rolling,

T_{sl} — micro-sliding,

T_{seal} — seal friction,

T_{drag} — hydrodynamic resistance.

Power losses are then obtained as

$$P_{SKF} = T_f \cdot \omega. \quad (3)$$

In all calculations, the bearing type, lubrication scheme, temperature, and lubricant parameters were kept constant to eliminate the influence of external factors. The online service provided by SKF makes it possible to automate the calculation. The figure presents a structured technical report generated using the updated version of the SKF online service (Fig. 1). It consists of three blocks: input parameters, settings, and calculation results.

The first block contains the operating conditions specified for the analysis: radial load of 7.496 kN, speed of 1093 rpm, and temperature of 100 °C. The second block includes the settings that affect the calculation: the selected bearing type — N 312 ECJ, lubricant — LGMT 2, and the CO₂ emission assessment period — one year under 100% utilization.

The third block presents the calculation results. The load level (C/P) equals 23.08. The nominal bearing life (L_{10} in mh) is 200,000 hours, and the static safety factor (s_0) is greater than 2. The relubrication interval is 1,230 hours, frictional losses amount to 36 W, and the total CO₂ emissions for one year of operation are 144.8 kg. The speed factor equals 104,000 mm/min. Overall, the structure of the report allows for a quick assessment of both operating conditions and the technical efficiency of the selected solution. A new feature is the inclusion of an environmental evaluation.

The second stage of the study was aimed at assessing how refined data on power losses in rolling bearings affect the calculation results of the efficiency of real transmissions. For this purpose, the simplest case — a two-shaft automotive gearbox — was selected.

Different variants of the power flow were considered sequentially, corresponding to various gear ratios and different combinations of gear engagements. For each variant, the composition of loaded and unloaded bearings, the number of active gear engagements, and the distribution of angular velocities across the shafts were determined. The total transmission power losses were regarded as the sum of flow losses in loaded contacts, idle rotation losses, hydrodynamic lubricant losses, and seal losses.

The overall power losses in the gearbox are presented as the sum of these components.

$$P_{\Sigma} = P_a + P_b + P_c + P_d, \quad (4)$$

where:

P_a — the flow losses (gear meshing + loaded bearings + clutches),

P_b — the losses in bearings and wheels during idle rotation,

P_c — the hydrodynamic losses due to oil splashing and mixing,

P_d — the seal losses.

Input power

$$P_{in} = T \cdot \omega. \quad (5)$$

The efficiency coefficient was determined by the ratio of total losses to the input power, which enabled the transition to a dimensionless form of indicators

$$\eta = \frac{P_{in} - P_{\Sigma}}{P_{in}}, \quad (6)$$

or

$$\eta(T, \omega) = 1 - \left(a + \frac{b}{T} + \frac{c \cdot \omega^2}{T} + \frac{d}{\omega} \right), \quad (7)$$

where the coefficients account for different types of losses

a — the flow losses,

b — the idle bearings and gears,

c — the hydrodynamic oil losses (splashing and mixing)

d — the seals [23, 24].

The coefficient a is a dimensionless fraction representing the flow losses of loaded contacts; coefficients b and d have the dimension of torque (N·m) and describe losses that are weakly dependent on load (idle bearings, seals); coefficient c has the dimension of N·m·s² and characterizes speed-dependent (hydrodynamic) losses proportional to the square of the angular velocity. Division by torque T converts all components into a relative form.

Losses in idle bearings were taken into account through an equivalent resisting torque

$$P_b = T_{idle} \cdot \omega \quad (8)$$

or represented in a relative form

$$P_b = b \cdot P_{in}, \quad (9)$$

where coefficient b scales with:

- the number of idle bearings,
- the shaft diameters,
- the temperature,
- the oil viscosity.

For hydrodynamic oil losses, a generalized power-law model was used

$$P_c(\omega) = P_0 + c, \omega^n \quad (10)$$

or

$$2 \leq n \leq 3. \quad (11)$$

The exponent was varied parametrically. In the research methodology, the sensitivity of the results to the choice of the exponent (n) was specifically examined, since the cubic model overestimates the reduction of losses at low speeds.

Seal losses were determined according to established dependencies

$$P_d = T_{seal} \cdot \omega, \quad (12)$$

$$T_{seal} = \text{const} \cdot D_{shaft}. \quad (13)$$

Representative coefficient values applied in automotive gearboxes are summarized in Table 1, which provides the reference ranges

Table 1. Reference range of coefficients.

Coef-ficients	Physical meaning	Reference range
a	Flow losses	0.07–0.08
b	Idle bearings	0.2–0.6 N·m
c	Oil splashing	(1–5)·10 ⁻⁴ N·m·s ²
d	Seals	0.1–0.3 N·m

Flow losses were determined through the product of the partial efficiencies of the power flow elements

$$\eta_a = \eta_{gear}^{n_g} \cdot \eta_{bear,load}^{n_{bl}} \cdot \eta_{clutch}^{n_c}, \quad (14)$$

where:

- n_g — the number of gear meshings in the power flow,
- n_{bl} — the number of loaded bearing supports,
- n_c — the number of clutches.

Corresponding power losses are

$$P_a = P_{in}(1 - \eta_a). \quad (15)$$

Relative power losses are

$$a = 1 - \eta_a. \quad (16)$$

To evaluate the difference between the traditional engineering approach and the use of the SKF online

service, a methodology was applied that involves dividing the flow losses into two main components. The first component covers power losses in gear transmissions and clutches, denoted as $P_{gear,clutch}$. The second component accounts for losses in bearings, denoted as P_{bear} . The total flow losses are determined by the following relation

$$P_a = P_{gear,clutch} + P_{bear}. \quad (17)$$

Thus, the power losses in bearings were distinguished as a separate part of the overall flow losses. In addition, losses in bearings operating in idle mode were considered. In the traditional engineering calculation, these are accounted for through a special component that includes coefficient b , provided in reference materials. When applying the SKF methodology, this component was replaced by a value obtained directly from the online service. In the engineering calculation, bearing losses were determined on the basis of the efficiency coefficient specified in handbooks, while for idle bearings coefficient b was used. In contrast, when using the SKF online service, all bearing losses—both loaded and idle—were determined from the results of the digital calculation.

This approach made it possible to assess the sensitivity of the gearbox loss model to the method of determining bearing losses. For this purpose, two indicators were introduced: engineering losses $P_{loss,eng}$ and losses determined by the SKF methodology $P_{loss,SKF}$. The criterion for comparing the methodologies was chosen as the relative difference of the results

$$\delta = \frac{P_{loss,eng} - P_{loss,SKF}}{P_{loss,eng}} \cdot 100\%. \quad (18)$$

This indicator provides a quantitative measure of the differences between the two approaches and clarifies how the application of the SKF online service improves the accuracy of total power loss calculations in automotive transmission systems.

4 Power loss in bearings

At the initial stage, the task was set to determine the power losses in bearings separately from other components of the transmission. For the correct application of the SKF methodology, it is necessary to consider a shaft with at least two supporting nodes. For this purpose, the simplest case was chosen — a shaft with a spur gear positioned between two supports.

As supporting elements, identical single-row cylindrical roller bearings SKF NUP310 ECJ/C3 were used, with the following main parameters:

- inner (bore) diameter — 50 mm,
- outer diameter — 110 mm,
- width — 27 mm,
- dynamic load rating — 127 kN,
- static load rating — 112 kN,
- fatigue load limit — 15 kN,
- reference speed — 6700 rpm,

- limiting speed — 8000 rpm.

The choice of this particular bearing size was not accidental. Preliminary calculations for different types and sizes of bearings showed that for the NUP310 ECJ/C3 model the difference between the two loading scenarios is most clearly manifested.

Power losses in the bearing were determined for two scenarios:

- First scenario: power increases at a constant torque due to the rise in angular velocity. This case is represented by the blue curve in Figure 2.
- Second scenario: angular velocity remains constant, while power increases as a result of higher torque. This case is represented by the green curve.

For comparison with the traditional engineering calculation, which is usually applied in transmission design, the figure additionally shows a straight yellow line. It corresponds to the bearing efficiency coefficient, taken as 0.996 (Fig. 2).

Thus, it became possible to compare the results obtained using the SKF online methodology with the classical engineering approach based on reference efficiency values. This allows the differences between the two approaches to be evaluated and their impact on the accuracy of predicting power losses in automotive transmission units to be determined.

The obtained results showed that, at the same power level, the bearing losses determined using the SKF methodology differ significantly depending on the loading operating mode. This indicates that the model is not invariant with respect to the way power is formed. This is precisely where the fundamental distinction of the SKF methodology from traditional engineering approaches and other previously applied calculation methods becomes evident.

The reason lies in the different nature of the components of the friction torque: some depend mainly on the load, while others depend on the speed. Thus, the

SKF digital model reveals sensitivity to the structure of the operating regime, which is an important refinement compared to classical methods that often rely on simplified linear dependencies.

In particular, in the case of constant torque, the power losses in the NUP310 ECJ/C3 bearing increase with angular velocity much more steeply than predicted by the linear model traditionally used in engineering practice. Only at high speeds do the losses calculated by the SKF methodology approach the values typically considered in reference engineering calculations. This demonstrates both the high quality of the bearing design and SKF's achievement in significantly reducing power losses in new generations of bearings.

In the other scenario, when angular velocity remains constant and power increases due to higher torque, a different picture emerges: power losses increase with speed less steeply than described by the simplified linear dependence. This phenomenon is explained by the specifics of the SKF methodology, according to which losses consist of several components: contact losses (depend on load), hydrodynamic oil losses (rise sharply with speed).

In the scenario with constant torque and increasing speed, oil-mixing losses dominate, exhibiting a nonlinear character. Conversely, in the scenario with constant speed and increasing torque, contact losses play the leading role, showing a more nearly linear behavior.

In both cases, the power losses turned out to be significantly lower than expected and lower than those traditionally considered in reference engineering calculations. This is especially noticeable at high torques and low speeds. What is important is how such a reduction in losses affects the overall efficiency of the automobile gearbox under different loading regimes, since this directly determines the energy efficiency of modern transport systems.

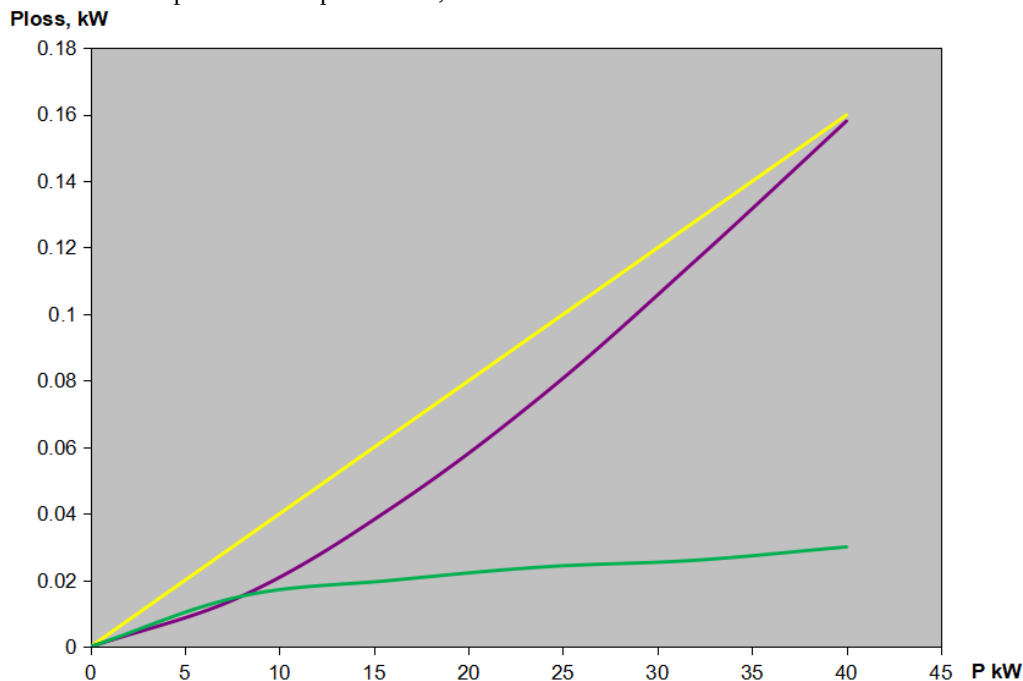


Fig. 2. Dependence of power losses on the load scenario.

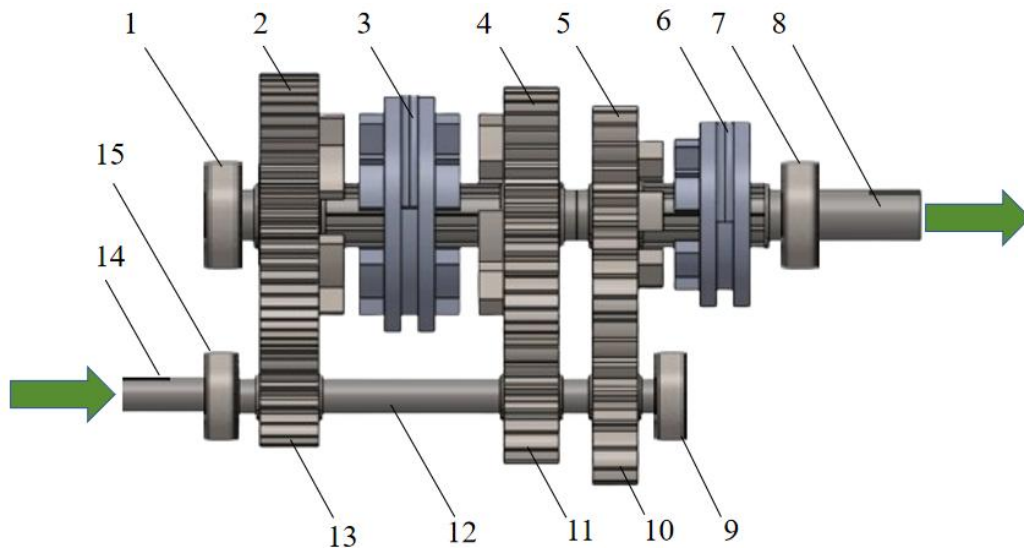


Fig. 3. Two-shaft automotive gearbox.

Table 2. Power loss comparison (two-shaft gearbox, gear ratio at $z_5 = z_{10}$).

Engineering SKF	Operating mode	P (kW)	ω (rad/s)	M (N·m)	P_{loss} (W)	η	%
Engineering	I	8	40	200	282	0.965	7.4
SKF		8	40	200	261	0.967	
Engineering	II	40	200	200	1660	0.9585	4.1
SKF		40	200	200	1592	0.960	
Engineering	III	40	40	1000	954	0.976	9.6
SKF		40	40	1000	862	0.9785	

Table 3. Power loss comparison (two-shaft gearbox, gear ratio at $z_2 = 2.5z_{13}$)

Engineering SKF	Operating mode	P (kW)	ω (rad/s)	M (N·m)	P_{loss} (W)	η	%
Engineering	I	8	40	200	260	0.9675	9.6
SKF		8	40	200	235	0.971	
Engineering	II	40	200	200	1110	0.972	8.9
SKF		40	200	200	1011	0.975	
Engineering	III	40	40	1000	932	0.977	12.3
SKF		40	40	1000	817	0.980	

5 Power loss in gearbox

Let us consider a vehicle gearbox with three forward gears (Fig.3). On the driving shaft (shaft 1), three gears (3, 4, and 5) are mounted. They are in permanent mesh with three gears (6, 8, and 9) on shaft 2. The direction of torque transmission is changed by the axial displacement of synchronizers 7 and 10.

The efficiency of such a gearbox was calculated under different operating modes. The efficiency of the gear transmissions, bearing supports, and synchronizers was taken into account. To ensure comparability, a set of characteristic operating modes was defined. The basic mode corresponded to moderate load — 8 kW at an

angular velocity of 40 rad/s and torque of 200 N·m. The high- power modes corresponded to a level of 40 kW, but were realized in two different ways: either at relatively high speed and moderate torque, or at high torque and lower speed. Thus, the same power was maintained, but the load distribution on the bearings and gear meshings varied.

For each mode, parallel calculations were performed — using the standard engineering method with semi- empirical loss coefficients and using the SKF digital model for each bearing support. Power losses occurred in the bearings of shaft 1 (positions 9 and 15) and shaft 2 (positions 1 and 7), as well as in the bearings supporting the gears. Two bearings on each shaft

operated without load, i.e., in idle mode. One bearing on each shaft was loaded by the force in the gear mesh transmitting torque in the selected operating mode. Losses also occurred in one synchronizer and in the mesh of one gear pair. Power losses in the other two gear pairs, which operated in idle mode, were not considered.

The calculation of gearbox efficiency under different operating modes revealed significant differences between the traditional engineering methodology and the SKF digital model. To ensure comparability, a set of characteristic operating modes was defined: a basic mode with moderate load (8 kW at $\omega = 40$ rad/s, $T = 200$ N·m), as well as two high- power modes (40 kW), realized through different combinations of speed and torque.

In the first scenario (Table 2), with the gear ratio provided by gears 5 and 10, the smallest difference between the engineering calculation and the SKF results was observed. At the combination ($T = 200$ N·m & $\omega = 200$ rad/s), both methods yielded nearly identical results. This indicates that at high angular velocities, the engineering methodology, based on semi- empirical loss coefficients, can be sufficiently accurate. At the same time, even a 4% difference is significant under modern conditions of increased requirements for vehicle energy efficiency and cannot be ignored.

In the second scenario (Table 3), when the gear ratio was 2.5 (meshing of gears 13 and 2), power losses increased as expected. The greatest losses were observed at maximum torque. Here, the key distinction between the methodologies emerged: the efficiency determined by the SKF digital model was considerably higher than that obtained by the engineering approach. The difference reached 12.3%, which may be critical for design calculations. This result indicates that the traditional methodology can substantially overestimate bearing power losses, especially under high loads and low speeds.

6 Conclusions

The study showed that the application of the SKF digital model of rolling bearing losses provides more accurate results compared to traditional engineering methods. The model revealed sensitivity to the way power is formed, which is a fundamental difference from simplified approaches that do not account for the dependence of losses on load structure.

It was established that at equal power levels, losses in the bearing calculated using the SKF methodology differ significantly between the two scenarios. The loss curves as functions of torque and speed had different shapes, indicating non- invariance of the model to the way power is formed. This is explained by the fact that some components of friction torque depend mainly on load, while others depend on speed. Thus, the SKF digital model proved to be sensitive to the operating regime structure, which is an important refinement compared to traditional methods.

It has been established that at constant torque, power losses in bearings increase with angular velocity much

more steeply than predicted by the traditional linear model used in engineering practice. But, only at high speeds do the losses calculated using the SKF methodology approach the values typically considered in reference engineering calculations. In the other scenario, when angular velocity remains constant and power increases due to rising torque, the losses grow with speed less steeply than described by the simplified linear dependence.

Comparison of results was carried out using the relative difference of calculated efficiencies. This indicator allowed quantitative assessment of the error arising when simplified engineering dependencies are used instead of the detailed digital model. It was found that the impact of the refined bearing loss model on overall gearbox efficiency depends not only on the power level but also on the way it is formed, the type of kinematic scheme, and the set of active bearing supports in the power flow.

The smallest error of 4 % between engineering calculation and SKF methodology was obtained for a gearbox with a unit transmission ratio and high speed, indicating close agreement of results. For a transmission ratio of 2.5 and high torque at low speed, the difference between the methods reached a maximum of 12.3 %, which is a critical value for transmission design calculations.

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